Distributed order PID optimization in comparison with fractional PID optimization based on performance and robustness indices

Boris B. Jakovljević, Tomislav B. Šekara, Milan R. Rapaić, Zoran D. Jeličić

***Abstract*—This paper presents a comparison of most frequently used criteria for optimizing distributed order PID and fractional PID controllers. The problem of interest is how to provide load disturbance rejection under constraints to performance and robustness. The comparison of optimal controllers via different optimization criteria has been performed for different non-integer processes. The optimal controllers are obtained via optimization by genetic algorithm.**

I. Introduction

PID controllers have been very popular among engineers and scientist, [1], [2]. Their popularity became significant due to their simplicity, small number of parameters to be tuned and the fact that it is not necessary to know the exact model of process in order to properly tune the controller. The development in the field of fractional calculus introduced a new PID-like structures that have proved themselves in different aspects [3], [4], [5]. These PID modifications introduce complexity in the controller's structure, but in the same way improve robustness in comparison with performance or vice versa. In order to tune an optimal controller some aspects must be taken into account. System in demand must conform itself to the constraints on robustness and performance as well to stay stable. These aspects of performance and robustness can be various and in this paper only the most commonly used will be presented.

The most usual problems for which the controllers are tuned are the reference tracking or disturbance rejection. The frequently considered system is given in Fig. 1. This paper treats the problem of finding an optimal controller for unit step disturbance rejection. The optimal controllers for different demands on robustness and performance are found and compared in the sense of response quality.

The paper is organized as follows: Section II gives an overview of PID-like structures, Section III presents the most commonly used performance and robustness indices, while Section IV presents the optimization problems and Section V shows the optimization results. Last section concludes the paper.

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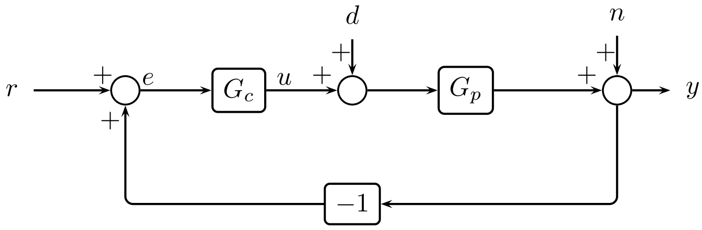

Fig. 1. Plant $G_p(s)$ with dPID controller $G_c(s)$. Reference signal is denoted by r (assumably r=0), load disturbance by d, measurement noise by n, control signal by u, error signal by e and system output by y.

II. PID-like structures

Classical PID controller can be described by different structures but the most usual is parallel controller which is defined by the following differential equation as

$$u_{pid}(t) = k_p e(t) + k_i \int_0^t e(\tau)d\tau + k_d \frac{de(t)}{dt} \ , \qquad (1)$$

where k_p, k_i, k_d represent proportional gain, integral gain, derivative gain, respectively. $e(t)$ represents error signal, while $u_{pid}(t)$ represents control signal.

The transfer function of the classical PID with noise cancellation filter is given by

$$G_{pid}(s) = \frac{k_p + k_i \frac{1}{s} + k_d s}{T_f s + 1} \qquad (2)$$

where T_f represents filter's time constant.

Compared to a classical PID controller, FPID has two additional degrees of freedom – the integration order α and the differentiation order β. These two newly introduced degrees of freedom provide more flexibility in applications but in the same time complicate the controller's structure.

$$G_{fpid}(s) = \frac{k_p + k_i \frac{1}{s^\alpha} + k_d s^\beta}{(T_f s + 1)^\beta} \ . \qquad (3)$$

By generalization of fractional PID controller, one can acquire a distributed order PID. The distributed order PID controller with noise cancellation filter is given by

$$G_{\text{dopid}}(s) = \frac{1}{T_f s + 1} \sum_{i=0}^{N-1} k_i s^{-1+i\Delta\alpha} \ . \qquad (4)$$

with k_i being the gains of differintegrators and *odd* integer $N > 1$ being the number of differintegrators and a positive step length calculated as $\Delta\alpha = \frac{2}{N-1}$

Proceedings of 4th International Conference on Electrical, Electronics and Computing Engineering, IcETRAN 2017, Kladovo, Serbia, June 05-08, ISBN 978-86-7466-692-0

The more details about the DOPID structure can be found in [3], [6].

