Measurement Uncertainty of One-bit A/D converter

Ljubica Župunski, Vladimir Vujičić, Ivan Župunski and Vesna Spasić Jokić

***Abstract*—Measurement uncertainty of an ideal one-bit analog-to-digital converter is presented. A low resolution of such converters is overcome by adding a specific random uniform noise, dither, to the measured signal, and by averaging the digital output on a time interval. High resolution of measurement result, and accuracy too, can be achieved when the measured quantity is defined as integral over the certain time interval, like effective value, active/reactive electrical power or energy, etc. For example, choosing time interval of approx. 1 s and sampling frequency of approx. 1 MHz, it is shown that (standard) measurement uncertainty due to limited resolution of the converter can be less than 0.05 %. The effective resolution for this type of converters is introduced in order to compare theirs metrological characteristic with other converters.**

***Index Terms*—A/D convertors; measurement uncertainty; resolution; effective resolution.**

I. Introduction

It is common, regardless of conversion method, that A/D converters are classified regarding their resolution, or number of bits. It is normally, because the high resolution is necessary condition for high converter accuracy. On the other hand, high resolution requires high converter circuits complexity, and, in general, slower operation [1].

Low resolution of digital devices (A/D converters, counter timers etc.) can be partly overcome by postprocessing of measuring results (averaging, for example). However, a lot of methods exist where the noise (random signal, uncorrelated with measuring signal, with specific characteristics, — dither) is intentionally added to the measured signal, before A/D conversion [2]. Then, increase of resolution is necessary followed by the prolongation of conversion time. But, some measured quantities (effective value, active/reactive electrical power and/or energy, Fourier coefficients of a non-sinusoidal waveforms, etc.) are just defined as mean value on certain time interval, so unavoidable prolongation of A/D conversion does not mean necessarily deterioration of the measurement performances.

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Methods where flash A/D converters with very low resolution (two, three, or four-bit) are used take special place [1]. Besides that they work with very high sampling frequencies (for example, hundreds of megahertz), their simple circuit design allow numerical processing just by hardware. So, the measurement result is available after only one period of sampling frequency (contrary to majority of other methods where it is necessary to apply floating point arithmetic, for example) [1].

Many published papers contain principles of dithering and device design with applications in the field of audio and video technics. In some of them, basic measurement terms like precision and accuracy are specially considered. However, they often lack reference to generally accepted standards and documents [3] which concern fundamental metrological terms like measurement uncertainty and it's consistent and comprehensive analysis.

In this paper it is chosen to analyze operation of the simplest A/D converter, one-bit one, in the ideal operating conditions, in order to establish basic relations between metrological characteristic which describe performances of this kind of converters. The emphasis is put on measurement uncertainty.

Purpose of this paper is further development of practical procedures for estimation of measurement uncertainty of multi-bit (two, three etc.) converters of this type (stochastic A/D converters). In addition, the aim is to cover non-ideal operation, in a natural and general enough manner, taking into account the effects of parasitic influence quantities, like non-ideal operation of dither generator, many offsets in electronic circuits, etc.

Even in the simplest one-bit version of such converters, applications exist where obtained metrological performances are comparable, or superior, with more complex and more expensive solutions.

II. Discriminator with one threshold level

The block scheme of device for measurement of the quantity v is shown in Fig. 1.

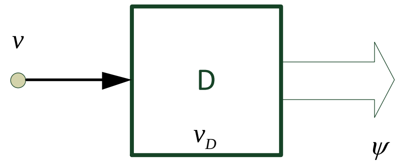

Fig. 1 Block scheme of device for measurement of the quantity *v*

Proceedings of 4th International Conference on Electrical, Electronics and Computing Engineering, IcETRAN 2017, Kladovo, Serbia, June 05-08, ISBN 978-86-7466-692-0 pp. MLI2.4.1-6

Discriminator D have one threshold level, v_D, and its output at the moment of decision is described by discrete function ψ (1).

$$\psi = \begin{cases} 0, & v < v_D \\ 1, & v \ge v_D \end{cases} \tag{1}$$

In this case it makes sense to discuss how to express the quantity value and associated measurement uncertainty [4], but this will not be considered in this paper.

A. Measurement model

Let the values of interest of measurement quantity v be in the range defined by its lower v_L and upper v_H boundary. In order to make simplification of analysis of measurement result and measurement uncertainty, but without loss of generality, let's adopt $v_D = (v_L + v_H)/2$.

It is useful to introduce the substitution:

$$x = \frac{v - v_L}{v_H - v_L} \tag{2}$$

by which the segment $[v_L, v_H]$ is mapped to segment $[0,1]$ and the measurement value $v = v_D$ is mapped to point $x_D = ½$.

