II. ZERO CROSSING TECHNIQUE

Among methods for measuring frequency in the time domain, the most widely used method is based on determining the transitions of the voltage signal through zero [2], [5], [6]. According to the definition, under the signal (voltage) period is meant the time for which one of its total oscillations occurs. Then the voltage period can be defined as the time between its transitions through a given level in the selected direction (from positive to negative or vice versa). As a level value, it is advisable to select zero, since in this case the maximum rate of change of the sinusoidal voltage (and the fundamental component of the polyharmonic voltage) is observed. Therefore, the moment of intersection can be found most accurately. The zero crossing can be considered for both directions of voltage variation. The event of a zero crossing is determined by the change in the sign of the voltage (for definiteness, in our case, from negative to positive). Figure 1 illustrates the main features of zero crossing technique.

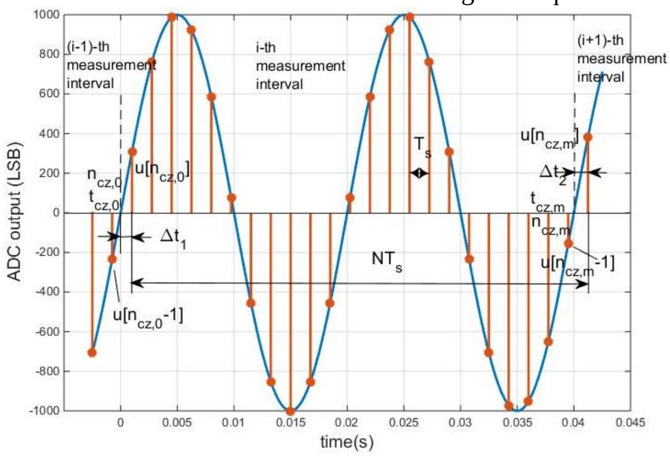

Fig. 1. Zero crossing technique demonstration.

The measurement interval consists of m periods of the fundamental voltage component, the beginning and end of which is determined by the voltage transitions through zero. Transitions through zero are determined by changing the sign of the count from negative to positive. The equation to determine the frequency (see Figure 1):

$$t_{cz,m} - t_{cz,1} = mT_1 = NT_S + \Delta t_0 - \Delta t_m, \qquad (1)$$

where m is the number of complete periods of the input voltage that fit within the measurement time; $T_1=1/f_1$ – the period of the fundamental component of the input voltage; N – the number of samples on the i-th measurement interval; $t_{cz,0}$, $t_{cz,m}$ – the time of the first and last crossing (respectively) by zero voltage for the measurement window under consideration; Δt_0, Δt_m – time intervals between the first voltage transition through zero and the first positive voltage sample, respectively, at the i-th and $(i+1)$-th intervals of the frequency measurement; T_S – sampling time.

In this case, the times of zero crossing of the input signal must correspond to one type of intersection (when the sign of the voltage samples changes from negative to positive in our case). Then the frequency of the fundamental component of the voltage f_1, as follows from (1), is calculated by the formula:

$$f_1 = \frac{m}{t_{cz,m} - t_{cz,1}} = \frac{mf_S}{N + \eta_0 - \eta_m} = \frac{mf_S}{N + \eta}, \qquad (2)$$

$$\eta = \eta_0 - \eta_m = \frac{\Delta t_0}{T_S} - \frac{\Delta t_m}{T_S}, \qquad (3)$$

where η_0, η_m – the values of the corrections to N, considering non-zero values Δt_0, Δt_m respectively; η – the total sum of the corrections; f_S – sampling frequency.

The disadvantages of the method under consideration include the possible dependence on the harmonics and noise of the input voltage. When the next transition through zero is reached, to avoid false transitions, it is recommended to enter a time delay to search for a subsequent transition. The delay value is reasonably chosen, for example, equal to three quarters of the nominal value of the input voltage period.

The error of the frequency discreteness in the case under consideration is determined by the duration of the zone of uncertainty of zero crossing of the input voltage. If the correction to N is not introduced, then the maximum error of discreteness is determined by the expression:

$$\delta_T = \delta_f \le \frac{T_S}{mT_1} = \frac{1}{N}. \qquad (4)$$

It can be seen from expression (4) that an increase in the number of observed periods and an increase in the sampling frequency (leading to an increase in the number of samples) leads to a decrease in the measurement error. However, the nominal number of observed periods is one of the terms of the requirements specification when performing the development, and the increase in the sampling rate involves several difficulties: a faster hardware components is required, and a greater consumption from the power source occurs.

