

# Partial Pose Measurements for Identification of Denavit-Hartenberg Parameters of an Industrial Robot

Zaviša Gordić, Kosta Jovanović

**Abstract**—This paper presents an approach to identification of parameters for modelling in robotics using Denavit-Hartenberg notation. It describes an automated procedure for obtaining parameters of industrial robots, with attention to implementation aspects. The procedure uses partial pose measurements, and it is adaptable to various configurations of manipulators, with different numbers and types of joints. Potential for industrial and practical application of the presented approach is considered together with its advantages and disadvantages.

**Index Terms**—Denavit-Hartenberg parameters; partial pose measurement; automated modelling; industrial robotics.

## I. INTRODUCTION

Accurate and reliable modelling of a robotic manipulator is of great importance for various reasons. One of most important uses of robotic modelling is related to direct kinematics and trajectory planning. Accurate model allows for better optimization of trajectories and precise movement of the robot, and enables many advanced operations, such as machine learning, predictive maintenance, calibration, etc.

In the middle of 20th century, Jacques Denavit and Richard S. Hartenberg presented the first minimal representation for a line, using four parameters [1]. Although there were numerous procedures and conventions created for modelling purpose since then, D-H notation is the most common one. Therefore, developing algorithms for identification of parameters for that method is of notable importance. This paper presents D-H parameters using homogenous transformations [2] - [4], and it describes a procedure for their calculation. Experimental parameter identification enables calibration and compensation of errors originating from differences in mathematical model and real robot [1], [5].

Second section describes D-H parameters on an understandable manner using homogenous transformations. From computing perspective, matrices are favoured as their reduce number of calculations.

Third section describes an algorithm that can be use to determine D-H parameters of the system in an intuitive way. With some use of analytical geometry, and simple robot programmes for elementary movement, they enable acquisition of information necessary for the calculation of

all D-H parameters. A similar idea of performing rotation movements is used in [6]. However, in mentioned paper it is used to obtain full-pose measurements of the robot's end effector using partial pose measurements. Additionally, rotations are performed only using the last joint. This paper presents an algorithm that requires only partial pose measurements, which greatly simplifies the measuring process. It applies the same movement to all joints, and it does not impose restrictions to the joint type. The algorithm is not time consuming, and it can be automated.

Conclusion is presented in fourth section, and it offers main observations and perspectives for future work on this and similar topics.

## II. DENAVIT - HARTENBERG PARAMETERS

This section will briefly explain some basics related to the Denavit - Hartenberg preconditions. It explains how homogenous transformations are used to describe relation between joints [2] - [4] and notation used throughout this paper, as shown on Fig 1- 2.

In order to describe a relation between two joints with indexes  $i$  and  $i-1$ , i.e. coordinate systems related to them, using D-H notation, two preconditions must be met [2]:

- axis  $x_i$  is perpendicular to axis  $z_{i-1}$
- axis  $x_i$  intersects axis  $z_{i-1}$ .

For the base coordinate system, the  $z$  axis goes along rotation axis of the joint. The  $x$  axis can be chosen in any suitable direction, as long as it is perpendicular to the  $z$  axis, and  $y$  axis is set in such way that it forms a right-handed Cartesian coordinate system together with previously set  $x$  and  $z$  axes.

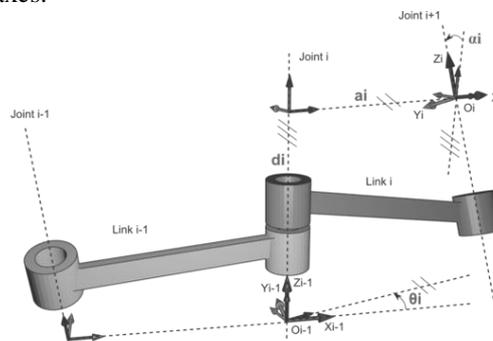


Fig. 1. Notation on neighbouring joints.

For a joint with index  $i$ ,  $z_i$  axis is also set along its rotation axis. However, the  $x_i$  axis is chosen in such way that it is positioned along the vector perpendicular to both  $z_i$  axis and  $z_{i-1}$  axis of previous joint, which is why it is also known as common normal.

Zaviša Gordić is with the School of Electrical Engineering, University of Belgrade, 73 Bulevar kralja Aleksandra, 11020 Belgrade, Serbia (e-mail: zavisa@etf.rs).

