Hybrid genetic optimization algorithm for target localization using TDOA measurements

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Abstract— This paper focuses on the determining the position of a target by measuring the time difference of arrival (TDOA) from a set of receivers whose positions are known. In the first step, weighted least squares (WLS) based optimization function is proposed to locate the target. In the second step, the TDOA target localization problem based on least squares (LS) is formulated as an optimization problem, with a highly nonlinear and multimodal objective function. There are several algorithms to estimate the position of a target. In this paper, we proposed a hybrid Genetic algorithm (GA) that combines the global search and local search abilities in an effective way in order to improve the performance and the solution accuracy. Gradient based local search algorithms such as the Newton-Raphson and Gauss-Newton are the most common methods used for hybridization. The paper investigates the performance of the hybrid genetic algorithm in modelling a TDOA positioning problem using a different local search optimization methods. The simulation results illustrate the performance comparison of these different optimization algorithms.

Index Terms— Localization, Optimization, Least Squares, Time Difference of Arrival, Wireless Sensor Networks.

I. INTRODUCTION

The problem of finding the unknown location of a target based on TDOA measurements has been one of the essential problems in many applications such as telecommunications, mobile communications, environmental monitoring, wireless sensor networks (WSNs), and many others [1-2].

Range measurements can be obtained from different techniques such as the time of arrival (TOA), the time difference of arrival, the received signal strength (RSS), or the angle of arrival (AOA), depending on the available hardware to localize the target using geometric approaches. The focus of the present paper is a target localization problem based on the TDOA measurements due to its simplicity and efficiency.

The target location can be estimated based on the least squares and the maximum likelihood (ML) as a powerful methods which can be employed successfully in a practical applications. Hence, the localization problem can be formulated as an optimization problem known as least squares

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that minimizes the sum of squared residuals between the estimated and the measured distances. The LS estimator can be divided into two classes: nonlinear least squares (NLS) and linear least squares (LLS) to estimate the target position. In this work, the NLS estimator is utilized to solve the problem of the target localization based on the TDOA measurements. However, the objective function of the NLS estimator of the presented localization problem is a highly nonlinear and multimodal. It is difficult to obtain its global optimal solution with a classical approach in which the convergence of these algorithms mainly depends on the selection of initial point and may not always converge to the global optimal solution. Therefore, in order to overcome this difficulty in the process of the optimization the Genetic Algorithm is hybridized with a gradient based local search methods. The GA has powerful global searching ability, which provides accurate and robust numerical results. The proposed hybrid GA algorithm acts in two phases in the process of the optimization. During the first phase, the GA explores the search space with the aim to find an initial global optimal solution, then, during the second phase, an initial solution from the first phase is improved using the gradient based local search methods.

The target location is usually obtained by the linear estimator as algebraic closed-form solution, however in the case of the hybrid GA we avoid the selection of initial solution [3]. The weighted least squares algorithm for estimation of the target is presented in this paper due to its computational efficiency and the linear closed-form solution in the WLS sense [4]. The Cramer-Rao Lower Bound (CRLB) of the TDOA measurements from all receivers provides a lower bound on the variance of any unbiased estimator [5] and it is a very useful tool for evaluating the localization accuracy.

The paper is organized as follows. The target localization problem based on the TDOA measurements from a set of receivers whose positions are known is reviewed in Section 2. Section 3 describes target localization problem which is modeled as a least squares estimation problem with NLS and WLS approaches. In Section 4, the hybrid GA algorithm is presented. The CRLB is given in Section 5. Section 6 gives the simulation results of the proposed hybrid GA against the WLS and the GA approaches. Finally, conclusion and future work are presented in Section 7.

II. PROBLEM FORMULATION

In this section, we present the two-dimensional (2-D) target localization problem using TDOA measurements under the line-of-sight (LOS) environment. In order to determine the

position of the unknown target, it is assumed that at least three receivers, $N \geq 3$ have to be placed on the known positions $\mathbf{x}_i = [x_i, y_i]^T, i \in \{1, 2, ..., N\}$, where $[\cdot]^T$ denotes matrix transpose, as shown in Fig. 1.
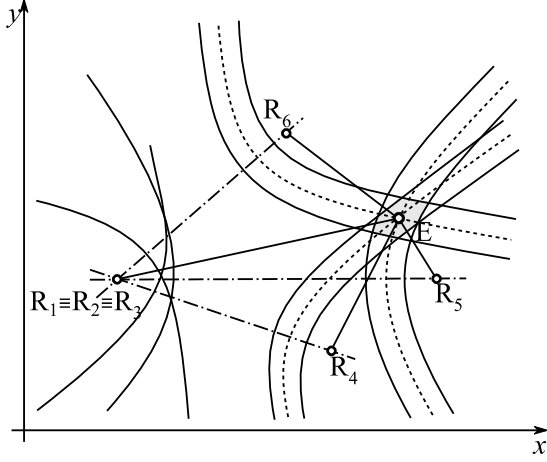

