Implementation of Monte Carlo method for the determination of uncertainty in measurement

Marina Bulat, Bojan Vujicic, *Member IEEE*, Dragan Pejic, *Member IEEE*, Ljubica Zupunski, *Member IEEE*, Marjan Urekar, *Member IEEE*, Djordje Novakovic

***Abstract* — This paper analyzes a comparison of two methods for determining a measurement uncertainty: as recommended by ISO - "International Organization for Standardization" and using Monte Carlo method. The first concept is accepted and required in modern metrology. However, there are special cases when it doesn't provide good results. In those cases, Monte Carlo method is used.**

***Key words* — measurement uncertainty, Monte Carlo method, reactive power.**

I. INTRODUCTION

In every measurement there is an error, so the result of a measurement is not represent by a number, but an interval in which the real value of the measurement is. Metrologically speaking, for a better quality of measurements, from an aspect of accuracy and precision, it is better to have a smaller interval in which the real value is. Looking back through history, different methods for expression the limits of intervals were used. Besides the well known concepts, such as secure and statistical limits of measurement errors, there were many other methods. Due to these ambiguities, problems during inter-laboratory and interstate comparisons had often occurred. In order to standardize the methods for defining measurement uncertainty, in 1995 International Standardization Organization has issued a document called "The **G**uide to the Expression of **U**ncertainty in **M**easurement" (hereinafter GUM).

GUM is a fundamental reference document for estimation and expression of measurement uncertainty. The purpose of GUM is to establish unique rules for the expression of measurement uncertainty for the needs of metrology, standardization, calibration etc. Furthermore it is to provide the complete information about a method of how measurement uncertainty has been obtained and to provide rules for international comparison of measurement results. Concepts on which GUM is based:

1. All components of measurement uncertainty are reported with standard deviation,
2. All systematic errors are corrected,
3. Intervals of all uncertainty components are symmetrical,
4. Components are grouped in two categories, depending on the source of data: measurement uncertainty type A – components are evaluated by statistical methods, and measurement uncertainty type B – components are evaluated by non-statistical methods.

The procedure for an assessment of measurement uncertainty in accordance with GUM:

1. Identify all the relevant components of measurement uncertainty,
2. Calculate standard a measurement uncertainty for each component,
3. Calculate a combined standard measurement uncertainty,
4. Calculate an expanded measurement uncertainty,
5. Express a result of measurement, consisting of, the best estimation of measured value and combined or expanded measurement uncertainty.

Although today compliance with GUM recommendations is practically an obligation for all metrological institutions, there are situations when GUM is not applicable: a) if probability distribution function of measured quantity is not normal, b) with asymmetrical and nonlinear problems and c) when expected measurement uncertainty, of a quantity we estimate, is of the same order as it's measured value, i.e. when the relatively stated measurement uncertainty is in order of tens of percentages.

This paper deals with a special case i.e. when it is not justifiable to express the measurement uncertainty in accordance with GUM recommendations.

In order to illustrate the applicability of GUM recommendations, in one specific case, simple example of determining a reactive power based on measurement of voltage, current and active power of consumer has been given. Measurement uncertainty has been determined using two methods: according to GUM recommendations and by Monte Carlo method. Based on measurements of voltage, current and active power used by consumer, reactive power is determined by a formula:

Marina Bulat – Faculty of Technical Sciences, University of Novi Sad, Trg Dositeja Obradovića 6, 21000 Novi Sad, Serbia (e-mail: marina.bulat@uns.ac.rs). Ljubica Zupunski – Faculty of Technical Sciences, University of Novi Sad, Trg Dositeja Obradovića 6, 21000 Novi Sad, Serbia (e-mail: ljubicaz@uns.ac.rs). Bojan Vujicic – Faculty of Technical Sciences, University of Novi Sad, Trg Dositeja Obradovića 6, 21000 Novi Sad, Serbia (e-mail: bojanvuj@uns.ac.rs). Dragan Pejić – Faculty of Technical Sciences, University of Novi Sad, Trg Dositeja Obradovića 6, 21000 Novi Sad, Serbia (e-mail: pejicdra@uns.ac.rs). Marjan Urekar – Faculty of Technical Sciences, University of Novi Sad, Trg Dositeja Obradovića 6, 21000 Novi Sad, Serbia (e-mail: urekarm@uns.ac.rs). Djordje Novakovic – Faculty of Technical Sciences, University of Novi Sad, Trg Dositeja Obradovića 6, 21000 Novi Sad, Serbia (e-mail: djordjenovakovic@uns.ac.rs).