Further decrease of this error can be achieved by polynomial approximation [2], [4] of the voltage in the zones of uncertainty of its transition through zero. The coefficients of the interpolation polynomial are calculated by solving the system of equations:

$$\begin{cases} a_k(n-0.5k)^k + a_{k-1}(n-0.5k)^{k-1} + \ldots + a_0 = u[n-0.5k], \\ \ldots \\ a_k(n+0.5k)^k + a_{k-1}(n+0.5k)^{k-1} + \ldots + a_0 = u[n+0.5k]; \end{cases} \qquad (5)$$

where $k+1$ – order of the approximating polynomial; $[a_k, a_{k-1}, \ldots a_0]$ – coefficients of the approximating polynomial; $(n-0.5k)$, ... $(n+0.5k)$ – number of samples by which the approximation is performed; $u[n-0.5k],\ldots,u[n+0.5k]$ – input voltage samples.

Then the moment of zero crossing can be found by solving the following equation:

$$a_k(n_{cz}+\eta_{cz})^k + a_{k-1}(n_{cz}+\eta_{cz})^{k-1} + \ldots + a_0 = 0, \qquad (6)$$

where n_{cz} – sample number (integer value) closest to the intersection by the input signal to zero, found per the equation (6); η_{cz} – correction of the reference number corresponding to the moment when zero intersects the input signal (n_{cz}).

If there are several roots in the solution of equation (6), the root corresponding to the intersection of zero by the input signal must satisfy the condition $n_{cz} \le (n_{cz} + \eta_{cz}) \le n_{cz} + 1$, where $(n_{cz} + \eta_{cz})$ – is the root of equation (6). The value of the time of zero crossing for the obtained roots of equation (6) is defined as:

$$t_{cz} = (n_{cz}+\eta_{cz})T_S = n_{cz}T_S + \Delta t_{cz}. \qquad (7)$$

Since the sinusoidal signal is close to linear upon crossing zero, it is justified to use approximating polynomials of the first order. The fraction of the sampling period between the voltage transition through zero and the first sample after this is determined by the expression [5]:

$$t_{cz} \cong T_S n_{cz} + T_S \eta = \frac{T_S n_{cz} u[n_{cz}+1] - T_S(n_{cz}+1)u[n_{cz}]}{u[n_{cz}+1]-u[n_{cz}]}. \qquad (8)$$

Then the correction values η_0, η_m for (2) and (3) can be found using the expression for the first transition for the i-th measurement interval:

$$\eta_{cz,0} = \frac{u[n_{cz,0}]}{u[n_{cz,0}]-u[n_{cz,0}-1]}, \qquad (9)$$

and for the first transition for (i+1)-th measurement interval:

$$\eta_{cz,m} = \frac{u[n_{cz,m}]}{u[n_{cz,m}]-u[n_{cz,m}-1]}, \qquad (10)$$

where $u[n_{cz,0}]$ – first positive voltage indication on the i-th measurement interval; $u[n_{cz,0} - 1]$ – last negative voltage indication on the (i – 1)-th measurement interval; $u[n_{cz,m}]$ – first positive voltage sample on the (i + 1)-th measurement interval; $u[n_{cz,m} - 1]$ – last negative voltage sample on the i-th measurement interval.

Then the correction value η:

$$\eta = \frac{u[n_{cz,0}]}{u[n_{cz,0}]-u[n_{cz,0}-1]} - \frac{u[n_{cz,m}]}{u[n_{cz,m}]-u[n_{cz,m}-1]}. \qquad (11)$$

To simplify further transformations, we take the following notation:

$$\begin{cases} X_1 = u[n_{cz,0}-1], & X_2 = u[n_{cz,0}], \\ X_3 = u[n_{cz,m}-1], & X_4 = u[n_{cz,m}]. \end{cases} \qquad (12)$$

With expressions (9), (10) and (12), the value of the frequency of the input voltage is determined by (see expression (2)):

$$\begin{cases} f = \dfrac{mf_S}{N+\eta}, \\ \eta = \dfrac{X_2}{X_2-X_1} - \dfrac{X_4}{X_4-X_3}. \end{cases} \qquad (13)$$

There are papers [2], [5], [6] devoted to the zero crossing technique and its modifications. However, there is no analysis of the sources of errors caused by the imperfection of the ADC in these papers. There are also no analytical relations for estimating additional errors caused by the linearization of the voltage near the crossing of zero. The estimates given in these studies were carried out by mathematical modeling. This makes it difficult to analyze and evaluate the measurement error when applying this method.

