# Analysis of a method for mitigating miscorrelations in target tracking algorithms

Asem Al-Hasaeri, Predrag Tadić, Aleksandra Marjanović and Željko Đurović

***Abstract*—Correct association of observations with objects is one of the most important and difficult tasks in algorithms for tracking multiple targets in the presence of false observations. We consider one modification of the standard Kalman filter which aims to reduce the tracking error by explicitly taking into account the fact that the probability of correct association is less than one. Through computer simulations, we analyze the performance of this method and assess if, and under what conditions, it can improve upon the standard Kalman filter.**

***Index Terms*—Target tracking, Kalman filtering, Correct correlation probability, Data association.**

## I. Introduction

RESEARCH in field of automatic target tracking started in the 1970s. Back then, the interest in this topic was driven mainly by aerospace applications: radar, sonar, navigation, guidance and air traffic control. Today, multiple target tracking (MTT) techniques are used extensively in many diverse arenas, such as image processing, oceanography, autonomous vehicles, robotics and biomedicine [1]. Many approaches to MTT have been proposed in the literature over the years. The main difference between them is in the way they solve the two main tasks which make up any MTT problem: *filtering* and *assignment*.

The task of *filtering* is to provide (preferably optimal) estimations of a state vector consisting of positions, velocities and possibly accelerations of individual targets, given some noisy measurements. Most commonly, Kalman filters (KF) are used for this purpose [3]. If the dynamics of the target or the mapping from states to measurements are described by non-linear functions, techniques other then the KF must be employed. These include the extended KF, unscented KF or particle filters [1, 2].

The sequence of estimates for one individual target constitutes a *track*. *Assignment* of observations to tracks is the second major task in MTT. Ideally, in each scan, every target produces exactly one observation. Even then, it might not be trivial to decide which observation originated from which target (if targets move in clusters, for example). Moreover, there is always the possibility that a target will not be detected in one or more scans, or that false detections might occur. There are several ways to tackle this problem. Typically, a prediction for the value of the state at time $t$ is made. Next, we define a region around the prediction, called the gate, in which we expect to record the next observation. If no observations are actually found within this gate, then we have a track with a missing observation. Otherwise, if the gate includes only one observation, we assign it to the considered track. Finally, if there are multiple observations within the same gate, we must decide which of them should be associated with the current track. The easiest way to do this is the sequential nearest neighbor (SNN) method. It consists of computing statistical distances between each possible prediction-observation pair at each scan, and choosing the assignment which minimizes the overall distance. The simplicity of SNN makes it appealing from a practical viewpoint. However, since only data from the current scan are considered, it is prone to *miscorrelation* – the erroneous assignment of extraneous observations to tracks, which results in poor tracking. The probability of miscorrelation can be reduced if we are willing to defer the final assignment decision until we obtain data from the next several scans. This is the basic idea behind the *track branching* and *multiple hypothesis testing* approaches [3], but these are not the focus of the present paper.

Several techniques have been proposed to account for the possibility of miscorrelation in the basic SNN method [3]. We consider one such methods, based on modifying the state and error covariance matrix estimates [4]. Obviously, if no miscorrelation occurs, modifying the estimates can only deteriorate the performance, since in this case the (unmodified) KF attains the minimum mean-squared error. This poses the following question: is the use of the method from [4] indeed justified and under what circumstances? The aim of this paper is precisely to provide an answer to this question.

In the remainder of this paper, we assume a KF-based MTT scheme with SNN assignment. We ignore the problems of track initiation, confirmation and deletion, and assume that the number of targets is constant and known in advance. We further assume that targets are detected independently of each other with some constant probability $P$ in each scan, and that there is a non-zero probability of false detections (observations not originating from any of the targets).

In the following section, we describe the basic MTT scheme in more detail. Then, in Section III we review the miscorrelation-mitigating technique from [4]. In Section IV we analyze the performance of these techniques through computer simulations. The final section concludes the paper.