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$$Q = \sqrt{(U \cdot I)^2 - P^2}$$

When power factor is close to 1, meaning when it is mainly about thermogenic impedance, it is shown that the first method creates high measurement uncertainty. This happens because of the subtraction of two close values: apparent power (defined by product of voltage and current) and active power. In situations like these, it is expected that GUM recommendations do not provide good evaluation of measurement uncertainty. Correctness of GUM recommendations in this example was evaluated by implementation of Monte Carlo method.

The aim of this paper is to determine the evaluation of reactive power *Q*, it's joint standard and expanded measurement uncertainty *u(Q)* and the smallest 95 % interval of expansion, according to GUM recommendations and by implementation of Monte Carlo method.

II. Determining a Measurement Uncertainty in Accordance with GUM

Reactive power of impedance is given by a formula:

$$Q = \sqrt{(U \cdot I)^2 - P^2} = 3.59 \text{ VAr}$$

where *U* is voltage measured by a voltmeter and is 230 V, *I* is current measured by a miliampermeter and is 50 mA, *P* is active power measured by a wattmeter and is 10.92 W.

Accuracy class of all instruments are the same and is 0,5.

According to GUM recommendations for evaluation of limits for measurement uncertainty the following steps are required:

1. Defining a measurement quantity (reactive power *Q*) and developing a mathematical model that describes the measurement,
2. Identification of all important components of measurement uncertainty, *U*, *I*, *P* and assigning probability distribution to each one of them,
3. Determining an absolute error of voltage, current and power, *ΔU*, *ΔI* and *ΔP,*
4. Calculation of standard measurement uncertainty for each component *u(ΔU)*, *u(ΔI)* and *u(ΔP),*
5. Calculation of combined measurement uncertainty, $u_c(Q)$,
6. Calculation of expended measurement uncertainty

$$u_{prošireno}(Q) = k \cdot u_c(Q)$$

   where *k* (coverage factor) has a value of 2, in order to have measurement uncertainty with 95% degree of trust.
7. Expressing the results of measurement in form:

$$Q \pm u_{prošireno}(Q).$$

For voltage and current we considered that distribution function is always uniform, and for the power there were three situations which we considered: uniform, Gauss, and triangular.

From formula for accuracy class:

$$kl_{X\%} = \frac{\Delta X}{X_{\max}} \cdot 100\%,$$

where *X* is a component of measurement uncertainty, absolute errors of voltage, current and power are as follows: 1,5 V, 0,25 mA and 0,075 W.

In table I measurement uncertainties for all three cases are given. Ranges of instruments 300 V, 50 mA and 15 W are also shown.

Combined measurement uncertainty for reactive power is calculated by a form:

$$u_c(Q) = \sqrt{\left(\frac{\partial Q}{\partial I}\right)^2 \cdot u^2(\Delta I) + \left(\frac{\partial Q}{\partial U}\right)^2 \cdot u^2(\Delta U) + \left(\frac{\partial Q}{\partial P}\right)^2 \cdot u^2(\Delta P)}$$

And calculated results for all three cases are given in the table II.