III. Measurement errors

The imperfection of the ADC leads to an inaccurate definition of η and, consequently, to the error f_1. We denote by $\Delta\eta$ the absolute error η due to the imperfection of the ADC. Using formula (13), we express the relative error of frequency measurement δ_f via $\Delta\eta$:

$$\delta_f = -\frac{\Delta\eta}{N+\eta} \cong -\frac{\Delta\eta}{N}. \qquad (14)$$

The approximate equation (14) can be used in calculations, since usually $\Delta\eta << N$. The error of the correction can reach 100 %, when the derivative of the voltage approaches zero when passing through zero. This situation can be caused by voltage harmonics. In the worst-case $X_2 = X_4 = 0$ (zero is considered positive); $X_1 = X_3 = q$ (q – value of the quantum of the ADC). To eliminate this, the ADC samples must be digitally filtered. To do this, it is sufficient to use a low-pass or bandpass filter. Since the derivatives of the harmonics are proportional to their number, the derivatives of the harmonics at the output of the second-order filter will be inversely proportional to their number. It is further assumed that preliminary digital filtering of ADC samples is used, which are then used in frequency measurement. During digital filtering, both IIR and FIR filters can be used to suppress noise and harmonics. It should be considered that the implementation of a FIR filter with a narrow bandwidth requires a much larger filter order than during IIR filter implementation. Express $\Delta\eta$ with ΔX_1, ΔX_2, ΔX_3, ΔX_4 – absolute sampling errors of X_1, X_2, X_3, X_4:

$$\Delta\eta = \sum_{i=1}^{4} \left| \frac{d\eta(X_1, X_2 \ldots X_4)}{d(X_i)} \right| \Delta X_i \,, \qquad (15)$$

where [X_1, X_2, X_3, X_4] – are the influencing values; ΔX_i – marginal deviation of the influencing value X_i.

With (13) and (15):

$$\Delta\eta = \frac{X_2 \cdot \Delta X_1 - X_1 \cdot \Delta X_2}{(X_2 - X_1)^2} - \frac{X_4 \cdot \Delta X_3 - X_3 \cdot \Delta X_4}{(X_4 - X_3)^2} \,. \qquad (16)$$

Then the measurement error of the frequency:

$$\delta_f \cong \frac{X_4 \cdot \Delta X_3 - X_3 \cdot \Delta X_4}{N(X_4 - X_3)^2} - \frac{X_2 \cdot \Delta X_1 - X_1 \cdot \Delta X_2}{N(X_2 - X_1)^2} \,. \qquad (17)$$

Relations between (16) and (17) do not consider the nature of the deviations of the influencing quantities and their relationship among themselves. That is why, the direct application of this approach will lead to an overestimation of the error. Considering the sources of error in measuring instantaneous voltage values and their effect on the error of frequency measurement. The following components are caused by the non-ideal ADC:

- the additive component caused by the zero offset of the ADC (Δ_{ADD});
- the multiplicative component caused by the deviation of the transfer coefficient of the ADC converter and the deviation of the ADC reference voltage source from the nominal value (the total error (δ_{MUL}));
- linearity caused by the nonlinearity of the ADC conversion function (Δ_{LIN});
- the quantization error of the ADC.

For the additive component, the error in measuring instantaneous voltage values:

$$\Delta X_1 = \Delta X_2 = \Delta X_3 = \Delta X_4 = \Delta_{ADD} \,. \qquad (18)$$

The values of the voltage readings for which the corrections of η_0 are calculated, along η_m modulus are close to each other. Since these samples are located near one level (zero), we can write:

$$|X_1| = |X_2| = |X_3| = |X_4| \,. \qquad (19)$$

From expression (17), (18) and (19), the error in measuring the frequency from the additive component of the ADC error is zero.

For the multiplicative component of the error, considering condition (19):

$$\Delta X_1 / X_1 = \Delta X_2 / X_2 = \Delta X_3 / X_3 = \Delta X_4 / X_4 = \delta_{MUL} \,. \qquad (20)$$

Thus, the error in measuring the frequency is affected only by the quantization errors and the linearity errors of the ADC. The linearity error can vary strongly (up to 100% in the case of pipeline ADC and successive approximation ADC), even for neighboring values of the ADC conversion function.

The error in measuring the frequency due to the aperture delay and the aperture bounce of the ADC can be neglected, as shown by the results of mathematical modeling.