## II. Basic Operation: Gating, Assignment, Filtering

Consider a target moving along the $x$-axis with unknown acceleration and assume that measurements $y$ of its position $x$ along the axis are sampled with period $T$. The corresponding state-space model is given by:

(1)

(2)

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where $q$ and $v$ are independent zero-mean, white Gaussian processes of known variances $\sigma_q^2$ and $\sigma_v^2$, modeling the unknown target accelerations and measurement noise, respectively, $\mathbf{x}$ is the state vector

(3)

(we use bold upright characters to denote vectors and matrices), and the matrices $\mathbf{A}$ and $\mathbf{C}$ are given by

(4)

Obviously, real-life targets move in 3 dimensions, but since the accelerations along each axis are typically regarded as mutually independent, it is numerically more convenient to consider three separate 2D models than one 6D model [3]. For the sake of clarity, we ignore the movement of targets in the other two dimensions from now on, noting that the required generalizations for the 3D case are trivial.

The general solution to the recursive, minimized mean-squared error estimation problem of the linear model (1)-(2) is given by the Kalman filter [5]:

(5)
(6)
(7)
(8)
(9)

where is the predicted state at time $t$ given the observations up to time 1, is the final estimation at time $t$, denotes the estimation error covariance matrix, is the Kalman gain, and

(10)

When $N$ targets are being tracked simultaneously, we have $N$ KFs running in parallel. We denote the estimates of the $i$-th filter as $\hat{\mathbf{x}}_i$ and $\hat{\mathbf{P}}_i$.

### A. Prediction and gating

Starting from the state and error covariance estimates from the previous scan at time $t-1$, the predictions are computed from (5)-(6) for all tracks. The predicted positions are $\hat{y}_i(t) = \mathbf{C}\hat{\mathbf{x}}_i(t|t-1)$. Assuming for a moment that the observations have already been properly assigned to each track, the variance of the $i$-th *residual* $r$ ( is given by

(11)

where denotes the (1,1) element of the covariance matrix.

An observation $y$ is said to be within the *gate* of track $i$ if

(12)

where is the gating constant. For a typical choice of $= 3$, the theoretical probability of a valid observation satisfying the gating test is 99.97%, assuming a Gaussian error model. Therefore, all observations outside the gate can safely be eliminated as candidates for assignment to the current track.

### B. Assignment

We now return to the problem of pairing observations $y_j$, $N$, with tracks (note that the number of observations $N_o$ need not equal the number of tracks $N$, due to missed detections and false alarms). The likelihood function associated with the assignment of the $j$-th observation to the $i$-th track is [3]

(13)

SNN assignment can now be posed as a problem of maximizing the $g_{ij}$ terms, which is equivalent to minimizing the following squared distances:

(14)

The optimal assignment is found by enumerating over all possible assignments and determining the one with the minimal sum of squared distances. Gating simplifies the procedure, since it eliminates *a priori* the combinations which violate the gating constraint.

### C. Filtering

Let binary values $\gamma_i$ be equal to 1 if some observation was paired with the $i$-th track, and 0 otherwise. For those tracks which had observations assigned to them, the update is given by (7)-(9). As for the remaining tracks, whose gates did not contain any observations, we have no choice but to set the final estimates to be equal to the predicted values. It can be shown that the error covariance matrix remains bounded, provided that the probability of missed observations is below a certain critical threshold [6]. Mathematically, the update step is given by

(15)

(16)

where is the observation associated with the $i$-th track.

## III. Accounting for Miscorrelation

It has been suggested in the literature that miscorrelation represents an additional source of error that should be accounted for in the KF covariance matrices [3]. In this section we describe one such technique from [4], and in the following section we assess if it indeed does improve the quality of tracking.

Let $P_{cc}$ be the probability of correct correlation, i.e. that the observation associated with a track truly did originate from the tracked target. Define the modified state estimate to be

$$\hat{\boldsymbol{\xi}} = P_{cc}\hat{\mathbf{x}}(\gamma = 1) + (1 - P_{cc})\hat{\mathbf{x}}(\gamma = 0) \tag{17}$$

(we have omitted the subscript $i$ and the dependence on time $t$ for notational convenience). This new value is the linear combination of two estimates: the first is computed assuming correct correlation, and the second corresponds to the case of a missed observation. The resulting covariance matrix is shown to be [3]

$$\boldsymbol{\Pi} = \sum_{\gamma=0}^{1} p(\gamma)[\mathbf{P}(\gamma) + \hat{\mathbf{x}}(\gamma)\hat{\mathbf{x}}^T(\gamma)] - \hat{\boldsymbol{\xi}}\hat{\boldsymbol{\xi}}^T \tag{18}$$

where $p(1) = 1 - p(0) = P_{cc}$, and allows for the increase in the covariance as a function of the difference in the estimates provided by the two correlation alternatives.