III. Performance and robustness indices

Integral of absolute error (IAE) is perhaps the most frequently used as performance measure, defined as

$$IAE = \int_0^\infty |e(t)|\, dt. \tag{5}$$

One more frequently used performance indexes when considering PID controllers is Integral of error (IE) and it is defined as

$$IE = \int_0^\infty e(t) dt. \tag{6}$$

There exists a direct dependence between integral gain, k_i, and integral of error (IE) in the case when load disturbance is constant unit signal [1].

Very often used performance index regarding noise influence is maximal sensitivity to measurement noise M_n

$$M_n = \max_{\omega \geq 0} \left| \frac{-G_c(j\omega)}{1 + G_c(j\omega)G_p(j\omega)} \right| \tag{7}$$

as it equals $\frac{k_d}{T_f}$ for classical PID controllers and DOPID controllers and $\frac{k_d}{T_f^\beta}$ for FPID controllers.

If one wants to guarantee robustness to model uncertainties, maximum sensitivity M_s was considered to be less than a specified value. M_s is given with

$$M_s = \max_{\omega \geq 0} \left| \frac{1}{1 + G_c(j\omega)G_p(j\omega)} \right|_{\omega=\omega_s}. \tag{8}$$

The index that constraints the maximum resonant peak is defined as

$$Q = \max_{\omega \geq 0} \left| \frac{k_i \frac{G_p(j\omega)}{j\omega}}{1 + G_c(j\omega)G_p(j\omega)} \right|_{\omega=\omega_q}. \tag{9}$$

Šekara and Mataušek showed in [8] that constraining Q to be less than 1.01 provides acceptable values of IAE.

One more frequently used robustness index, often in combination with M_s is Maximum complementary sensitivity defined as

$$M_p = \max_{\omega \geq 0} \left| \frac{G_c(j\omega)G_p(j\omega)}{1 + G_c(j\omega)G_p(j\omega)} \right|_{\omega=\omega_p}. \tag{10}$$

Perhaps, one not so conventional performance index is the maximization of minimum of amplitude characteristic of controller, of course over complete frequency range, as it reduces the influence of load disturbance. This can be described as

$$M_{min}A = \max_{\omega \geq 0}(\min |G_c(j\omega)|)\ . \tag{11}$$

The last index coincides with maximization of proportional gain k_p at classical PID controllers [7].

IV. Optimization problems

For all optimization problems in this research the issue was to find an optimal controller FPID and DOPID in order to reject load disturbance and closed loop system is stable. The optimization in all cases was performed by particle swarm optimization algorithm. The processes that were analyzed are the following non-integer processes

$$G_{p1}(s) = \frac{-(1-s)\ln(s)}{(s+1)^3}, \tag{12}$$

$$G_{p2}(s) = \frac{-(1-s)\ln(s)}{(s+1)^2}. \tag{13}$$

The following optimization problems were analyzed.

A. Optimization problem 1 (OP1)

This optimization problem is formulated for DOPID controller with $N = 7$ differintegrators of the form

$$G_{\text{dopid},7}(s) = \frac{1}{sT_f + 1}\left(k_0 \frac{1}{s} + k_1 \frac{1}{s^{2/3}} + \ldots + k_5 s^{2/3} + k_6 s\right). \tag{14}$$

The controller is optimized with the following concerns

$$\max_{k_0, k_1, \ldots, k_6, \omega_q, \omega_s} k_3\ , \tag{15}$$

$$T_f = \frac{k_d}{M_n^{\max}}\ , \tag{16}$$

$$M_s = \left| \frac{1}{1 + G_c(j\omega)G_p(j\omega)} \right|_{\omega=\omega_s} \leq M_s^{\max}\ , \tag{17}$$

$$Q = \left| \frac{k_i \frac{G_p(j\omega)}{j\omega}}{1 + G_c(j\omega)G_p(j\omega)} \right|_{\omega=\omega_q} \leq Q^{max}\ , \tag{18}$$

$$k_i \geq k_i^{min}, i \in \{0, ..., N-1\}\ . \tag{19}$$

The same was observed for FPID controller, so the only difference in this case is in the first equation, i.e. in the controller structure.