TABLE I
MEASUREMENT UNCERTAINTY OF ABSOLUTE ERRORS OF VOLTAGE CURRENT AND ACTIVE POWER FOR ALL THREE CASES

case	*u(ΔU)*	*u(ΔI)*	*u(ΔP)*
first	$\Delta U/\sqrt{3}$	$\Delta I/\sqrt{3}$	$\Delta P/\sqrt{3}$
second	$\Delta U/\sqrt{3}$	$\Delta I/\sqrt{3}$	$\Delta P/3$
third	$\Delta U/\sqrt{3}$	$\Delta I/\sqrt{3}$	$\Delta P/\sqrt{6}$

TABLE II
EXPENDED UNCERTAINTY IN MEASUREMENT, UNCERTAINTY GIVEN IN PERCENTAGES AND INTERVAL CONTAINING ALL MEASUREMENTS RESULTS FOR ALL THREE CASES ACCORDING TO GUM

case	$u_{\exp ended}(Q)$ (VAr)	$u_c(Q)$ (%)	Interval (VAr)
first	0.43	12.19	(3.15 ;4.03)
second	0.38	10.61	(3.21 ;3.97)
third	0.39	11.03	(3.19 ;3.99)

Fig. 1. shows the intervals containing true value of reactive power calculated according to GUM for all three cases. Surface areas below the curves are normalized, so for that reason wider intervals are lower and vice versa.

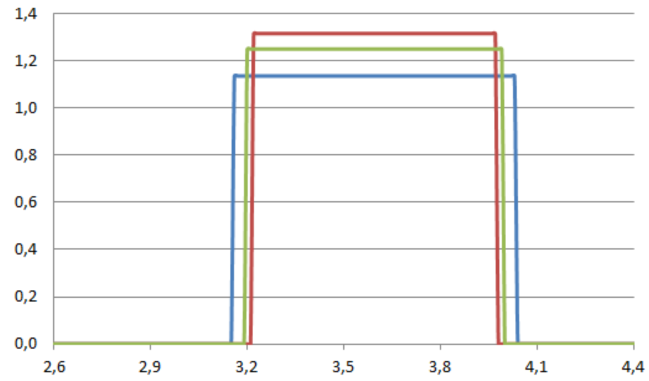

Fig. 1. Presentation of intervals containing true value obtained by the recommendation of GUM (first case is shown by blue, second by red, third by green color)

Fig. 2. shows three normal distributions which, by GUM recommendations, are obtained for distribution function of reactive power in all three cases.

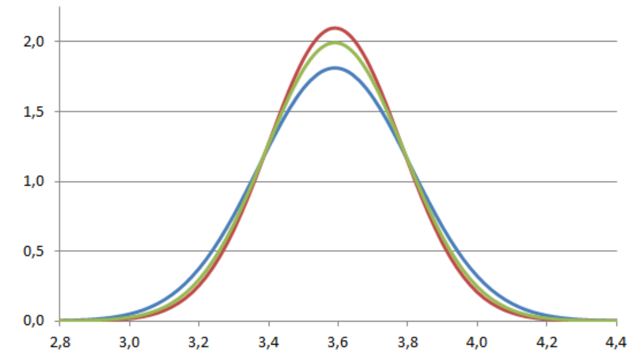

Fig. 2. Distributions of reactive power obtained by recommendations of GUM (first case is shown by blue, second by red, third by green color)

III. Assessment of Uncertainty in Measurement by Monte Carlo Method

Monte Carlo method is a procedure that, based on random sampling of pre-defined PDF of measured values (*U*, *I*, and *P*), generates distribution function of an output quantity (reactive power).

According to the procedure, we randomly choose measured values from a large number of consecutive measurements (million measurements considered here), that fulfill the requirement, over the whole ensemble, measured values have expected error distribution. PDF of an output quantity (reactive power). Using an interval from 2.5 % to 97.5 %, measurement uncertainty limits are calculated for coverage factor k = 2.

In first case we considered, all three input quantities have uniform distribution. Based on Monte Carlo method, the interval containing true value of reactive power with level of confidence of 95 % is (2.68; 4.31) VAr. Fig. 3 shows a histogram of simulated results.

