A Survey on Sequential Monte Carlo Particle PHD Algorithm

Zvonko Radosavljević, Branko Kovačević

***Abstract*— This paper presents and evaluates a Sequential Monte Carlo (SMC) approach for target estimating. In a surveillance situation, the origin of each measurement is uncertain. Each measurement may be a false (clutter) measurement, or it may be a target detection. Probabilistic Hypothesis Density (PHD) methods are usually used to discriminate between the clutter and the target measurements. Clutter measurement density is an important parameter in this process. The values of the clutter measurement density in the surveillance space are rarely known a-priory, and are usually estimated using sensor data and track information. This method enhances target tracking performance, compared with standard particle filter. Simulation results validate this approach.**

***Index Terms*—PHD filter, target tracking, SMC methods.**

I. Introduction

In a target tracking, the number of targets and their trajectories vary with time due to targets appearing and disappearing. In this article we only consider the standard setting where sensor measurements at each instance have been preprocessed into a set of points or detections [1]. The tracker receives a random number of measurements due to detection uncertainty and false alarms. Tracks are initialized and updated using random measurements of unknown origin, thus each track may be a true track (following a target) or a false track [2]. For nonlinear and non-Gaussian models, particle filtering [3], (or Sequential Monte Carlo - SMC) [4], [5], has become a practical and popular numerical technique to approximate the Bayesian tracking recursions. This is due to its efficiency, simplicity, flexibility, ease of implementation, and modelling success over a wide range of challenging applications. It represents the target distribution with a set of samples, known as particles, and associated importance weights, which are then propagated through time to give approximations of the target distribution at subsequent time steps.

A number of target tracking algorithms are used at present in various tracking applications, with the most popular being the Joint Probabilistic Data Association Filter (JPDAF), Multiple Hypothesis Tracking (MHT) [6,7,8], and Random Finite Set (RFS) based multitarget filters [9-12]. The MHT tracker attempts to keep track of all the possible association hypotheses over time. In this paper, we proposed a sequential Monte Carlo methodology [13,14].

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The paper is organized as follows: after introduction considerations, a problem statement is presented in Section II. Section III derives the PHD filter approach, followed by the one recursion of SMC Particle PHD filter approach which is presented in Section IV. Concluding remarks are presented in Section V.

II. Problem Statements

Target tracking is a dynamic state estimation problem, in which the state varies with time. This procedure involves determining the existence and the trajectory of possible targets in the surveillance space, by comparing random measurements received by the sensor with the applicable stochastic models. We use superscripts τ to denote tracks, and also targets followed by tracks.

A. Targets model

In this paper we use the Markov Chain model [15] for the propagation of the probability of target existence [16]. This model assumes that a target may exist and when it does it is always detectable with a given probability of detection P_D, or it may not exist. During the targets maneuvering, the motion can be changed at random times. The trajectory of a target can be described at any time by one of predefined dynamic models. A linear model is considered. The targets trajectory state, for the linear system, at time k, evolves by:

$$x_k^\tau = F_k\, x_{k-1}^\tau + v_k^\tau \tag{1}$$

where F_k is the propagation matrix, and the process v_k^τ noise is a zero mean and white Gaussian sequence with covariance. At each scan k, the sensor returns a random number of the random target and clutter measurements. The measurement of the existing and detectable target is taken with a probability of detection.

B. Sensors model

At each scan the sensor returns a random number of the random target measurements and a random number of the random clutter measurements. The measurement of the existing and detectable target is taken with a probability of detection P_D is given by the following equation:

$$y_k^\tau = Hx_k^\tau + w_k^\tau \tag{2}$$

where H is measurements matrix and the measurements noise w_k^τ is zero mean and white Gaussian sequence with covariance matrix R.

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C. Measurements model

Measurements may originate from the targets as well as from other objects [17]. The clutter measurements follow the Poisson distribution. We assume that the uniform intensity of the Poisson process at point y in the measurement space, termed here the clutter measurement density and denote by $\rho(y)$ is a priori known, or can be estimated using the sensor measurements. At time *k*, one sensor delivers a set of measurements denoted by $y_k = \{y_{k,j}\}_{j=1}^{M_k}$. Denote by Y^k the sequence of selected measurement sets up to including time *k*, $Y^K = \{Y^{k-1}, y_{k,1}, ..., y_{k,j}, ..., y_{k,M_k}\}$.