B. Optimization problem 2 (OP2)

This optimization problem is formulated for DOPID controller of the same structure as for optimization problem 1.

$$\max_{k_0, k_1, \ldots, k_6, \omega_q, \omega_s} k_0\ , \tag{20}$$

The controller is optimized with the following concerns

$$T_f = \frac{k_d}{M_n^{\max}}\ , \tag{21}$$

$$M_s = \left| \frac{1}{1 + G_c(j\omega)G_p(j\omega)} \right|_{\omega=\omega_s} \leq M_s^{\max}\ , \tag{22}$$

$$Q = \left| \frac{k_i \frac{G_p(j\omega)}{j\omega}}{1 + G_c(j\omega)G_p(j\omega)} \right|_{\omega=\omega_q} \leq Q^{max}\ , \tag{23}$$

$$k_i \geq k_i^{min}, i \in \{0, ..., N-1\}\ . \tag{24}$$

The same was observed for FPID controller, so the only difference in this case is in the first equation, i.e. in the controller structure.

C. Optimization problem 3 (OP3)

This optimization problem is formulated for DOPID controller of the same structure as for optimization problem 1.

$$\max_{k_0,k_1,\ldots,k_6,\omega_q,\omega_s} k_0 \ , \tag{25}$$

The controller is optimized with the following concerns

$$T_f = \frac{k_d}{M_n^{\max}} \ , \tag{26}$$

$$M_s = \left| \frac{1}{1+G_c(j\omega)G_p(j\omega)} \right|_{\omega=\omega_s} \leq M_s^{\max} \ , \tag{27}$$

$$M_p = \max_{\omega \geq 0} \left| \frac{G_c(j\omega)G_p(j\omega)}{1+G_c(j\omega)G_p(j\omega)} \right|_{\omega=\omega_p} \leq M_p^{\max} \ , \tag{28}$$

$$k_i \geq k_i^{min}, i \in \{0, ..., N-1\} \ . \tag{29}$$

The same was observed for FPID controller, so the only difference in this case is in the first equation, i.e. in the controller structure.

D. Optimization problem 4 (OP4)

This optimization problem is formulated for DOPID controller of the same structure as for optimization problem 1.

The controller is optimized with the following concerns

$$\max_{k_0,k_1,\ldots,k_6,\omega_q,\omega_s} k_3 \ , \tag{30}$$

$$T_f = \frac{k_d}{M_n^{\max}} \ , \tag{31}$$

$$M_s = \left| \frac{1}{1+G_c(j\omega)G_p(j\omega)} \right|_{\omega=\omega_s} \leq M_s^{\max} \ , \tag{32}$$

$$Q = \left| \frac{k_i \frac{G_p(j\omega)}{j\omega}}{1+G_c(j\omega)G_p(j\omega)} \right|_{\omega=\omega_q} \leq Q^{max} \ , \tag{33}$$

$$M_p = \max_{\omega \geq 0} \left| \frac{G_c(j\omega)G_p(j\omega)}{1+G_c(j\omega)G_p(j\omega)} \right|_{\omega=\omega_p} \leq M_p^{\max} \ , \tag{34}$$

$$k_i \geq k_i^{min}, i \in \{0, ..., N-1\} \ . \tag{35}$$

This optimization problem has one more constraint compared to all other optimization problems considered so far. For this problem it was impossible to find neither optimal FPID nor classical PID controller.