Fig. 1. Geometrical model based on TDOA.

This paper assumes that range difference errors $\{n_i\}$ are independent Gaussian random variables with zero mean and known variance σ_i^2, i.e., $\mathcal{N}(0, \sigma_i^2)$. Without loss of generality, we set the first receiver R_1 to be the reference receiver.

The unknown position of a target is obtained using geometrical relations between a target and the receivers R_i $i \in \{2, 3, ..., N\}$. Multiplying the obtained times by the velocity of the light yields unknown distances denoted by, $\{r_{i,1}\}$ which can be evaluated by

$$r_{i,1} = d_{i,1} + n_{i,1}, \quad i \in \{2, ..., N\}, \tag{1}$$

Where $d_{i,1} = d_i - d_1$. Here, distances between the target and the receiver pair R_1 and R_i can be expressed as follows

$$d_1 = \sqrt{(x - x_1)^2 + (y - y_1)^2}, \\ d_i = \sqrt{(x - x_i)^2 + (y - y_i)^2}. \tag{2}$$

where $\mathbf{x} = [x, y]^T \in \mathbb{R}^2$ is the unknown position of a target.

Hence, the hyperbola is characterized by the fact that the difference, $d_i - d_1$, between any point on it and the two foci at R_i and R_1, respectively, is constant as shown in Fig. 1.

In the absence of noise, the geometric model for determining the unknown true coordinates of the target E using TDOA measurements is given by the intersection of two hyperbolas in 2-D, as depicted in Fig. 1.

In practical environments where noise exist, more than two hyperbolas do not intersect at the same point and corresponding optimization method is needed to minimize the localization error.

III. LEAST SQUARE METHODS

In this section, we describe LS methods for a target localization based on the TDOA measurements. In the first approach, NLS minimizes the objective function $J_{NLS}(\tilde{\mathbf{x}})$ which is defined as the sum of squared residuals between the estimated and the measured TDOA values, i.e.

$$J_{NLS}(\tilde{\mathbf{x}}) = \min \sum_{i=1}^{N} R_{es,i}^2(\tilde{\mathbf{x}}), \tag{3}$$

where $\tilde{\mathbf{x}}$ is optimization variable and residual $R_{es,i}(\tilde{\mathbf{x}})$ is given by

$$R_{es,i}(\tilde{\mathbf{x}}) = \tilde{r}_{i,1} - r_{i,1}, \tag{4}$$

where $\tilde{r}_{i,1}$ is a measured distance. Thus, the optimal solution $\hat{\mathbf{x}}$ can be obtained from (3), which can be written as

$$\hat{\mathbf{x}} = \arg \min_{\mathbf{x} \in \mathbb{R}^2} J_{NLS}(\tilde{\mathbf{x}}). \tag{5}$$

The nonlinear hyperbolic equations can be transformed into a set of linear equations through the following process. Substituting (2) into (1) results in

$$r_{i,1} + \sqrt{(x - x_1)^2 + (y - y_1)^2} = \sqrt{(x - x_i)^2 + (y - y_i)^2} + n_{i,1}, \\ i \in \{2, 3, ..., N\}. \tag{6}$$

Squaring both sides of (6) and introducing an additional variable

$$R_f = d_1 = \sqrt{(x - x_1)^2 + (y - y_1)^2}, \tag{7}$$

and after some algebraic manipulation, we can show that

$$(x_i - x_1)(x - x_1) + (y_i - y_1)(y - y_1) + r_{i,1}R_f \\ = 0.5\left[(x_i - x_1)^2 + (y_i - y_1)^2 - r_{i,1}^2\right] + d_i n_{i,1} + 0.5 n_{i,1}^2, \tag{8} \\ i \in \{2, 3, ..., N\}.$$

In the second approach, the system (8) can be linearized by

$$(x_i - x_1)(x - x_1) + (y_i - y_1)(y - y_1) + r_{i,1}R_f \\ = 0.5\left[(x_i - x_1)^2 + (y_i - y_1)^2 - r_{i,1}^2\right] + m_{i,1}, \quad i \in \{2, 3, ..., N\}, \tag{9}$$

where the second-order term $n_{i,1}^2$ is neglected and $m_{i,1} = d_i n_{i,1}$.