The function ψ becomes

$$\psi = \begin{cases} 0, & x < ½ \\ 1, & x \ge ½ \end{cases} \tag{3}$$

By reduction the result to only two possible values, the measuring device is unable to resolve mutually close values of quantity *x*, and this is recognized as error of measurement result due to its limited resolution (amplitude discretization error).

Measurement result x_m can be written as

$$x_m = \frac{1}{2}\psi + k \tag{4}$$

Quantity k is introduced as correction of measurement result due to limited resolution. This is random quantity, determined by its Probability Density Function, PDF [2,3]

If any specific knowledge about the related PDF doesn't exist, (like in our case), the Uniform distribution, with lower and upper boundary of 0 and ½ is adopted, respectively. Standard deviation of that PDF is taken as a standard measurement uncertainty $u(k)$ of correction *k*: $u(k) = \sigma(k) = 1/(4\sqrt{3})$.

The type of evaluation of standard uncertainty is Type B [3].

Estimation of the value of measurement quantity for $x < ½$ is $x_m = ½\,\psi + \mu(k) = ¼$ and combined measurement uncertainty is $1/(4\sqrt{3}) \approx 0.14$, or, approximately 14 %.

Measurement result is:

$$x_m = ½ \cdot \psi + \mu(k) \pm u(k) = 0 + (0 + ½)/2 \pm (½ - 0)/(2\sqrt{3}).$$

The probability that the value of quantity x will be in the range $¼ \pm 1/(4\sqrt{3})$,

$p\left[¼ - 1/(4\sqrt{3}) \le x \le ¼ + 1/(4\sqrt{3})\right] \approx 0.58$, is (under given circumstances) approximately 58 %.

In a similar way, for $x \ge ½$,

$x_m = ½ \cdot \psi + \mu(k) \pm u(i) = ¾ \pm 1/(4\sqrt{3})$ is obtained.

Error of measurement result, *e*, is

$$e = x_m - x \tag{5}$$

For x values from segment $0 \le x \le 1$, this function is well-known and is given in Fig. 2.

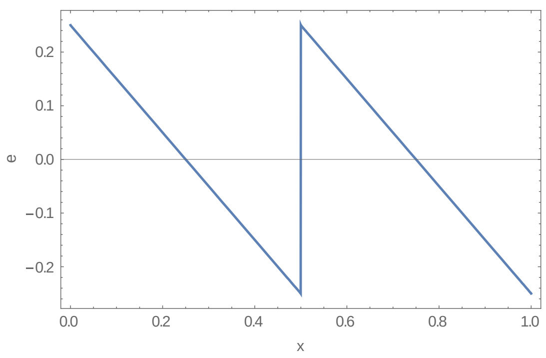

Fig. 2 Error of measurement result x_m, as function of value *x*

B. Measurement during the time

Values of the measurement quantity $v(t)$ are compared with value v_D at discrete moments in time t_i, $i = 1, 2, \ldots$, by sampling frequency f_s. Consecutive values of the function $\psi(t_i)$ are accumulated in counter/accumulator A, as it is shown in Fig. 3.

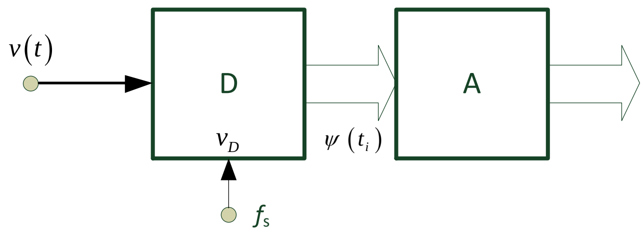

Fig. 3. Block scheme of device for measurement the quantity *v*(*t*)

Let the value $\psi(t_i)$ be equal zero N_0 times, and equal one N_1 times, $N = N_0 + N_1$.