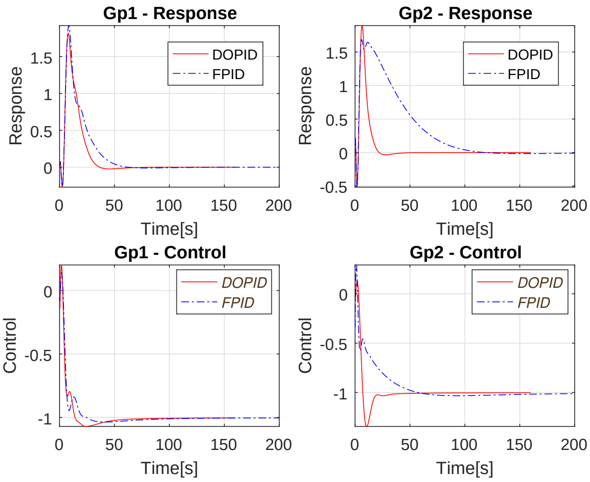

Fig. 2. Optimization problem 1: Response and control signal on unit step load disturbance d(t) for process G_{p1} on the left and response and control signal for process G_{p2} on the right controlled by FPID and DOPID under constraints $M_s^{max} = 2$, $M_n^{max} = 20$ and $Q^{max} = 1.01$.

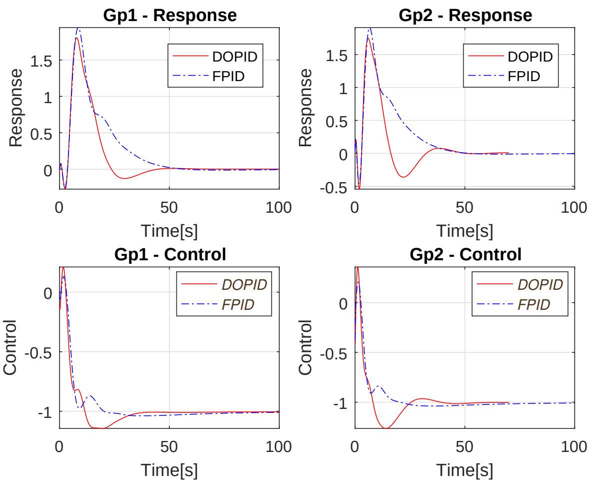

Fig. 3. Optimization problem 2: Response and control signal on unit step load disturbance d(t) for process G_{p1} on the left and response and control signal for process G_{p2} on the right controlled by FPID and DOPID under constraints $M_s^{max} = 2$, $M_n^{max} = 20$ and $Q^{max} = 1.01$.

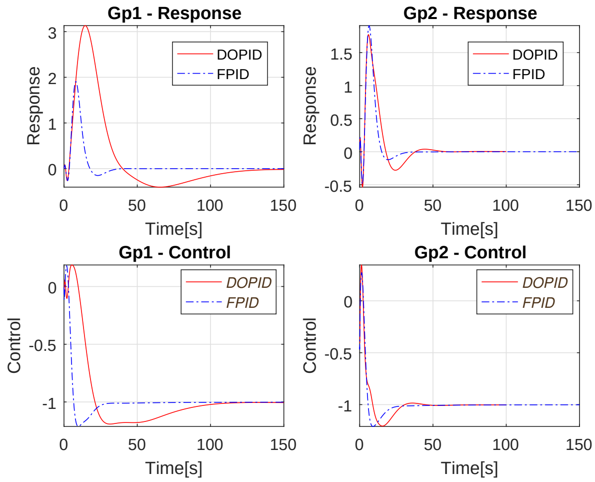

Fig. 4. Optimization problem 3: Response and control signal on unit step load disturbance d(t) for process G_{p1} on the left and response and control signal for process G_{p2} on the right controlled by FPID and DOPID under constraints $M_s^{max} = 2$, $M_n^{max} = 20$ and $M_p^{max} = 1.3$.

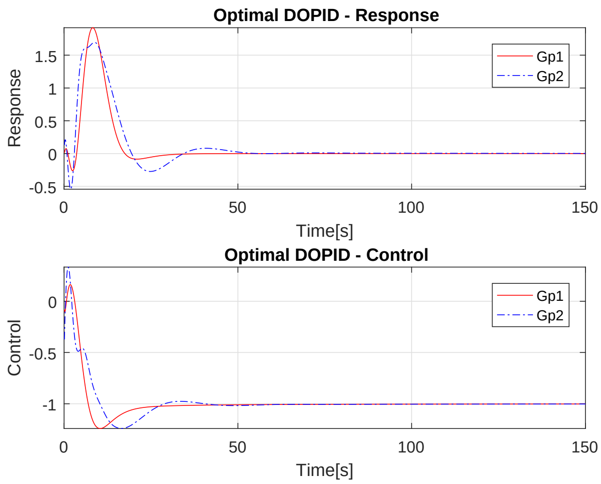

Fig. 5. Optimization problem 4: Responses on unit step load disturbance d(t) for processes G_{p1} and G_{p2} above and control signals for processes G_{p1} and G_{p2} on bottom, for optimal DOPID controller under constraints $M_s^{max} = 2$, $M_n^{max} = 20$ and $M_p^{max} = 1.3$.