Kosta Jovanović is with the School of Electrical Engineering, University of Belgrade, 73 Bulevar kralja Aleksandra, 11020 Belgrade, Serbia (e-mail: kostaj@etf.rs).

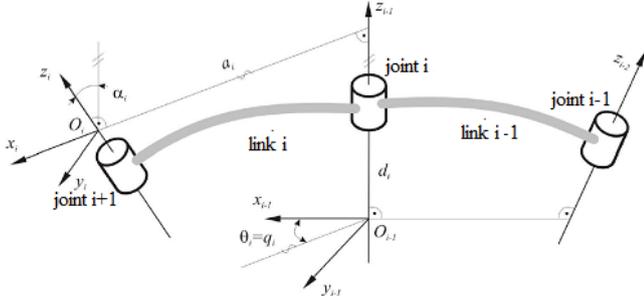


Fig. 2. Notation used for calculation of Denavit-Hartenberg parameters [2].

To Denavit and Hartenberg, common normal served as the main geometrical concept which enabled them to find a minimal representation [1]. This normal also represents the shortest distance between axes  $z_i$  and  $z_{i-1}$ . The origin  $O_i$  of coordinate system is located at the intersection of  $z_i$  axis and the previously determined axis  $x_i$ . The  $y_i$  axis completes the right-handed Cartesian coordinate system.

In order to match coordinate systems of two neighbouring joints with indexes  $i$  and  $i-1$ , a set of two translations and two rotations was used. First, the coordinate system with index  $i-1$  is translated along axis  $z_{i-1}$  to the point where it intersects with axis  $x_i$ . The distance of translation represents parameter  $d_i$ . Second operation rotates the coordinate system with index  $i-1$  until axis  $x_{i-1}$  is aligned with axis  $x_i$ . The angle of rotation is equal to parameter  $\alpha_i$ . Third step is to move the coordinate system with index  $i-1$  along axis  $x_i$  until the origins  $O_i$  and  $O_{i-1}$  match. Distance travelled along  $x_i$  axis is equal to parameter  $a_i$ . The final step is to rotate coordinate system with index  $i-1$  around axis  $x_i$  until axes  $z_i$  and  $z_{i-1}$  match. The angle of rotation represents parameter  $\theta_i$ .

All four steps of matching two coordinate systems can be described with set of four acquired parameters  $\theta_i$ ,  $\alpha_i$ ,  $d_i$ , and  $a_i$ , and homogenous transformation matrix (1) - (2) [2]:

$$H_{i-1}^i = Rot_{z,\theta_i} \cdot Trans_{z,d_i} \cdot Trans_{x,a_i} \cdot Rot_{x,\alpha_i} \quad (1)$$

$$H_{i-1}^i = \begin{bmatrix} C\theta_i & -S\theta_i C\alpha_i & S\theta_i C\alpha_i & a_i C\theta_i \\ S\theta_i & C\theta_i C\alpha_i & -C\theta_i S\alpha_i & a_i S\theta_i \\ 0 & S\alpha_i & C\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2)$$

### III. OBTAINING PARAMETERS

This chapter describes an algorithm that can be used in order to identify Denavit - Hartenberg parameters. The concept is based on gathering partial pose measurements of a single point attached onto robot's end effector. During the acquisition of measurements, the robot performs elementary movements, and therefore it does not require any complex programming. Additionally, this approach can be fully automatic. Although similar procedure is used by company Scape Technologies to extract D-H parameters from robot itself, the concept is not used for calibration purposes.

The idea is to gather exactly the information which is needed to calculate D-H parameters, and that is relative position and rotation angles between neighbouring axes.

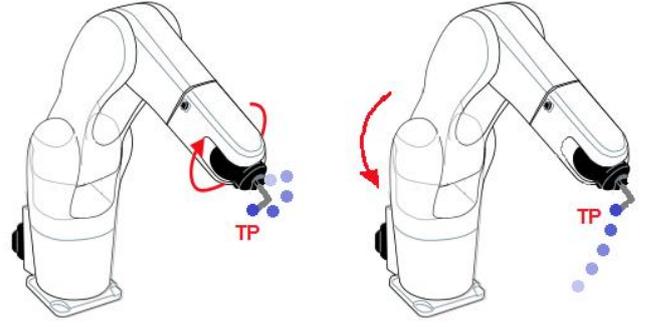


Fig. 3. Examples of robot movements and measured positions.