Mean value of sequence of values ψ is,

$$\bar{\psi} = \frac{1}{N}\sum_{i=1}^{N}\psi(i) = \frac{N_1}{N} \quad (6)$$

and the standard deviation of the quantity $\bar{\psi}$ is standard uncertainty $u(\bar{\psi})$ of measurement result [3]:

$$u(\bar{\psi}) = \sqrt{\frac{1}{N-1}\bar{\psi}(1-\bar{\psi})} \quad (7)$$

Besides the substitution (2), it is also useful to introduce:

$$i = t_i \ f_s \quad (8)$$

Now, function $\psi(t_i)$ has the form

$$\psi(i) = \begin{cases} 0, & x(i) < ½ \\ 1, & x(i) \geq ½ \end{cases} \quad (9)$$

Let the measurement procedure repeats with frequency f_s during the time interval T. For that time it will be collected $N = f_s T$ samples of quantity x. Using (4), averaged value $\bar{x}_m$ of values $x_m(i)$, is

$$\bar{x}_m = \frac{1}{N}\sum_{i=1}^{n}\left(\frac{1}{2}\psi(i) + k(i)\right) = \frac{1}{2}\bar{\psi} + k \quad (10)$$

Variance of quantity $\bar{x}_m$, $V(\bar{x}_m)$, is given as

$$V(\bar{x}_m) = \left(\frac{1}{2}\right)^2 V(\bar{\psi}) + V(k) = \left(\frac{1}{2}\right)^2 \frac{1}{N} V(\psi) + V(k) \quad (11)$$

Standard measurement uncertainty of quantity $\bar{x}_m$ is

$$u(\bar{x}_m) = \sqrt{V(\bar{x}_m)} = \sqrt{\left(\frac{1}{2}\right)^2 \frac{1}{N} V(\psi) + \left(\frac{1}{2\sqrt{3}}\frac{1}{2}\right)^2} \quad (12)$$

Therefore, device provides measurement result as mean value of quantity x on the time interval $T = f_s N$, with measurement uncertainty $u(\bar{x}_m)$.

For fast changing quantity x, device acts as a low-pass filter.

If measured quantity x is constant over period of measurement T, then $V(\psi) = 0$, and only source of uncertainty that remains is uncertainty due to finite resolution of measurement results, $u(\bar{x}_m) = 1/(4\sqrt{3}) \approx 0.14$.

C. *Measurement in presence of noise*

Let the noise $n(t)$ be superimposed over the measurement quantity $v(t)$. Further, let the noise be characterized by Uniform distribution, in the boundaries of $\pm n_g$, and uncorrelated with $v(t)$. In order to make investigation of noise influence on measurement result easier, suppose that n_g is known value.

Figure 3 represents output of the discriminator in the presence of noise. For measured quantity that have constant value x and noise limits $\pm n_g$, the probability of getting value $\psi(i) = 1$ is the ratio of shaded area to the total area of corresponding column.

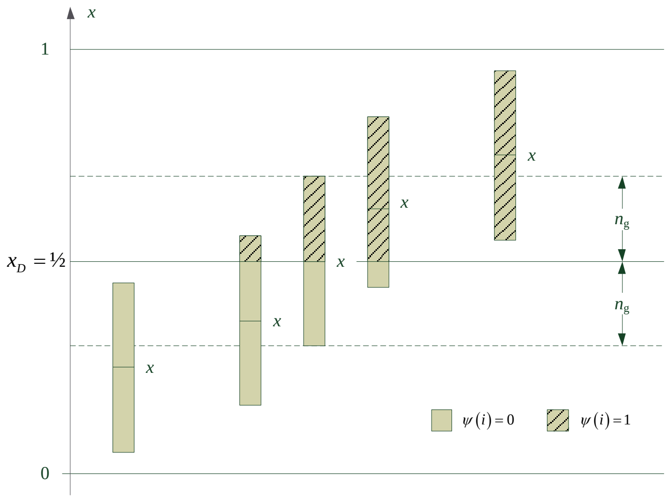

Fig 4. Graphical presentation of discriminator operation when the $x(t)$ is superimposed by noise $n(t)$ in the boundaries $\pm n_g$

The relevant functions describing the operation of device are derived from Fig. 4. directly.

Mean value of function ψ, depending on the value of measured quantity x, and noise boundaries n_g, is

$$\bar{\psi}(x, n_g) = \begin{cases} 0 & 0 \leq x < ½ - n_g \\ \dfrac{x + n_g - ½}{2n_g} & ½ - n_g \leq x \leq ½ + n_g \\ 1 & ½ + n_g < x \leq 1 \end{cases} \quad (13)$$

Result of the measurement x_m, depending on the value of measured quantity x, and noise boundaries n_g, is

$$x_m(x, n_g) = \begin{cases} (½ - n_g)/2 & 0 \leq x < ½ - n_g \\ \left[2\bar{\psi}(x, n_g) - 1\right] n_g + ½ & ½ - n_g \leq x \leq ½ + n_g \\ 1 - (½ - n_g)/2 & ½ + n_g < x \leq 1 \end{cases} \quad (14)$$