Hence, the system (9) is linear and can be written in the following matrix form

$$A\theta = b + m, \quad (10)$$

in which

$$A = \begin{bmatrix} x_2 - x_1 & y_2 - y_1 & r_{2,1} \\ x_3 - x_1 & y_3 - y_1 & r_{3,1} \\ \vdots & \vdots & \vdots \\ x_N - x_1 & y_N - y_1 & r_{N,1} \end{bmatrix}, \quad (11)$$

$$\theta = \begin{bmatrix} x - x_1 & y - y_1 & R_f \end{bmatrix}^T, \quad (12)$$

$$b = 0.5 \begin{bmatrix} (x_2 - x_1)^2 + (y_2 - y_1)^2 - r_{2,1}^2 \\ (x_3 - x_1)^2 + (y_3 - y_1)^2 - r_{3,1}^2 \\ \vdots \\ (x_N - x_1)^2 + (y_N - y_1)^2 - r_{N,1}^2 \end{bmatrix}, \quad (13)$$

$$m = \begin{bmatrix} m_{2,1} & m_{3,1} & \cdots & m_{N,1} \end{bmatrix}^T. \quad (14)$$

From (10)-(14) respectively, we define the WLS objective function, which can be written as

$$J_{WLS}(\theta) = (A\theta - b)^T W (A\theta - b). \quad (15)$$

where is $W = (E\{mm^T\})^{-1}$ the weighting matrix. Thus, the unconstrained optimization problem can be formulated as follows

$$\min_{x \in R^2} J_{WLS}(\theta). \quad (16)$$

The WLS method provides the algebraic closed-form solution $\hat{x}_{WLS}$ for target localization using TDOA measurements which minimizes the objective function $J_{WLS}(\theta)$. It can be shown that $\hat{x}_{WLS}$ can be obtained from (16) by the following equation [4]

$$\hat{x}_{WLS} = (A^T W A)^{-1} A^T W b. \quad (17)$$

The WLS method is usually chosen in practice due to its easy implementation and higher computational efficiency.

IV. HYBRID GA ALGORITHM

In this section, the implementation of a hybrid GA incorporating effective local search techniques in GA is provided in order to improve the convergence properties and accuracy of the algorithm, and to reduce the computation time

[3]. The proposed hybrid method is based on the global search GA and gradient based local search methods such as Newton-Raphson and Gauss-Newton. The framework of the proposed hybrid algorithm is depicted in Fig. 2, with the detailed description of its components given in the following subsections. The overall procedure of the proposed hybrid approach is described as follows

Step 1 Initialize the parameters of the hybrid algorithm.
Step 2 Randomly generate the GA population over the solution space.
Step 3 Evaluate the fitness of the individuals in the population.
Step 4 Check if the termination criterion is satisfied, then go to the *Step 8*, otherwise proceed with algorithm and go the *Step 5*.
Step 5 Start the global search GA procedure.
Step 5.1 Using genetic operators: selection, crossover and mutation, the population is evolved towards the global optimal solution.
Step 6 Apply the gradient based local search methods to the solutions found by GA in order to improve the quality of the solution.
Step 7 Set the $k = k+1$ and go to *Step 3*
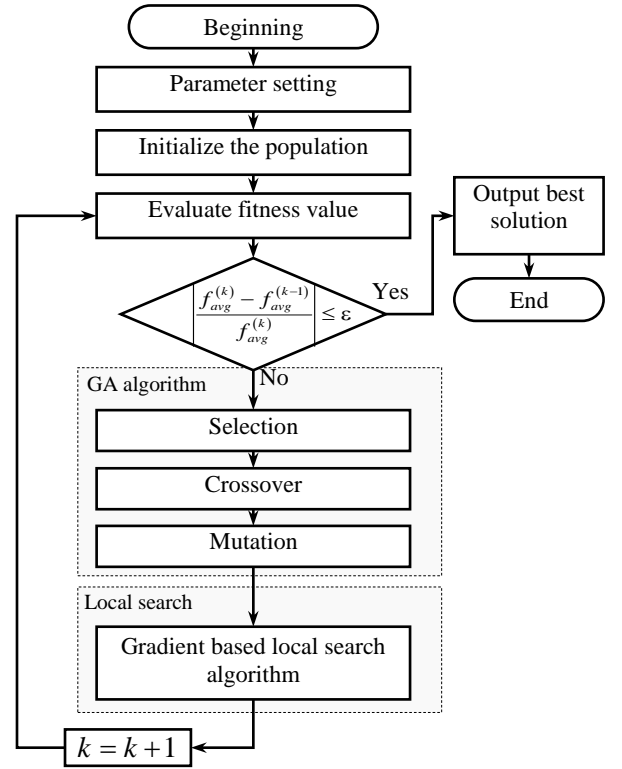**Step 8** Output the best solution and end the procedure.

