# Operability of an interval criterion for determining the steady state of the transient in the measuring circuit

Anton Volkov, Anastasia Umnyagina, Yury Bekhtin, Alexey Lupachev, Tatiana Kovaleva

***Abstract* — This paper deals with the measuring circuit which model is represented by a dynamic element of the first order. It is shown that the determining the end of the transient for the element of the automatic control system is different with respect to the instrument's measuring circuit. The most common criteria to detect the end of the transient in the measuring circuit are considered using different types of information. The standardized interval criterion is shown to be widely used. The dependence determining the critical value of the maximum time constant of the measuring circuit is found.**

***Index Terms* — transient analysis; measuring circuit; duration of the transient; dynamic element of the first order; the critical time constant.**

## I. Introduction

The effective specialized measuring means (MM) [1] is assumed to be separated in accordance with a measurement type (static or dynamic) and an operation mode (steady or transient).

For example, the digital thermometers from Testo AG use the Auto Hold function, when the instrument automatically keeps the measurement readings on the display after the expiration of the temperature equalization time [2].

The formal criterion of the transient end in the measuring circuit (MC) makes sense only if there is a priori information about the parameters of the MC model and the specified value of the admissible dynamic error (in absolute $\Delta_0$ or relative $\delta_0$ representations). Then, the setting time $T_Y$ is determined as follows:

$$T_Y = \tau \ln(1/\delta_0) = \tau\beta, \quad (1)$$

where $\beta$ is the factor that depends on the permissible dynamic error of the first kind, $\beta = \ln(1/\delta_0)$.

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The criterion (1) which associates the duration of the transient with the required error is considered to be “the classical method” to determine the transition time for a signal at the output of a filter in the input circuits of digital voltmeters. This method is widely used to estimate operability of digital instruments [3].

Determining the transient time (i.e., the steady state) is an urgent task for many branches of technology [4, 5].

The postulate of the measuring object model is very important for improving the accuracy and operability of measuring means. The measuring object model in the form of the transient response that can be represented as the sum of the exponents is very important for dynamic mode (DM) in practice. The dynamic element of the first order with the largest time constant $\tau$ is dominant among the components of this sum [6].

These models are commonly used in MC for temperature tests of power transformer windings in a regular mode [7] when measuring the direct current (DC) resistance of power transformer windings [8].

One of the measurement technology tasks is developing a criterion of the end of the transient (ET) using instantaneous values $a_k = a(t_k)$ of the transient at equally spaced discrete moments $t_k = k\Delta T_D$; $\Delta T_D = t_k - t_{k-1}$; $1 \leq k \leq T_Y/\Delta T_D + 1$.

During analysis of quality parameters of the transient with known $A_\infty$, the transient time is a posteriori determined by a registered process curve as the time of achieving the given transient accuracy ($1.00 \pm \Delta$), where $\Delta$ is an allowable transient error and $A_\infty = 1.00$ [1]. Usually, the limits of this approximation are 1…5 % of the steady-state value.

The steady-state value $A_\infty$ of the transient signal is unknown a priori and determined during the measuring experiment. Therefore, it is impossible to determine the current dynamic error $\Delta_{DYN}(t) = A(t) - A_\infty$ before the experiment; hence, the one can be calculated a posteriori only.

## II. On-line Criteria for ET Determining

Determining the moment of the ET in the on-line mode taking into consideration the current information is based on criteria which provide with an analysis of the instantaneous values of the transient for a given time interval [5, 9]. These criteria are defined in [10] as interval criteria (IC) and widely used in tests for electric machines. In the steady-state case, the following measurements are carried out.

- The resistance of windings of a power transformer under a constant current according to the Russian standards GOST 3484.1-88 [11] using the ammeter and voltmeter in a practically cold state. It requires a steady-state of working current in the MC.
- Temperature of the power transformer windings. The corresponding test is used for heating of the windings according to the GOST 3484.2-88 [11]; the test must be carried out in a steady-state thermal regime.

Another example of information about transients is the results of the thermo-balance and thermo-vacuum tests for space vehicles [12]. Significant errors in accuracy of determining the time of steady-state can lead to significant errors for the parameters' values of the object model or excessive energy costs [9].

The Russian standard GOST 3484.1-88 [11] considers that the criterion of ET under lack of a priori information about the transformer windings time constant and of the steady-state operating current in the measuring circuit is as follows: *"The steady-state reading of the instrument should be considered as a reading that varies by no more than 1% of the counted value for at least 30 seconds."* The time constant of the power transformer (PT) is changed in the range [10…500] seconds depending on the rated power of PT (from 10 to 100000 kVA).