III. PROBABILITY HIPOTHESYS DENSITY FILTER

The Probability Hipothesys Density filter of an Random Finite Sets is the analog of the expectation of a random vector. The *Random Finite Set* Γ can be represented by a random counting measure $M_\Gamma(S)$ defined by the number of elements in set $\{Y\}$. Assuming there are *n* targets in the multi target system, each having state **s**, the density p_Γ of M_Γ can be used to represent the *RFS* Γ:

$$p_\Gamma(X) = \sum_{i=1}^{n} \delta(\mathbf{s}_i - X_i) \quad (3)$$

where $\delta(\mathbf{s}_i - X_i)$ denotes the Dirac delta function centered at *X*. The *PHD* is then the first moment of the above is $D_\Gamma(X) = E\{p_\Gamma(X)\}$. The first moment density *PHD* is given by the equation:

$$D_{k|k}(X|Y^k) = \int_{X_k \in \mathbf{X}} f(k|k)(X|Y^k)dX \quad (4)$$

The expected number of targets in region *S* is then

$$N^{k|k} = E \cdot NoE\{\Gamma(k) \cap S\} = \int_S D_{k|k}(X|Y^k)dX \quad (5)$$

The *PHD* filter recursion is given in [10] and [11]. The predicted *PHD* is

$$\begin{aligned} &D_{k|k}(Y|Y^{k-1}) = \\ &= b_{k|k-1}(Y) + \int D_{k|k-1}(Y|X) D_{k-1|k-1}(X|Y^{k-1})dX \end{aligned} \quad (6)$$

where

$$D_{k|k-1}(Y|X) = d_{k|k-1}(X) f_{k|k-1}(Y|X) + b_{k|k-1\rangle}(Y|X) \quad (7)$$

and

- $b_{k|k-1}(Y)$: *PHD* of the spontaneous target birth,
- $d_{k|k-1}(X)$: probability of target survival,
- $f_{k|k-1}(YX)$: transition probability density,
- $b_{k|k-1}(Y|X)$: *PHD* of the targets spawned by existing targets.

After the new scan *k*, arives neasurements data $Y^k = \{y_1, ..., y_m\}$, the updated *PHD* is given by:

$$\begin{aligned} D_{k|k}(X|Y^k) = &\sum_{Y \in Y^k} \frac{P_D(X) D_k(Y)}{\lambda_k c_k(Y) + P_D(X) D_k(Y)} \times \\ &D_k(X|Y) + (1 - P_D(X) D_{k|k-1}(X|Y^{k-1}) \end{aligned} \quad (8)$$

where

$$D_k(Y) = \int f(k)[Y|X] D_{k|k-1}(X|Y^{k-1})dX \quad (9)$$

$$D_k(X|Y) = \frac{f(k)[Y|X] D_{k|k-1}(X|Y^{k-1})}{D_k(Y)} \quad (10)$$

and

- $P_D(X)$ is probability of detection,
- λ_k is average number of false alarms per scan, assuming a Poisson distribution,
- $c_k(Y)$ is distribution of each of the false alarms,
- $f(k)[Y | X]$ is sensor likelihood function.

At each time step, the *PHD* filter propagates not only the *PHD*, but also the expected number of targets.

Consequently, estimation of the multitarget state is accomplished by searching for the $\min\{N^{k|k}\}$ largest peaks of $D_{k|k}(X|Y^k)$.

IV. SEQUENTIAL MONTE CARLO PHD RECURSION

The *PHD* propagation involves multiple integrals that have no computationally tractable closed form expressions even for the simple case where individual targets follow a linear Gaussian dynamic model. Particle filtering techniques permit recursive propagation of the full posterior and have been used for near-optimal Bayesian filtering. The *SMC* implementation of the *Particle PHD* filter was given from [6]. For any $k \geq 0$ let $\{w_i(k), \xi_i(k)\}_{i=1}^{L(k)}$ denote a particle approximation of the *PHD*. The algorithm is designed such that the concentration of particles in a given region of the state space represents the expected number of targets in this region. We start with $L(k-1)$ particles, which are predicted forward to time *k*. At time *k*, we generate additional $J(k)$ new particles for exploring newborn targets. In the prediction step, we can get particle representation:

- $\{w_i(k|k-1), \xi_i(k)\}_{i=1}^{L(k-1)+J(k)}$

The update step maps the function with particle representation

- $\{w_i(k|k-1), \xi_i(k)\}_{i=1}^{L(k-1)+J(k)}$

into one with particle representation

- $\{w_i(k|k), \xi_i(k)\}_{i=1}^{L(k-1)+J(k)}$

by modifying the weights of these particles. Note that when implementing the resampling step, the weights are not normalized to 1 but sum to $\hat{N}^{k|k}$, the expected number of targets at time *k*. The procedure of the particle PHD filter is given as follows.