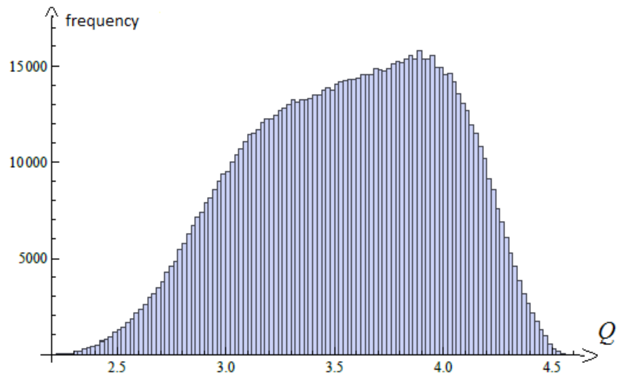

Fig. 3. Results distribution by Monte Carlo method for uniform distribution of voltage, current and active power

Second case we analyzed, PDF for voltage and current is uniform, and for active power is Gauss. Based on Monte Carlo method, the interval containing true value of reactive power with level of confidence of 95 % is (2.97; 4.12) VAr. Fig. 4 shows a histogram of simulated results.

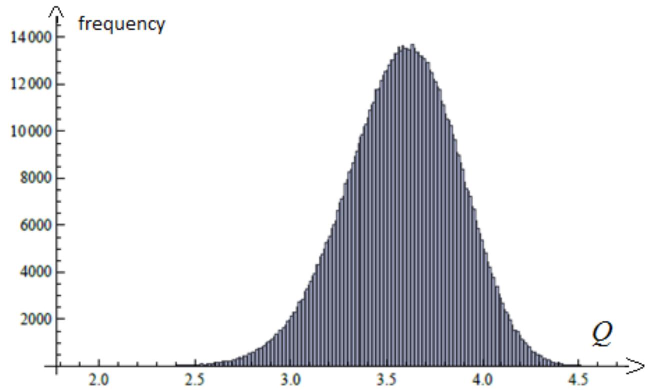

Fig. 4. Results distribution by Monte Carlo method for uniform distribution of voltage and current, Gauss distribution of active power

In final third case, PDF for voltage and current is uniform, and for active power is triangular. Based on Monte Carlo method, the interval containing true value of reactive power with level of confidence of 95 % is (2.87; 4.19) VAr. Fig. 5 shows a histogram of simulated results.

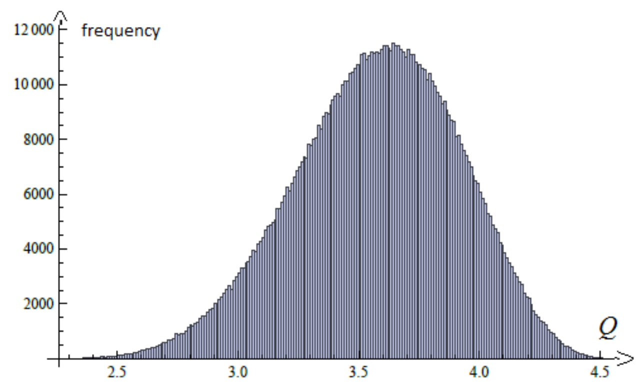

Fig. 5. Results distribution by Monte Carlo method for uniform distribution of voltage and current, triangular distribution of active power

In table III overall results, simulated by Monte Carlo method, are presented.

Fig. 6 is the graphic presentation of the results by Monte Carlo method.

TABLE III
RESULTS BY MONTE CARLO METHOD

case	Interval (VAr)	u (%)
first	(2.68 ; 4.31)	23.32
second	(2.97 ; 4.12)	16.22
third	(2.87 ; 4.19)	18.70

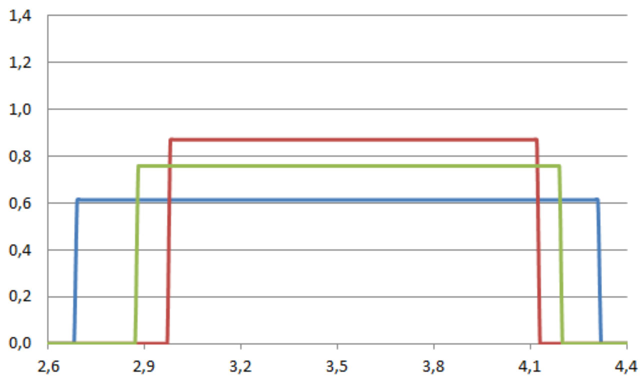

Fig. 6. Graphic presentation of intervals obtained by Monte Carlo method (first case is shown by blue, second by red, third by green color)

Table 4. shows the results obtained according to GUM recommendations and Monte Carlo method, presented in percentages.