This criterion uses information on the permissible value of changes in the signal of the transient at a given observation interval. The IC assumes that the result is independent of the MC time constant value. It is assumed here that the derivative of the transient has a monotonous and decreasing character in modulus.

The duration constant of the $j^{th}$ observation interval of the transient dynamics is designated as $\Delta T_O(j)$, as it is shown on Fig. 1. According to the criterion, the allowable change of the signal $A(t)$ at this interval is $\Delta A_O(j)$ and

$$\Delta T_O(j) = t_2 - t_1 = \text{const } \Delta T_O;$$
$$\Delta A_O(j) = B_{\%} A_\infty/100, \qquad (2)$$

where $t_1$ is the beginning of the $j^{th}$ observation interval;
$t_2$ is the end of the $j^{th}$ observation interval;
$B_{\%}$ is the maximum relative deviation that is appointed by the criterion, $B_{\%} = 1$ % [11].

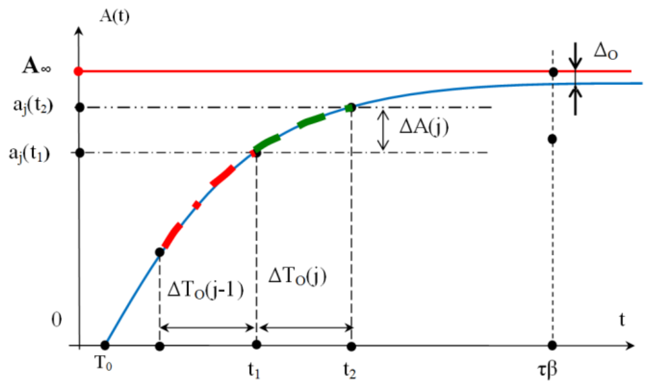


Fig. 1. Position of the observation interval on the transient curve.

The time of ET $T_{TR}$ which is defined by the interval criterion can be attributed as both the beginning $t_1$ and the end $t_2$ of the observation interval $\Delta T_O$. The relative increment of the transient signal in the time $\Delta T_O$ can be determined for two end points of the $j^{th}$ observation interval as follows:

$$\rho_{j1} = 100[a_j(t_2) - a_j(t_1)]/a_j(t_1)$$
$$\rho_{j2} = 100[a_j(t_2) - a_j(t_1)]/a_j(t_2), \qquad (3)$$

it yields

$$\rho_{j2} = \rho_{j1}/\nu_j, \quad \nu_j = a_j(t_2)/a_j(t_1) > 1.$$

The steady-state occurs when the criterion $B_{\%} \geq \rho_{j\kappa}$ is fulfilled at the $j^{th}$ interval at the moment $T_{TR}$

$$T_{TR} = \min\{\arg[B_{\%} \geq \rho_{j2}]\}. \qquad (4)$$

The figure 2 shows on how signal increments $\rho_{j2}$ are changed in dependency on the number of adjacent non-overlapping observation intervals. Selection a set of time constants for numerical simulation is conducted from the set $\{\tau_1 = 6.25$ s; $\tau_2 = 12.5$ s; $\tau_3 = 25$ s; $\tau_4 = 50$ s, $\tau_5 = 100$ s; $\tau_6 = 200$ s$\}$; the choice is determined by the following inequalities:: $\tau_1 < \tau_2 < \tau_3 < \Delta T_H < \tau_4 < \tau_5 < \tau_6$. Thus, all the curves are joined in the point $j(\tau)$ with a specific value. So, for example:

$\tau_1 = 6.25$s: $T_Y \geq 29$ s and hence, the steady-state is reached at the second observation interval and $j(\tau_1) = 2$;

$\tau_2 = 12.5$ s: $T_Y \geq 57.5$ s and $j(\tau_2) = 3$, etc.