The implementation of (17) and (18) requires the knowledge of $P_{cc}$. A simple way of approximating this probability is given by [7]:

$$\hat{P}_{cc} = \frac{g_{ij}}{\sum_{l=1}^{N_T} \min(g_{lj}, g_{ij}) + \beta} \tag{19}$$

where $\beta$ represents the new observation density, i.e. the expected number of new observations (true targets or false alarms) that arise per unit volume per unit scan time, and $N_T$ is the number of tracks for which the gating condition is satisfied. In other words, $\hat{P}_{cc}$ is the ratio of the likelihood associated with the chosen observation-to-track assignment to the sum of the likelihoods associated over all possible assignments for that observation (see [3] for an intuitive explanation on the use of minimization in the denominator).

## IV. Simulation Results and Discussion

Throughout this section, we denote the original KF as algorithm A1, and the modification given by (17)-(18) as algorithm A2. The test case is a single object moving in a plane, along a trajectory shown in Fig. 1. The total duration of the experiment was 150 seconds, with a sampling period of 1 second. Target dynamics correspond to those of a military aircraft, with speeds of 300 m/s and accelerations of up to 2G.

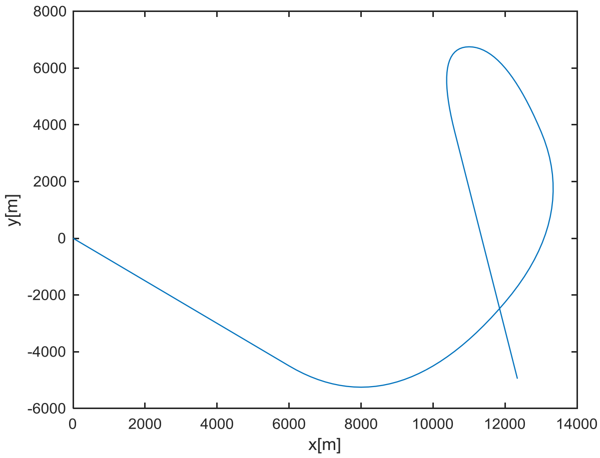


Fig. 1. True trajectory of the tracked target.

At each scan, a response from the true target is received with probability $P_D < 1$, and false observations appear with some density $\beta$. Although only one target is present, there is still the possibility of none to several observations being received within the gate at each scan. SNN was used for data association.

Figs. 2 and 3 show the true and estimated values of the $x$ coordinate for each of the two algorithms. The parameters were $\beta = 4.5 \cdot 10^{-5}$ and $P_D = 0.8$. Rather than estimating the correct correlation probability $P_{cc}$ on-line, we adopted a fixed value of $P_{cc} = 0.9$ for algorithm A2.

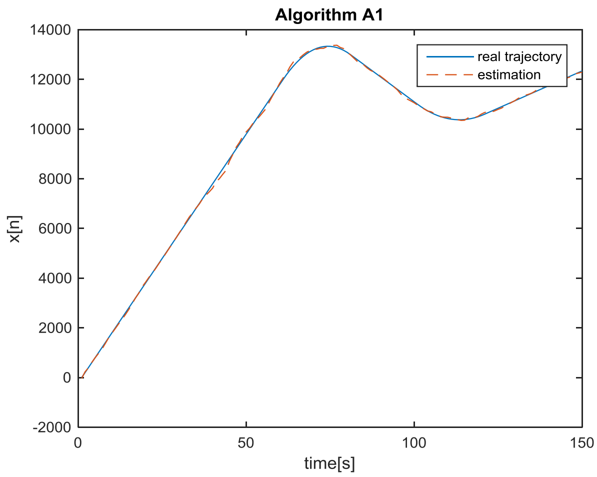


Fig. 2. Performance of the standard Kalman filter.

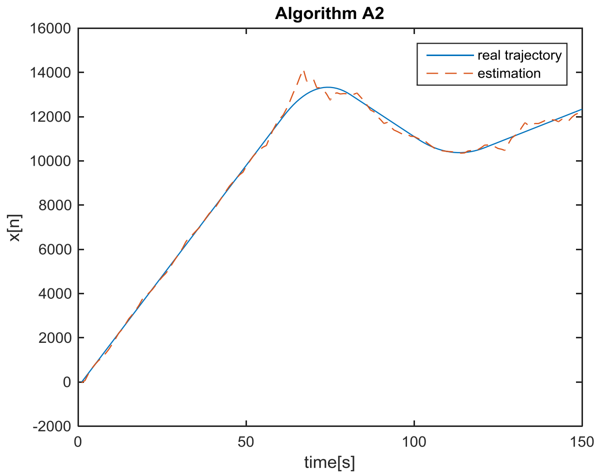


Fig. 3. Performance of the modified Kalman filter.