A. Prediction Step

For $i = 1,...,L(k-1)$ sample $\tilde{\xi}_i(k) \approx q(k)[\cdot|\tilde{\xi}_i(k-1), Y^k]$ and compute the predicted weights:

$$\tilde{w}_i(k|k-1) = \frac{\tau(k)(\tilde{\xi}_i(k), \xi_i(k-1))}{q_k(\tilde{\xi}_i(k)|\xi_i(k-1), Y^k)} w_i(k-1) \qquad (11)$$

For $i = L(k-1)+1,...,L(k-1)+J_k$, sample $\tilde{\xi}_i(k) \approx p(k)[\cdot|Y^k]$ and compute the weights of newborn particles:

$$\tilde{w}_i(k|k-1) = \frac{1}{J(k)} \frac{b(k)[\tilde{\xi}_i(k)]}{p(k)[\tilde{\xi}_i(k)|Y^k]} \qquad (12)$$

Where $\tau(k)[\cdot,\cdot] = d(k|k-1)[\cdot] f(k|k-1)[\cdot|\cdot] + b(k|k-1)[\cdot|\cdot]$ with $d(k|k-1)[\cdot]$, $f(k|k-1)[\cdot|\cdot]$ and denoting $b(k|k-1)[\cdot|\cdot]$ the same meaning as in (14), $p(k,\cdot)$ and $q(k,\cdot)$ are proposal densities and $b(k,\cdot)$ denotes the *PHD* of the spontaneous target birth.

B. Update Step

For each $y \in Y^k$ compute

$$C(k,y) = \sum_{j=1}^{L(k-1)+J(k)} \psi[k, y, \tilde{\xi}_j(k)] \tilde{w}_j(k|k-1) \qquad (13)$$

For $i = 1,...,L(k-1)+J(k)$ update weights

$$\tilde{w}_i(k) = \\ = [1 - P_D + \sum_{y \in Y^k} \frac{\psi[k, y, \tilde{\xi}_i(k)]}{\lambda(k)c(k,y) + C(k,y)}] \tilde{w}_i(k|k-1) \qquad (14)$$

where $\psi(k, y, \xi) = P_D f(k, y|\xi)$ and $\lambda(k), c(k,\cdot), P_D$ and $f(k,\cdot|\cdot)$ denote the same thing as in (8-10).

C. Resampling step

Compute the total mass

$$\hat{N}^{k|k} = \sum_{j=1}^{L(k-1)+J(k)} \tilde{w}(k, j) \qquad (15)$$

Resample $\{\frac{\tilde{w}_i(k)}{\hat{N}^{k|k}}, \tilde{\xi}_i(k)\}_{i=1}^{L(k-1)+J(k)}$ to get $\{\frac{w_i(k)}{\hat{N}^{k|k}}, \xi_i(k)\}_{i=1}^{L(k-1)+J(k)}$.

In this filter, since the *PHD* is obtained for a frame at each scan, there is no state-to-state correlation between consecutive scans, so the choice of the proposal densities q_k and p_k is rather subjective. Besides, all the measurements including target-originated and clutter-originated measurements are used equally weighted in the update step. This is not efficient for the use of the particles.

In next section, we introduce a track labeling technique combined with the *PHD*, so that we can use the information from the previous scan and choose better proposal densities and adjust the weights for the measurements accordingly in the update step. A flow chart of *Particle PHD* algorithm is given by the *Figure* 1.