Fig. 2. Flowchart of hybrid GA algorithm.

In the following subsections the GA and the gradient based local search algorithms are presented, respectively.

A. Genetic Algorithm

The genetic algorithm is one of the most powerful metaheuristic method based on the evolutionary ideas of

natural selection and natural genetics by David Goldberg [6]. Introducing the bound-constraints, the optimization problem (3) is modified into the following form

$$\min_{\mathbf{x}^l \leq \mathbf{x} \leq \mathbf{x}^h} J_{NLS}(\mathbf{x}), \qquad (18)$$

in which $\mathbf{x}$ is a vector of decision variables, $\mathbf{x}^l$ and $\mathbf{x}^h$ are the lower and upper bounds of $\mathbf{x}$, respectively.

The GA starts with a randomly generated population of N_p individuals, each of which is represented by a chromosome. GA uses three basic operators to manipulate the genetic composition of a population: selection, crossover, and mutation.

In GA, roulette wheel is the simplest selection approach. The selection operator is used to select the individuals according to the objective function (3), in which individuals with the smaller objective function in the current generation are reproduced in the next generation. The selection probability P_i of the ith chromosome is computed as follows

$$P_i = \frac{F_i}{\sum_{j=1}^{N_p} F_j}, \quad i \in \{1,...,N_p\}, \qquad (19)$$

where F_i is the corresponding fitness value. The cumulative probability C_i of the ith chromosome is obtained as follows

$$C_i = \sum_{j=1}^{N_p} F_j. \qquad (20)$$

Based on a randomly generated nonzero floating-point number $r \in [0 \quad 1]$ for each individual, the chromosome is selected if $C_{i-1} < r \leq C_i, i \in \{1,2,...,N_p\}$ and $C_0 = 0$.

In order to create new chromosomes for the next generation a crossover operator is used. Two random crossover points, c_1 and c_2, are chosen from the chromosomes. Then, the chromosome parts between crossover points are exchanged and two new chromosomes are created as shown in Fig. 3.
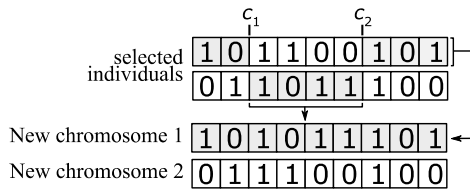

Fig. 3. Two point crossover.

Finally, the mutation process selects a random variable of a random individual and negates the bit value in order to introduce new unexplored solutions into the GA population and avoid a premature convergence to a local minima.

The implementation of the genetic operators is repeated until the average population fitness converges to a stable value, which can be expressed as

$$\left| \frac{f_{avg}^{(k)} - f_{avg}^{(k-1)}}{f_{avg}^{(k)}} \right| \leq \varepsilon, \qquad (21)$$

where ε is a small positive real number, in which

$$f_{avg}^{(k)} = \frac{1}{N_p} \sum_{i=1}^{N_p} f_i, \qquad (22)$$

represents the average fitness value of the entire population in kth iteration.