In order to perform measurement, it is needed to measure position of a point rigidly fixed to the last segment of the robot. Let us name that point of interest as tracked point, or TP. The only restriction to the position of TP is that it may not be located on the rotation axis of the last joint. The restriction is imposed by the principle of the algorithm itself.

If the TP is not collinear with the axis of the rotation of the last joint, when rotation of that particular joint occurs, the tip of the vector connecting any point on joint axis with TP will have a circular trajectory. If position of the TP is measured during this movement, measurements can be fitted to a circle, and its centre can be determined, as shown on Fig. 3. The line perpendicular to the plane in which the circle lays, and containing its centre is actually the axis of rotation of the last joint. Parameters of that axis in space can be determined and recorded in various ways. One of them is to form two vectors, both of which originate in the centre of circle, but whose tips are two measurements that belong to a circle, as shown on Fig. 4. Result of vector product of those two vectors corresponds to direction of the joint's rotation axis, which at the same time represent respective  $x$  axes of each joint.

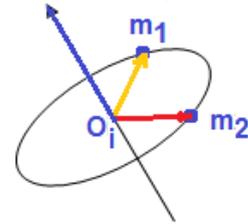


Fig. 4. Calculation of the joint rotation axis.

The procedure can be repeated for the second-to last joint, with all other joints being fixed. Measurements from second rotation result in another circle. In similar manner, its centre can be identified together with direction of its  $z$  axis, which means that rotation axes of two neighbouring joints are known in 3D space.

When points and direction vectors of two axes are known, shortest distance can be calculated as a vector connecting two points on axes, while being perpendicular to both axes. This vector is also known as common normal, and its length represents parameter  $a_i$ . Point of intersection of vector with axis  $z_i$  determines the origin of the coordinate system of joint  $i$ , and its direction determines the direction of axis  $x_i$ .

Distance between coordinate origin  $O_{i-1}$  of joint  $i-1$  and point closest to the axis  $z_i$  represents offset  $d_i$ .

When coordinate origins have been matched, axes  $x_i$  and  $x_{i-1}$  lay in the same plane. Therefore the angle  $\theta$  can easily be calculated from scalar product of unit vectors along  $x_i$  and  $x_{i-1}$  (3).

$$\theta_i = \arccos(\hat{c}_i \cdot \hat{c}_{i-1}) \quad (3)$$

After rotation of axis  $x_i$  to match  $x_{i-1}$ , angle between  $z_i$  and  $z_{i-1}$  can also be calculated using scalar product of unit vectors along respective axes. Calculated angle represents angle  $\alpha_i$ .

Calculated values form D-H parameters for one set of joint, which can be incorporated into homogenous transformation matrix (2). When values for all neighbouring joints have been determined, the final transformation matrix is equal to product of all matrices (4).

$$H_1^n = H_{final} = \prod_{i=1}^n H_{i-1}^i \quad (4)$$

When final transformation matrix has been obtained, the model can be used to accurately represent the real robot. Parameters  $\alpha_i$ ,  $d_i$  and  $a_i$  are constant in case of rotary joints, while  $\theta_i$  are actually internal coordinates  $q_i$ , used to calculate the position of segments. While the described procedure has been explained on example for robots with rotary joints, it is also applicable for robots with linear axis with simple modifications.

If the robot has linear joints, there are a few differences, some of which simplify calculation. One difference is that axis  $z_i$  is set along the axis in which the linear joint moves. Value  $a_i$  is considered to be zero, since it can be chosen arbitrarily. Axis  $x_i$  is set to be normal to the plane in which  $z_i$  and  $z_{i-1}$  lay, i.e. to be in direction of  $z_{i-1} \times z_i$ , or the opposite direction. Axis  $y_i$  is set so that it forms a right-handed Cartesian coordinate system with  $x_i$  and  $z_i$ . Value  $d_i$  is now internal coordinate  $q_i$ , and it is equal to zero at the point where  $O_i$  and  $O_{i-1}$  match. Parameters  $\theta_i$  and  $\alpha_i$  are constant in case of linear joint.

From the described procedure, it is possible to conclude that the approach can be applied for any number and type of joints with single degree of freedom. Therefore, it can be used with any given configuration of the robot, including any external axes that may be used to extend its robot's working range or to introduce redundancy, as long as they form a kinematic chain with robot itself.