TABLE IV
RESULTS OF EXPENDED MEASUREMENT UNCERTAINTIES BY MONTE CARLO METHOD AND GUM RECOMMENDATIONS SHOWN IN PERCENTAGES

case	MC u (%)	GUM u (%)	MC / GUM
first	23.32	12.19	1.91
second	16.22	10.61	1.53
third	18.70	11.03	1.70

According to GUM, for all three cases, small variation of measurement uncertainties: 12.19 %, 10.61 % and 11.03 %, have been noted. Identical evaluation of measurement uncertainty, for reactive power regardless of variations distributions of *U*, *I* and *P*, has been obtained.

According to Monte Carlo method, major differences are created when PDF vary for all three cases and they are 23.32 %, 16.22 % and 18.70 %. Different evaluations of reactive power value are obtained: 3.49 VAr, 3.54 VAr and 3.53 VAr. Distribution of results obtained deviate from normal because they are not symmetrical. Monte Carlo method cause greater measurement uncertainties than GUM, by order: 1.91, 1.53, 1.70 times for all three cases. Considering the fact that GUM is not intended for use in cases like these, as well as how Monte Carlo method function, it is logically to accept measurement uncertainties simulated by Monte Carlo method. In other words, measurement uncertainties someone would get, by wrongfully implementing GUM recommendations would be smaller, which means that the interval containing expected true value of measured quantity would be smaller.

IV. CONCLUSION

Based on the example of evaluation of reactive power (with a power factor close to 1, which is the reason why obtained measurement uncertainty is so large, order of 10 %) we estimated measurement uncertainty two ways: by recommendation of International Standardization Organization, and by implementation of Monte Carlo method. According to GUM recommendations, obtained results are smaller in comparison to the results obtained by Monte Carlo method.

If someone were to determine measurement uncertainty by blindly following recommendations of GUM, it would be almost two times lower, and the interval containing true value would be two times smaller than the one that Monte Carlo method provides. In these critical situations, it is not correctly to use GUM for determination of measurement uncertainty. Much more realistic results are obtained by use of Monte Carlo method.

The aim of this paper is by no means to question the correctness of GUM recommendations, but to point out a special case where those recommendations do not provide good results.

REFERENCES

[1] *„Evaluation of measurment data – Supplement 1 to"The guide to the Expression of Uncertainty in Measurement" - Propagation of distributions using a Monte Carlo method", JCGM 101:2008*

[2] Stojaković M., "*Uvod u teoriju verovatnoće i matematičke statistike"*, Novi Sad, Stylos, 1995

[3] Stojaković M., "*Verovatnoća, statistika i slučajni procesi"*, Novi Sad, Symbol, 2007.

[4] Gentle, James E., "*Random Number Generation and Monte Carlo Methods"*, New York, , Springer - Verlag 2005.

[5] Jevremović, V. i J. Malixić, "*Statističke metode u meteorologiji i inženjerstvu"*, Beograd, Savezni hidrometeorološki zavod, 2002.

[6] Mladenović P., "*Elementaran uvod u verovatnoću i statistiku"*, Beograd, Društvo matematičara Srbije, 1998.

[7] Stojaković M., "*Matematička analiza 2"*, Novi Sad, Symbol, 2007.

[8] Mališić J., "*Verovatnoća i matematička statistika"*, Beograd, Krug, 1999.

[9] Simonović V., *Uvod u teoriju verovatnoće i matematičku statistiku"*, Bor, Građevinska knjiga, 1995.

[10] Brezinšćak M., "*Procjenjivanje mjerne nesigurnosti"*, Beograd, Savezni zavod za mjere i dragocjene kovine, 1976.