Figs. 4 and 5 show the mean squared errors of A1 and A2, given by

$$J = \frac{1}{N}\sum_{i=1}^{N}\left((x[i] - \hat{x}[i])^2 + (y[i] - \hat{y}[i])^2\right) \tag{20}$$

as a function of $P_{cc}$ (the operation of A1 is independent of $P_{cc}$ and thus its mean squared error curve is flat in both figures). Detection probability was kept at a constant value of $P_D = 0.8$. Results in Fig. 5 were obtained for $\beta = 2 \cdot 10^{-5}$. They show that the performance of A2 improves for high values of $P_{cc}$, but is constantly worse than that of A1. This can be attributed to a relatively low density of false alarms in this case - since the possibility of miscorrelation is rare, it is better to ignore it altogether, which is precisely what A1 does.

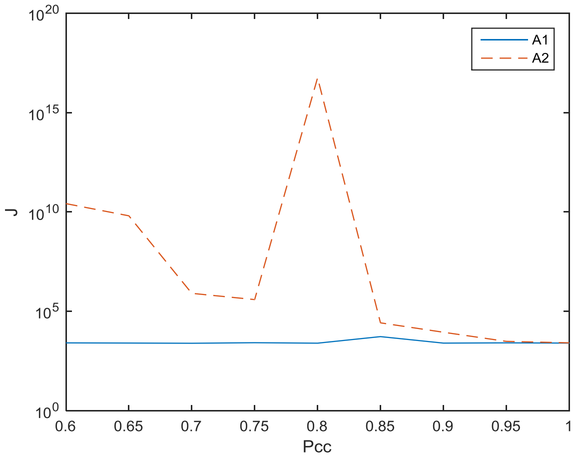


Fig. 4. Mean squared errors for $\beta = 2 \cdot 10^{-5}$.

In Fig. 6 the false observation density was increased to $\beta = 6 \cdot 10^{-5}$. As expected, the performance of A1 deteriorates due to the higher number of false observations. It is interesting to note that A2 is still inferior to A1 for most of the values of $P_{cc}$, but for $P_{cc} > 0.95$ A2 actually outperforms A1. This emphasizes the importance of properly choosing $P_{cc}$ and suggests that special care must be taken to construct an effective way of estimating this parameter on-line.

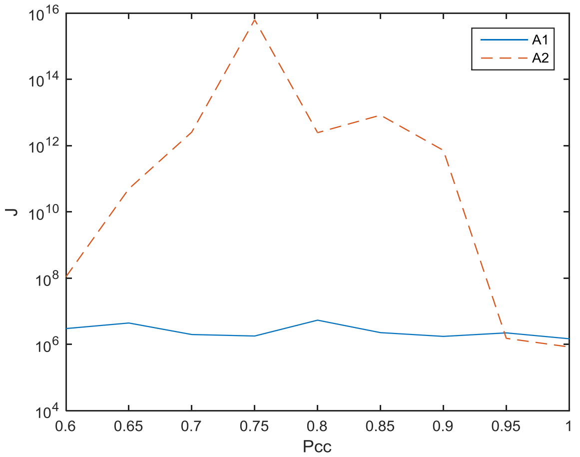


Fig. 5. Mean squared errors for $\beta = 6 \cdot 10^{-5}$.

## V. Conclusion

The paper analyzes the possibility of improving the performance of Kalman filters for target tracking, in the presence of both missed detections and false observations originating from noise, clutter etc. Modifications of the basic KF algorithm, which explicitly account for the possibility of imperfect data association, have the potential to improve tracking performance. However, their performance strongly depends on the proper choice of the probability of correct correlation. The simplest approach is to choose a constant value *a priori*, but a wrong choice can actually lead to a significant deterioration in performance compared with the standard, non-modified Kalman filter. It is therefore crucial to have reliable methods for on-line estimation of this key parameter.


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