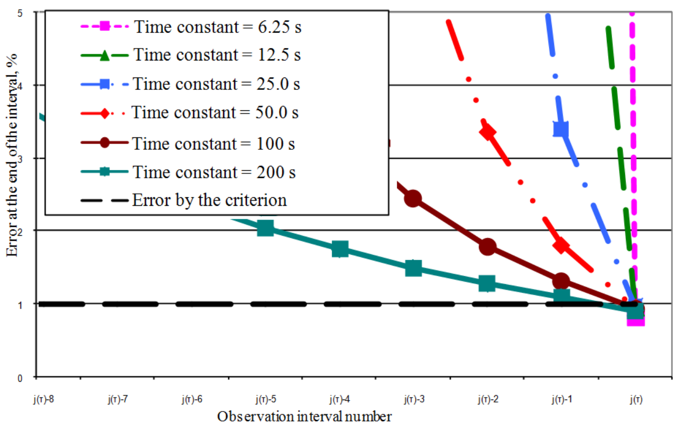


Fig. 2. Transient increment changing at the end of the observation interval.

We consider the application of IC of the transient end in modern microprocessor systems.

The interval criterion of the steady-state measuring mode is used, for example, in the "Transformer testing instrument PIT" [14]. It determines the steady-state operating current in the MC automatically. The current measuring by the method of "the voltmeter-ammeter" is terminated if the values of 30 consecutive observations (carried out in 1s) do not differ by more than 1%.

In the "testo 105" digital thermometer of Testo AG [2], the duration of the observation interval is set manually from a number of values: 5; 10; 15 or 20 s at an allowable level of transient change at 0.2 ° C.

## III. Interval Criterion for ET

The work [14] transforms the interval criterion for linear MC as: *"no more than 1% of the counted value for at least 30 seconds."*:

$$B_{\%} \geq 100[a_j(t_1 + \Delta T_O) - a_j(t_1)]/a_j(t_1) \qquad (5)$$

The figure 3 shows the transient curves with different values of $\tau$ and the dedicated observation intervals satisfying the interval criterion (5).

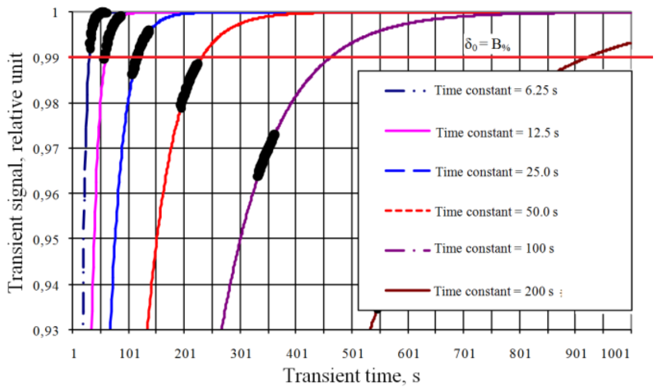


Fig. 3. Observation interval position on the transient curves with different time constants.

The position of these intervals beyond the allowable dynamic error illustrates the fact that the interval criterion (6) does not guarantee the steady-state at this interval with the error $\delta_0 = B_{\%}$ required by this criterion. As can be seen from Fig. 3 for $\tau \geq 50$ s, the desired interval lies beyond the allowable error.

The relative errors are designated a posteriori at the ends of observation interval as $\delta_1(\tau, j)$ at the beginning and $\delta_2(\tau, j)$ at the end:

$$\delta_1(\tau, j) = 100[A_\infty - a_j(t_1)]/A_\infty; \qquad (6)$$
$$\delta_2(\tau, j) = 100[A_\infty - a_j(t_2)]/A_\infty. \qquad (7)$$

The figure 4 shows the changes $\delta_2(\tau, j)$ depending on the number of adjacent non-overlapping observation intervals. All the curves (similar to Fig. 2) are also joined at the same point $j(\tau)$.

Table 1 presents the error $\delta_2(\tau,j)$ at the end of the observation interval for the analyzed changing range of $\tau$. It is approximated by a linear dependence with accuracy no more than ± 1% of:

$$\delta_2^*(\tau,j) = 0{,}0299\tau - 0{,}328, \%, \text{при } \tau \geq 25\text{s}. \qquad (8)$$

For reference, Table 1 shows the duration of the analyzed transient obtained by "the classical criterion" (1) with $\delta_0 = 1$ %, what gives as it is known $\beta = 4.6$.

An analysis of the influence of possible values rates of the MC on the relative error $\delta_\kappa(\tau, j)$ for the j[th] observation interval at which the IC conditions are fulfilled makes it possible to distinguish two subranges of $\tau$. In the lower subrange, for $\tau_{min} < \tau < \tau_{cr}$, the ET is correctly executed. In the upper subrange, $\tau_{cr} < \tau < \tau_{max}$, the ET is not executed correctly. Therefore, the considered IC does not have invariance to the perturbing action in the form of parametric uncertainty.