B. Newton-Raphson optimization method

The Newton-Raphson method is one of the best-known and most powerful methods for the solution of nonlinear problems. This method requires the calculation of the gradient of the objective function and the Hessian matrix, in each iteration, which are expressed as follows

$$\nabla J_{NLS}(\mathbf{x}) = \begin{bmatrix} \dfrac{\partial J_{NLS}(\mathbf{x})}{\partial x} \\ \dfrac{\partial J_{NLS}(\mathbf{x})}{\partial y} \end{bmatrix} \in R^2, \qquad (23)$$

$$\nabla^2 J_{NLS}(\mathbf{x}) = \begin{bmatrix} \dfrac{\partial^2 J_{NLS}(\mathbf{x})}{\partial x^2} & \dfrac{\partial^2 J_{NLS}(\mathbf{x})}{\partial x \partial y} \\ \dfrac{\partial^2 J_{NLS}(\mathbf{x})}{\partial y \partial x} & \dfrac{\partial^2 J_{NLS}(\mathbf{x})}{\partial y^2} \end{bmatrix} \in R^{2 \times 2}. \qquad (24)$$

Using the Taylor series expansion, the second-order approximation of the objective function $J_{NLS}(\mathbf{x} + \Delta \mathbf{x})$ is expressed as

$$J_{NLS}(\mathbf{x} + \Delta \mathbf{x}) \approx \varphi(\mathbf{x}) = J_{NLS}(\mathbf{x}) + \Delta \mathbf{x}^T \left[\nabla J_{NLS}(\mathbf{x}) \right] + \frac{1}{2} \Delta \mathbf{x}^T \nabla^2 J_{NLS}(\mathbf{x}) \Delta \mathbf{x}. \qquad (25)$$

Then, the step size is obtained from the optimality condition of (25) as follows

$$\nabla \varphi(\mathbf{x}) = \mathbf{0} \Rightarrow \\ \Delta \mathbf{x} = -\left[\nabla^2 J_{NLS}(\mathbf{x}) \right]^{-1} \nabla J_{NLS}(\mathbf{x}). \qquad (26)$$

Iterative update rule of the Newton-Raphson method in the (k+1)th iteration is given as

$$\mathbf{x}^{(k+1)} = \mathbf{x}^{(k)} + \Delta \mathbf{x}^{(k)}, \qquad (27)$$

where $(\cdot)^{(k)}$ is $(\cdot)$ in the kth iteration.

A stopping criteria for the NR method is to check if a norm

$$\left\| \nabla J_{NLS}\left(\mathbf{x}^{(k+1)}\right) \right\| \le \varepsilon, \tag{28}$$

of the gradient of the objective function $J_{NLS}(\mathbf{x})$ is less or equal than sufficiently small positive constant ε.

The Newton-Raphson method has quadratic convergence, but may fail to converge to an optimal solution if the Hessian matrix is not positive definite. Therefore, to overcome this problem, the Gauss-Newton method, is applied and represented in the next subsection.

C. Gauss-Newton method

The Gauss-Newton method is an approximation of the Newton-Raphson method [7], in which the calculation of the second derivatives of the objective function $J_{NLS}(\mathbf{x})$, is not required, and is computationally less expensive in each iteration.

The iterative update rule of the GN method is denoted by

$$\mathbf{x}^{(k+1)} = \mathbf{x}^{(k)} - \left[\mathbf{J}\left(J_{NLS}\left(\mathbf{x}^{(k)}\right)\right)^{T} \cdot \mathbf{J}\left(J_{NLS}\left(\mathbf{x}^{(k)}\right)\right) \right]^{-1} \\ \cdot \mathbf{J}\left(J_{NLS}\left(\mathbf{x}^{(k)}\right)\right)^{T} \cdot \mathbf{e}\left(\mathbf{x}^{(k)}\right), \tag{29}$$

where $\mathbf{e}\left(\mathbf{x}^{(k)}\right)$ is a residual vector and $\mathbf{J}\left(J_{NLS}\left(\mathbf{x}^{(k)}\right)\right)$ is the Jacobian matrix evaluated at $\mathbf{x}^{(k)}$, which can be expressed as

$$\mathbf{J}\left(J_{NLS}\left(\mathbf{x}\right)\right) = \begin{bmatrix} \dfrac{\partial \|\mathbf{x}_1 - \mathbf{x}\|_2}{\partial x} & \dfrac{\partial \|\mathbf{x}_1 - \mathbf{x}\|_2}{\partial y} \\ \vdots & \vdots \\ \dfrac{\partial \|\mathbf{x}_N - \mathbf{x}\|_2}{\partial x} & \dfrac{\partial \|\mathbf{x}_1 - \mathbf{x}\|_2}{\partial y} \end{bmatrix}. \tag{30}$$

The iteration procedure is terminated, when the norm of the gradient of the objective function $J_{NLS}(\mathbf{x})$ becomes less or equal than sufficiently small positive constant ε, according to (28).