Let us proceed to estimate $\Delta\eta_{max}$ – maximum value of the module $\Delta\eta$. That is why we formulate a number of propositions concerning $\Delta\eta$, on which the assessment will be based $\Delta\eta_{max}$. First, all absolute errors in ADC samples are pairwise independent of each other and can be positive or negative. Therefore, in the worst case, they are summed up. The maximum value of these errors (ΔX_x) in the least significant bit (LSB) of the ADC are determined by the quantization error (1/2) and *INL* - the integral nonlinearity of the ADC:

$$\Delta X_x = 0.5 + INL \,. \qquad (21)$$

Secondly, the increments in the output signal of the ADC are the same and are determined by the derivative of the sinusoidal voltage at the zero point, q is the LSB of the ADC and the sampling frequency f_s:

$$X_2 - X_1 \cong X_4 - X_3 \cong \frac{2\pi f_1 U_m}{q \cdot f_S} \,, \qquad (22)$$

where U_m – amplitude of sinusoidal voltage.

A LSB, in units of voltage, is defined:

$$q = \frac{2U_{ref}}{2^{N_{ADC}-1}} = \frac{2U_m}{2^{N_{ADC}-1}} \,, \qquad (23)$$

where N_{ADC} – ADC resolution; coefficient 2 in the numerator considers the requirements of normative documentation: the upper limit of the voltage measurement range equals to twice the nominal value of the voltage.

With (16), (21) – (23) equation for $\Delta\eta_{max}$:

$$\Delta\eta_{\max} = \frac{(1 + 2INL)\, f_S}{\pi f_1 2^{N_{ADC}-1}} \,. \qquad (24)$$

Then (14) и (24), $\delta_{f,max}$ – maximum relative error of the frequency measurement, caused by the imperfection of the ADC:

$$\delta_{f,\max} = \frac{\Delta\eta_{\max}}{N} = \frac{(1 + 2INL)\, f_S}{\pi N f_1 2^{N_{ADC}-1}} \,/ \qquad (25)$$

With (2), approximately expressing N:

$$N \cong \frac{m f_s}{f_1} \,. \qquad (26)$$

And substituting its expression in (25). Thus, we finally find:

$$\delta_{f,\max} = \frac{\Delta\eta_{\max}}{N} = \frac{1+2INL}{\pi m 2^{N_{ADC}-1}}. \quad (27)$$

IV. Simulation Results

Equation (27) shows that $\delta_{f,max}$ is determined only by the ADC parameters characterizing its nonideality, and the duration of the measurement interval in the periods of the fundamental voltage component. Table 1 presents the calculated values of $\delta_{f,max}$ for some types of ADC.

TABLE I
ADC PARAMETERS AND FREQUENCY MEASUREMENT ERROR

ADC Parameter	ADC		
	ADS7812PB	ADS7279	ADS8513IB
Resolution	12	14	16
Offset error, max (mV)	6	1.25	6
Full-scale error, max (%)	0.25	-----	0.25
Gain error, max (%)	-----	0.25	-----
INL, max (LSB)	0.5	1.0	2.0
Relative frequency measurement error, δf (%)			
Mathematical modeling (Simulink 8)	$6{,}3\cdot10^{-4}$	$8{,}1\cdot10^{-7}$	$2{,}6\cdot10^{-5}$
Equation (27)	$3{,}1\cdot10^{-3}$	$1{,}2\cdot10^{-3}$	$4{,}9\cdot10^{-4}$

The error is calculated for a number of modern ADC from Texas Instruments (ADS7812PB, ADS7279, ADS8513IB). When performing the calculation, it is assumed that the only source of instrumental error is the non-ideality of the ADC: the quantization error and non-linearity of the conversion function. The parameters of the considered ADC are taken from the technical documentation posted on the manufacturer's website – Texas Instruments (www.ti.com). The results corresponding to the line "Mathematical modeling" were obtained by applying by Simulink 8.

The results are obtained with the following values of the parameters: the sampling frequency – 10240 Hz; measurement time – 0,2 sec (10 nominal periods of input voltage); fundamental frequency is 50 Hz; range of input voltage variation corresponds to the input range of the ADC.

V. Conclusion

For zero crossing technique, a modification is considered that includes linear approximation of the voltage near the zero crossing to calculate the correction to improve the accuracy of the frequency measurement. An analytical expression (27) is obtained that allows one to estimate the maximum error of frequency measurement caused by the imperfection of the ADC: the quantization error and the nonlinearity of the conversion function. The results of mathematical modeling show that the application of the expression (27) obtained allows us to estimate from above the value of the measurement error in the frequency using this method. The expression can be used to select the ADC resolution that provides voltage frequency measurements with a given error.

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