From the implementation point of view, procedure is very simple, and it can be divided into two main phases. First phase is dedicated to acquisition of measurements. The robot needs to move only one joint for each set of measurements. Therefore, its program only needs to move the arm in joint coordinates in certain angular increments. Angular increments and number of samples in general depend on the physical capabilities of the robot and computational requirements of the algorithm, but they must be chosen properly in order to cover widest range of movement of each joint with adequate resolution. Robot-intended program is repeatable for each joint, so one function can be reused, cutting down on programming time, and making it easier to adapt for various brands of robots.

Second phase of model acquisition is the analysis of measurements and calculation of D-H parameters.

Homogenous transformation greatly simplifies computation requirements, as it reduces number of operations. Fitting measurement data to a curve is one of most important aspects of the algorithm, since its quality directly or indirectly influences the accuracy of many other values. It depends on the sampling resolution, more samples generally resulting in better outcome. However, simply increasing the number of samples while measuring them in limited range of joint's movement cannot bring optimal results by itself. As mentioned before, measurements should be taken from entire range of motion of one joint, in order to get more robust fitting. Results of the fitting also influence accuracy of calculating  $z$  axis direction, since it depends on the norm of vectors shown in Fig 4, both of which are in fact radii of the circle obtained by fitting.

It is possible to note that the described procedure only requires partial pose measurements, i.e. positions of points in space. Orientation of points is not necessary any of the calculations, as all the needed information can be extracted from position of points.

#### IV. CONCLUSION

This paper offered an approach on robot modelling. It presented an applicable and practical algorithm for determining Denavit - Hartenberg parameters. The presented procedure relays on performing simple movements with only one axis active at a time. Therefore, it can be performed within short periods of time, and with relatively simple program. It does not depend on the configuration of the robot, number and/or type of its joints, and the whole algorithm can be fully automated. The procedure can be used on various brands of robots, but it is not limited to industrial manipulators, as humanoid and service robots can also benefit from such algorithm. One of the key benefits is also that it can be achieved with partial pose measurement, making it feasible to perform with various measuring devices. Some of the implementation aspects were discussed, along with general recommendations for realization.

The main drawback of this algorithm is the accuracy and resolution of devices that provide measurements. Tolerances of measurements can in some cases be greater or in similar range like the inaccuracies of the model [7]. In those cases, usability of the approach is limited to situations where encoder information was lost, or the when robot has suffered deformation of some of its links. For some measuring devices, accuracy is not an issue, but they often have a limited measuring volume. However, although measuring devices limit the reach of the procedure, their performance is constantly getting better, which can be seen from datasheet of Creafom's MaxSHOT [8], parts of which are shown in Table I.

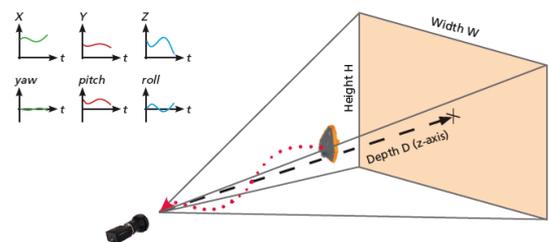


Fig. 5. Nexonar IR Single Camera Tracking measurement volume [7].

TABLE I  
CREAFORM'S MAXSHOT NEXT ELITE TECHNICAL SPECIFICATIONS [8]

Volumetric accuracy	0.015 mm/m
Average deviation	0.005 mm/m
Volumetric accuracy when combined with HandySCAN 300™ HandySCAN 700™	0.020 mm + 0.015 mm/m
Weight	0.79 kg
Dimensions	104 mm x 180 mm x 115 mm

Potential of this algorithm is notable, and its certain aspects extend to robot calibration, predictive maintenance, reconfigurable robot programming. Robot calibration is a very actual issue which rises with the growth of offline robot programming and automatic code generation. Data gathered with presented algorithm is suitable for online or offline use in various applications. Fields such as machine learning, automatic reconfiguration of machine parameters, self-diagnosis etc., can greatly benefit from observing model parameters. All of the stated are in accordance with concept of Industry 4.0, making this topic very interesting for further research.

#### ACKNOWLEDGMENT

The work on this project was partly supported by the Ministry of education, science, and technological development, Republic of Serbia, grant No. TR35003.

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