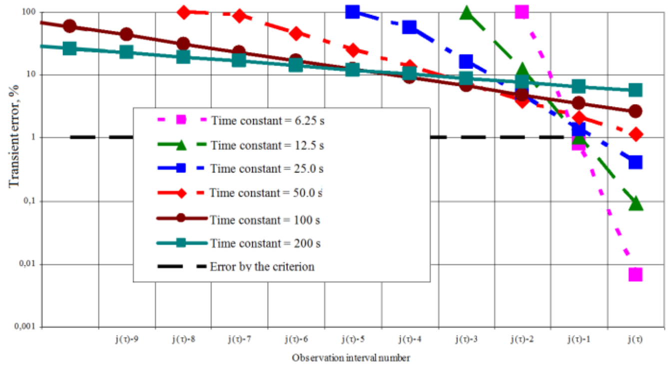


Fig. 4. Transient error changing.

TABLE I

THE ERROR AT THE END OF THE OBSERVATION INTERVAL

| τ, s: | 6.25 | 12.5 | 25 | 50 | 100 | 200 |
|---|---|---|---|---|---|---|
| $j(\tau)$ | 2 | 3 | 4 | 8 | 12 | 19 |
| Ratio of the relative errors of transient | $\rho_{j2} \leq B_{\%}$ | | | | | |
| $\Delta_2(\tau, j)$, %: | 0.007 | 0.094 | 0.42 | 1.2 | 2.8 | 5.8 |
| $T_{TR}$, s | 60 | 87 | 137 | 222 | 360 | 571 |
| $T_Y$, s | 28.8 | 57.5 | 115 | 230.3 | 460.5 | 921 |
| Ratio of the dynamic errors | $\Delta_2(\tau, j) < \delta_0$ | | | $\Delta_2(\tau, j) > \delta_0$ | | |
| ET | $T_Y < T_{TR}$ | | | $T_Y > T_{TR}$ | | |

## IV. Working Area IC

The limits of the interval criterion has been found when the MC time constant exceeds the certain critical time $\tau_{CR}$. As the first approximation, this constant can be found from (8) if to consider $\delta_2^*(\tau,j)$ as the allowable relative error at the end of the interval, i.e., $\delta_2^*(\tau_{CR}, j) = 1$ %. Thus, $\tau_{CR.1\%} = 44.4$ s. It is possible to prove that there is the relation $\Delta T_O = \lambda\tau_{CR}$ when the interval criterion (5) can be still applied. This ratio is $\lambda = 30\text{s}/44.4\text{s} \approx 0.68$.

Nevertheless, our studies have shown that it is possible to find analytically the dependence between the critical time constant and the influencing parameters by finding the extreme value of the functional:

$$\varphi = \{(\delta_\kappa(\tau, j) - B_{\%})/B_{\%}\}^2 \rightarrow \min. \qquad (9)$$

Usage of the least squares criterion (9) gives strictly positive value with a pronounced extreme value equal to zero for $\Delta T_{O.Opt}$.

Thus, the refined critical value of the time constant MC has to be calculated as follows:

$$\tau_{CR} = \Delta T_O / \ln [2/(1 + 0.01B_{\%})]. \tag{10}$$

If $B_{\%}$ = 1 % then the value of the parameter $\lambda^* = \Delta T_{O.Opt}/\tau_{CR} = 0.683$.

The value $\lambda^*$ which is found analytically differs no more than 1.5 % of the maximum value $\lambda^*_{MAX} = \ln 2 = 0.693$.

For example, the value of the critical time constant (10) is equal to $\tau_{CR.an}$ = 43.9 s for the interval criterion (1% / 30s) [11]. Hence, the analytical value of $\tau_{CR.an}$ differs from the previously found value $\tau_{CR.1\%}$ not more than by 1.1%.

It should be noted that there is no definition of the critical time constant for the measuring circuit in [15-18], and moreover, the steady-state value is not determined in the on-line mode.

## V. SUMMARY

It has been discovered that the interval criterion recommended by some modern standards does not define the required moment throughout the range of possible time constant of the MC model.

In spite, the relation between the transient observation interval duration and the critical value of the time constant was found, our next research will be concentrated to provide the reliability of the suggested criterion. Moreover, it is necessary to extend the interval criteria for ET in measuring circuits which are characterized by parametric and structural uncertainty.

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