Measurement error e, depending on the value of measured quantity x, and noise boundaries n_g, is

$$e\left(x,n_g\right)=x_m\left(x,n_g\right)-x \qquad (15)$$

Measurement uncertainty $u\left(x,n_g,N\right)$ of quantity x_m, depending on the value of measuring quantity x, noise boundaries n_g, and number of collected samples N:

$$u\left(x,n_g,N\right)=\begin{cases}\dfrac{½-n_g}{2\sqrt{3}}, & 0\le x<½-n_g\\ \sqrt{\dfrac{1}{N-1}\bar{\psi}\left(x,n_g\right)\left[1-\bar{\psi}\left(x,n_g\right)\right]}, & ½-n_g\le x\le ½+n_g\\ \dfrac{½-n_g}{2\sqrt{3}}, & ½+n_g<x\le 1\end{cases} \qquad (16)$$

Dependence of the standard measurement uncertainty u on the number of samples N in a range of one decade, for $x=½$, is shown in Fig. 5.

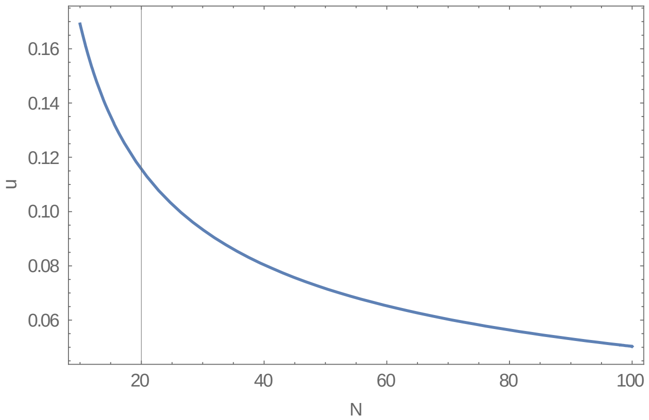

Fig. 5. Dependence of the standard measurement uncertainty *u* on the number of samples *N*

Error of the measurement e, combined with measurement uncertainty u (as shaded area around e), that depends on the measuring quantity x, for $n_g=0.2$ or $n_g=0.5$, and $N=100$, are graphically represented in Fig. 6, and Fig. 7.

We confirm the well-known result that the error *e* will be suppressed by adding a noise (Uniform PDF, $n_g=0.5$, uncorrelated with measuring signal [1,2,5,6]), and by averaging a digital output of discriminator. Besides, we attach the information on related measurement uncertainty, according to [3].

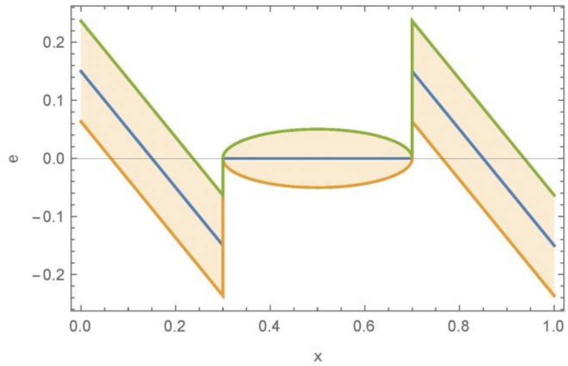

Fig. 6. Dependence of the measurement error e on the value of measured quantity *x*, and area of uncertainty around *e*, for $n_g=0.2$ and $N=100$

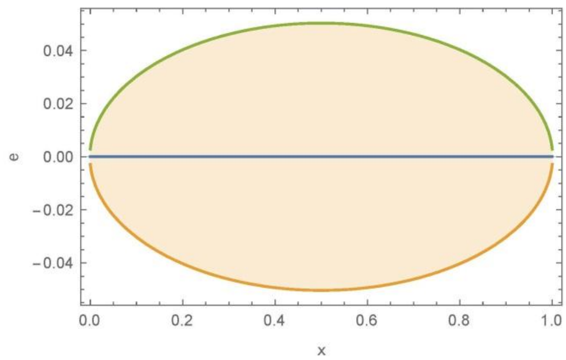

Fig. 7. Dependence of the measurement error e on the value of measured quantity *x*, and area of uncertainty around *e*, for $n_g=0.5$ and $N=100$

D. Dither

Block scheme of device, where random signal $d\left(t\right)$ characterized by Uniform distribution in the range $\left[-n_g,n_g\right]$ is added to the measured signal $v\left(t\right)$, is shown in Fig. 8.