V. CRAMER-RAO LOWER BOUND

The Cramer-Rao Lower Bound of the TDOA measurements provides a lower bound on the covariance of any unbiased estimator $\hat{\mathbf{x}}$ of $\mathbf{x}$, and can be used as a benchmark for performance comparison. Thus, the relationship between the CRLB and the variance can be expressed as

$$E\left[(\hat{\mathbf{x}} - \mathbf{x})(\hat{\mathbf{x}} - \mathbf{x})^{T} \right] \ge CRLB(\mathbf{x}) = \text{trace}\left\{ \mathbf{I}(\mathbf{x})^{-1} \right\} \tag{31}$$

where $E[\cdot]$ is the expectation operator and $\mathbf{I}(\mathbf{x})$ is the Fisher information matrix (FIM) given by

$$\mathbf{I}(\mathbf{x}) = -E\left[\frac{\partial^2 \ln\left(f\left(\mathbf{r}|\mathbf{x}\right)\right)}{\partial \mathbf{x} \partial \mathbf{x}^T} \right]. \tag{32}$$

The probability density function $f(\mathbf{r}|\mathbf{x})$ can be defined as

$$f(\mathbf{r}|\mathbf{x}) = \frac{1}{(2\pi)^{(N-1)/2} |\mathbf{C}|^{1/2}} \\ \cdot \exp\left(-\frac{1}{2}(\mathbf{r} - \mathbf{d}(\mathbf{x}))\right)^T \mathbf{C}^{-1}\left(-\frac{1}{2}(\mathbf{r} - \mathbf{d}(\mathbf{x}))\right), \tag{33}$$

where $\mathbf{C}$ is covariance matrix is given as

$$\mathbf{C} = \begin{bmatrix} \sigma_1^2 + \sigma_2^2 & \sigma_1^2 & \cdots & \sigma_1^2 \\ \sigma_1^2 & \sigma_1^2 + \sigma_3^2 & & \sigma_1^2 \\ \vdots & & \ddots & \vdots \\ \sigma_1^2 & \sigma_1^2 & \cdots & \sigma_1^2 + \sigma_N^2 \end{bmatrix}. \tag{34}$$

After performing differentiation on the natural logarithm of (33) with respect to $\mathbf{x}$, the FIM can be obtained as

$$\mathbf{I}(\mathbf{x}) = \left[\frac{\partial \mathbf{d}(\mathbf{x})}{\partial \mathbf{x}} \right]^T \mathbf{C}^{-1}\left[\frac{\partial \mathbf{d}(\mathbf{x})}{\partial \mathbf{x}} \right], \tag{35}$$

where $\mathbf{d}(\mathbf{x})$ the true distance vector.

VI. SIMULATION RESULTS

In this section, numerical simulations have been conducted to evaluate the performance of the hybrid GA-NR and hybrid GA-GN methods by comparing with the GA and the closed-form WLS approaches. The simulation environment is represented by a set of five receivers with known coordinates $[340, 440]^T$ m, $[1120, 240]^T$ m, $[670, 1310]^T$ m, $[1380, 1250]^T$ m and $[1520, 700]^T$ m, while the target is located at $[940, 850]^T$ m.

The localization performance is evaluated in terms of root mean square error (RMSE), which is expressed as

$$RMSE = \sqrt{\frac{1}{N}\sum_{n=1}^{N} \|\hat{\mathbf{x}}(n) - \mathbf{x}\|_2^2}, \tag{36}$$

where $N = 500$ is a number of Monte Carlo simulation samples, and $\mathbf{x}$ and $\hat{\mathbf{x}}(n)$ are the true and estimated positions of the target, respectively.

To evaluate the performance of the considered localization algorithms the cumulative distribution functions (CDFs) of the GA, hybrid GA-NR, hybrid GA-GN and WLS localization errors are observed with different levels of signal-to-noise ratio (SNR), SNR = 20 dB and SNR = 40dB, respectively.