TABLE I
OPTIMAL PARAMETERS OF FPID AND DOPID CONTROLLER FOR PROCESSES G_{p1}, G_{p2} FOR CONSTRAINTS VALUES $M_n^{max} = 20$, $M_s^{max} = 2$, $M_p^{max} = 1.3$ AND $Q^{max} = 1.01$ FOR OPTIMIZATION PROBLEMS 1, 2, 3; RESULTS FOR FPID CONTROLLER ARE MARKED WITH *.

OP	**Proc.**	k_0/k_i	k_1/α	k_2/β	k_3/k_p	k_4	k_5	k_6/k_d	T_f	**IAE**	M_p **/ Q**
OP1	G_{p1}	4.96e-2	-2.15e-3	2.53e-2	0.317	-3.58e-2	4.81e-3	6.22e-1	3.11e-2	22.33	1.06
OP1	$G_{p1}*$	3.68e-2	1	1	0.378	/	/	4.85e-1	1.89e-2	29.37	1.03
OP1	G_{p2}	8.19e-2	7.09e-3	4.14e-3	0.361	1.23e-2	7.35e-4	2.74e-1	1.37e-2	14.48	1.13
OP1	$G_{p2}*$	1.66e-2	1	0.94	0.256	/	/	3.56e-2	1.34e-2	64.87	1.04
OP2	G_{p1}	6.14e-2	7.90e-3	4.23e-2	2.19e-1	2.15e-2	-5.74e-4	6.66e-01	3.33e-2	19.61	1.16
OP2	$G_{p1}*$	4.05e-2	1	1	3.84e-1	/	/	2.8e-1	1.39e-2	26.78	1.06
OP2	G_{p2}	7.04e-2	2.65e-1	-7.65e-1	1.26	-5.11e-1	4.42e-4	5.00e-01	2.5e-2	18.32	1.42
OP2	$G_{p2}*$	4.50e-2	1	1	3.79e1	/	/	1.71e-1	8.53e-3	24.70	1.03
OP3	G_{p1}	3.82e-2	-8.86e-2	5.91e-2	3.53e-1	-4.08e-1	-6.20e-1	4.65e-1	2.32e-2	78.12	**2.47**
OP3	$G_{p1}*$	9e-2	1	1	3.86e-1	/	/	4.85e-1	2.42e-2	15.37	**1.16**
OP3	G_{p2}	7.27e-2	2.24e-1	-7.49e-1	1.32	-4.93e-1	-6.80e-2	5.13e-1	2.57e-2	18.42	**1.05**
OP3	$G_{p2}*$	1.05e-1	1	1	3.76e-1	/	/	2.88e-1	1.44e-2	13.48	**1.13**

V. RESULTS

Table I shows the optimal parameters of all controllers for all optimization problems considered in this paper as well as the values of indices of interest. Figs. 2, 3, 4 show the responses and control signals for all optimization problems for all considered processes. Fig. 5 shows the results of response and control signals for optimal DOPID controllers for both observed processes.

It can be seen that for most cases it is obvious to consider optimal DOPID instead of optimal FPID controller as one obtains either less fluctuating, either faster response. Fo optimization problem 4 that includes more constraints it is only possible to find optimal DOPID controller.

VI. CONCLUSION

This paper shows the comparison of optimal DOPID and FPID controllers for different optimization problems. It is shown that there are benefits of using DOPID controller for most cases with regular number of constraints. For the case for larger number of constraints it is only possible to find optimal DOPID controller.

ACKNOWLEDGEMENTS

The authors would like to gratefully acknowledge the financial support of Serbian Ministry of Science and Education, under grants TR33020 (T.B.Š), TR32018 (M.R.R, B.B.J, Z.D.J.), TR33013 (M.R.R), III44008 (B.B.J).

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