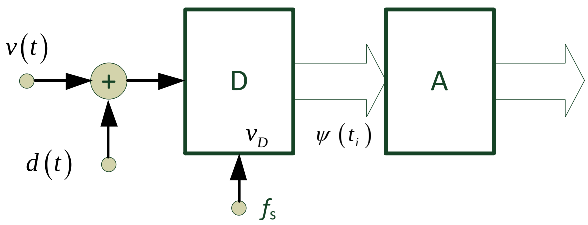

Fig. 8. Block diagram of the device for measurement of the quantity *v*(*t*) with added dither *d*(*t*)

Adopting $n_g=½$ and $x_D=½$, from (14), result of the measurement is

$$x_m=\left(2\bar{\psi}\left(x\right)-1\right)/2+½=\bar{\psi} \qquad (17)$$

and standard measurement uncertainty is

$$u=\sqrt{\frac{1}{N-1}\bar{\psi}\left(1-\bar{\psi}\right)} \qquad (18)$$

Discretization per amplitude of the measured signal is replaced with the discretization per time of sampling. Influence of discretization per amplitude, which is common limiting factor for low resolution A/D converter applications, is substituted with influence of discretization per time, which can be neglected for significant number of applications by proper choice of sampling frequency f_s and/or measurement duration T.

E. Effective resolution of stochastic one-bit A/D converter

The function $\bar{\psi}$ is discrete function, with resolution of $1/N$ and, in principle, device would be able to detect the minimum difference between two close values, which is $1/N$. However, standard measurement uncertainty decreases only by factor $\sqrt{N}$ (16) and, generally, it is much higher than $1/N$. Thus, it is more convenient to adopt some other way for expressing resolution for these stochastic A/D converters. In order to practically compare metrological performances of different types of converters, just standard uncertainty can be taken as effective resolution [4].

If one notice that $\mathrm{Max}\left(\bar{\psi}\left(1-\bar{\psi}\right)\right)=¼$ for x from interval $0 \le x \le 1$, and $N-1 \approx N$ for $N >> 1$, from (16) follows $u \le 1/\left(2\sqrt{N}\right)$, effective number of bits b_e for this type of converters would be

$$b_e \approx \log_2\left(2\sqrt{N}\right) \tag{19}$$

For example, for $N = 10^6$, $b_e \approx 11$ bits is obtained.

III. Conclusion

Resolution of measurement is effectively increased and measurement uncertainty due to limited resolution is decreased by superimposing random signal, dither $d(t)$, to measuring signal and by taking the quantity $\bar{\psi}$ (with $N > 1$) as estimate of measuring quantity x.

Quantity $\bar{\psi}$ is a measure of mean value $\bar{x}$ of the measured quantity x during time period of the measurement.

Therefore, it is necessary to elapse some time in order to obtain the measurement result, “measurement over an interval” [1,5,8,10,11], contrary to concept “measurement in a point” where the result should represent measuring quantity at a moment of comparison with some referent value [1,9].

It is worth mentioning again that a lot of important measuring quantities are suitable for “measurement over an interval”, every time when the quantity is defined as mean value over a time interval (effective value, active/reactive electrical power and/or energy, harmonics of non-sinusoidal signals, etc.).

This paper is mainly dealing with estimation of measurement uncertainty of the stochastic one-bit A/D converter in the case of repetitive and non-repetitive measurement. As it is expected, without dither, dominant source of uncertainty is low resolution of the converter and practically there is no application where such measurement result can be satisfactory. However, by adding the certain noise, dither, to measuring signal, mentioned limitation can be prevailed. In this manner one can obtain valuable results in many of practical applications.

For example, if we measure DC signal with amplitude $x \approx 0.5$ and with $N \approx 100$ samples (over time period of $100\ \mu\mathrm{s}$ and sampling and dither frequency of $1\ \mathrm{MHz}$), standard measurement uncertainty is nearly 5 % (Fig. 7.). Measurement of the signal with the same characteristics but with $N \approx 10^6$ samples (over time period of 1 s with sampling and dither frequency of 1 MHz), gives standard measurement uncertainty of nearly 0.05 %.

Mathematical model of the measurement result (4) has convenient form where many parasitic quantities that have influence on the real converter operation can be included (for example, if x_D is a little bit different from ½, and/or n_g slightly differs from ½, and/or if there are some offsets in unit for summation quantity x and dither, and/or if dither generator is not ideal (including a low resolution of D/A converter that make dither), and/or if measuring signal is corrupted by real noise, etc.).

Acknowledgment

This research is a part of the project III45006 (Physics and Chemistry with Ion Beams) and III 43011 (Joint research of measurements and effects of ionizing and UV radiation in medicine and environmental protection). The authors are sincerely grateful to Ministry of Education and Science, Republic of Serbia, for supporting this study.

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