The CDFs of localization error using the proposed algorithms are illustrated in Fig.4. for SNR level set to 20 dB.
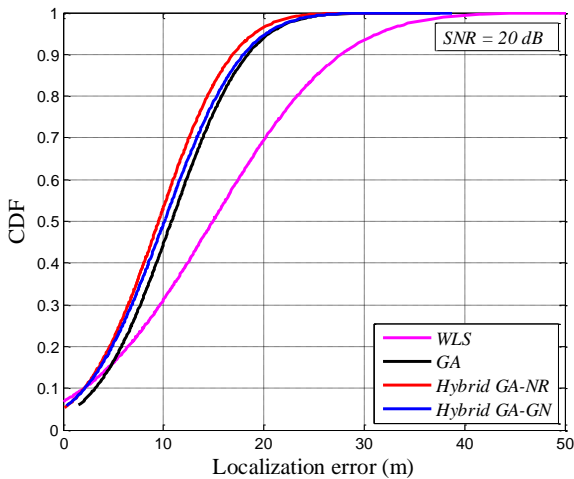

Fig 4. CDFs of the localization error for SNR = 20 dB.

The results presented in Fig.4 show that the hybrid GA-NR estimator has a superior performance in comparison to the other estimators. The proposed hybrid GA method incorporating NR local search procedure performs very close to the one using the GN local search method. This is because the local search estimators are initialized with potentially good solution provided by the GA.

In Fig. 5, the CDFs of target localization error of the GA, hybrid GA-GN, hybrid GA-NR and WLS estimators are compared, when SNR is set to 40 dB.
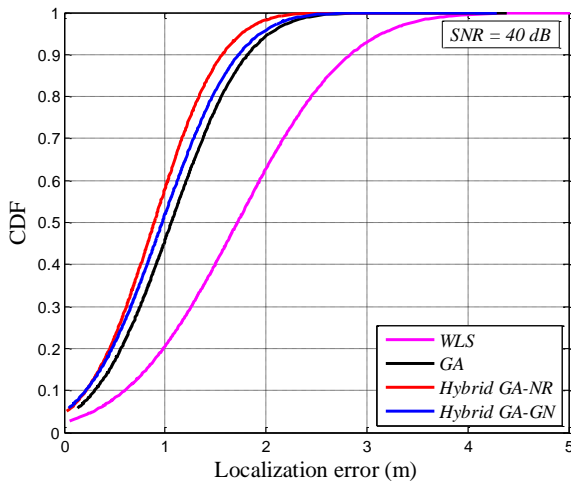

Fig 5. CDFs of the localization error for SNR = 40 dB.

From Fig. 5, it is observed that in the case when the SNR is larger the CDFs of both hybrid estimators have similar performance and outperform the both GA and WLS estimators.

Comparing the numerical results from Fig. 4 and Fig. 5, it can be observed that the higher level of SNR leads to significantly improved performance.

Finally, the impact of SNR on the estimation performance is analyzed for the considered estimators and compared to the derived CRLB, as depicted in Fig. 6.
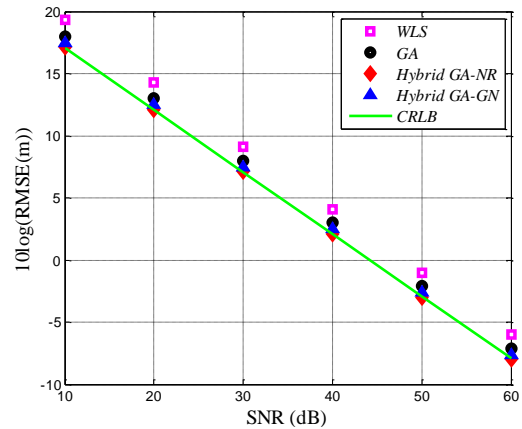

Fig 6. Comparison of RMSE versus SNR levels.

It is evident from Fig. 6 that all considered estimators except WLS are able to approach the CRLB, for wide range of SNR. As the SNR increases the proposed hybrid estimators achieve considerably better performance. Moreover, we see that there is no obvious difference between the performances of the proposed hybrid methods, which aligns with our pervious analyzes.

VII. CONCLUSION

In this paper the localization model using TDOA measurements has been proposed to locate the unknown position of a target. The hybrid GA is developed to solve highly nonlinear and multimodal localization problem. Simulation results show that the proposed hybrid GA method gives significantly more accurate results than the GA and WLS approaches. The results also showed that our hybrid methods accelerate the convergence speed and improve the performance of GA. Future work will focus on the application of the proposed hybrid GA on the optimal receivers placement in the WSNs based on the TDOA measurements.

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