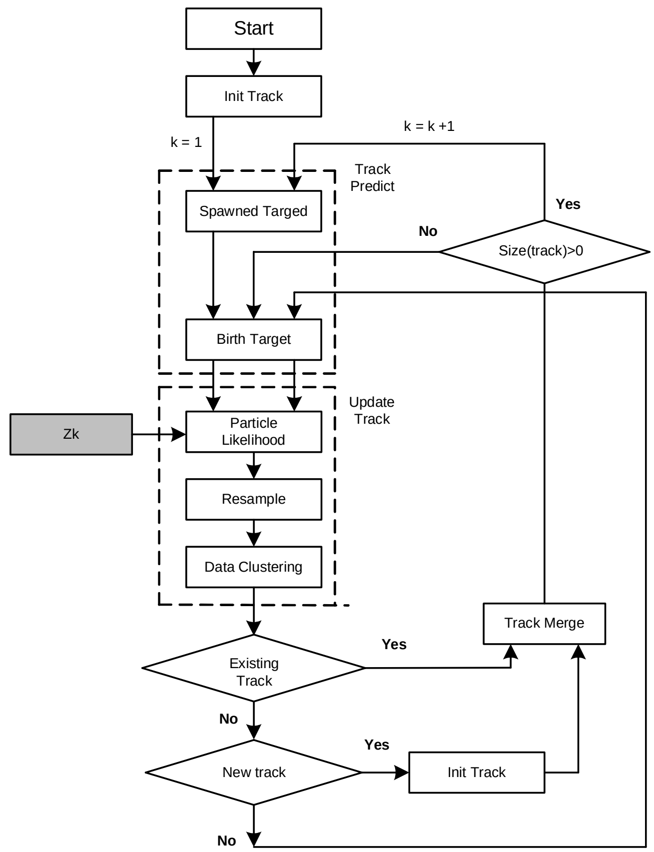

Fig. 1. Flow chart of Particle PHD algorithm .

V. RESULTS OF SIMULATIONS

The application selected for the study was a two dimensional (positions and velocities), four-state aircraft tracking problem in which the sensor observes both position coordinates. The area under surveillance was x=[0;1000][m] long and y=[0;1000][m] wide. Simulations have been performed by choosing 1000 particles. In the initialization process, creating a total of *N=1000* particles on the track, and schedules are to: position have to Gaussian distribution, and the speeds have uniform distribution in a circle with the center at zero and radius v_max. Both dimensions were assumed independent. The *SMC* Particle *PHD* parameters are calculated on-line according to the appropriate equations. Period of scaning is *T=1s*. Experiment were conducted on the basis of single target tracking (STT). Transition and process noise matrix are given by:

$$F=\begin{bmatrix}1 & T & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & T\\ 0 & 0 & 0 & 1\end{bmatrix}\ \mathrm{Q}=\begin{bmatrix}T^3/3 & T^2/2 & 0 & 0\\ T^2/2 & T & 0 & 0\\ 0 & 0 & T^3/3 & T^2/2\\ 0 & 0 & T^2/2 & T\end{bmatrix} \quad (16)$$

Measurements governed by the matrix:

$$H=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0\end{bmatrix} \quad (17)$$

The measurements noise vector are independent Gaussian noise with constant covariance matrix, given by the $R=\begin{bmatrix}\sigma_x^2 & 0\\ 0 & \sigma_y^2\end{bmatrix}$, where $\sigma_x^2=\sigma_y^2=25[m^2]$. The set of simulation with five targets and 100 Monte Carlo runs are presented. Each run lasted for 60 scans.

Results of simulation are presented by the diagrams of diagrams of confirmed true tracks over time and Root Mean Square target position error, respectivelly, compared proposed SMC Particle PHD and standard Particle Filter (*Figure 1.* and *Figure 2.*).

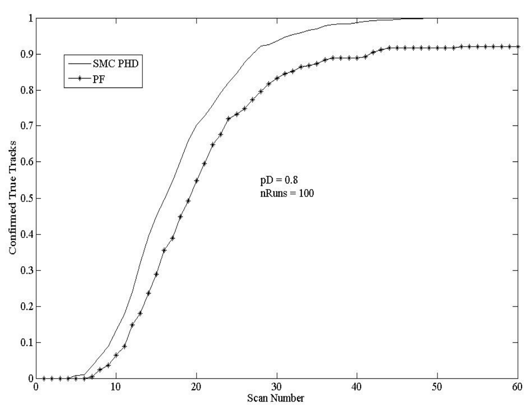

Fig. 2. Diagram of number of confirmed true track over time.

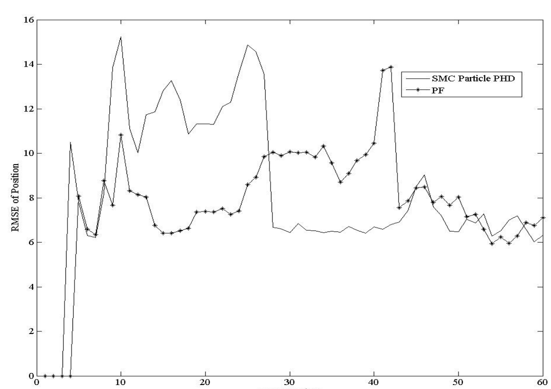

Fig. 3.Diagram of Root Mean Square Error of target position .

VI. CONCLUSION

In this paper, we suggested and compared a single target tracking *SMC* aproach named *Particle PHD* algorithms. Through an single target tracking numerical simulations with clutter, we showed that proposed methodology have good tracking performance (diagram of number of confirmed true track over time and root mean square error of target postion), compared with standard *Particle* Filter.

Although, a number of open questions remain, and various avenues are available for future research. Problems are the modification of the algorithms to deal with an unknown and variable number of targets, and the development of automatic initialisation (or detection) procedures.

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