Improved efficiency of matrix fill in higher order modeling of axially symmetric antennas

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***Abstract*—Analysis-time requirements of a method for analysis of axially symmetric metallic antennas are considered when using 1) modified and 2) max-ortho basis functions. Several improvements in implementation of max-ortho basis functions are inspected in order to increase the efficiency of the algorithm and reduce the matrix fill-in time. Benefits of these improvements are shown on the example of a thick dipole antenna.**

***Index Terms*—Axially symmetric antenna, integral equations, matrix fill time, max-ortho basis functions, method of moments.**

I. Introduction

DEMANDS for efficient and accurate full wave EM analysis of electrically large structures in frequency domain are continuously growing. Therefore, many researchers focus on inventing various techniques that can reduce memory and CPU time resources needed for these simulations.

Efficiency of EM simulations can be improved by using higher order basis functions, since it is shown that the same accuracy can be achieved with fewer number of unknowns if higher order bases are used instead of increasing the segmentation of the system [1]. However, increasing the maximum expansion order results at some point in poorly conditioned system matrix and unstable solution. This condition number can be significantly reduced if higher order basis functions with orthogonality properties are used. In [2] it is shown that condition number for max-ortho bases practically does not depend on used expansion order.

In [3], [4] a new method for analysis of axially symmetric metallic antennas is developed which uses electric field integral equation (EFIE) based on exact kernel in combination with Galerkin test procedure to specify the problem, truncated cones for precise geometrical modeling and modified higher order basis functions for approximation of the unknown surface currents. In [5] implementation of max-ortho bases is examined from the aspect of analysis-time requirements and average number of integration points per system matrix element.

In this work the goal is to reduce duration of matrix fill in case of max-ortho bases and to compare these results with results for modified bases. In Section II implementation of modified and max-ortho bases in the method from [4] is given. In Section III matrix fill-in time requirements of the method in case of modified and max-ortho bases are analyzed on the example of a thick dipole antenna for various expansion orders in single and double precision, and several improvements are considered for implementation of max-ortho bases.

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II. Description Of The Method

The method from [4] analyzes axially symmetric metallic antennas placed in vacuum, which can consist of one or more bodies. They are modeled with n_e building elements in the form of perfectly conducting right-truncated cone surfaces. The system is driven by voltage, delta generators of angular frequency ω. In inset of Fig. 1 thick dipole antenna is shown as an example of such structure. The unknown current along each of antenna building elements is approximated with modified higher order basis functions given with

$$f_i(s) = \begin{cases} \dfrac{1 \mp s}{2}, & i = 0,1 \\ s^i - s^{i-2}, & i = 2,...n \end{cases}, \quad (1)$$

where s is local coordinate that goes along the generatrix of the element and n is the expansion order of the current at considered element. Impedance integrals that figure in MoM system matrix based on exact kernel of EFIE, that correspond to the coupling between the l^{th} and k^{th} element, are given with

$$Z_{ljki} = \frac{\mathrm{j}\beta Z}{2}\int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1}\Big\{\Delta a_k\,\Delta a_l f_i(s_k) f_j(s_l)\cos(\pi p) + \Delta z_k\,\Delta z_l f_i(s_k) f_j(s_l) - \frac{1}{\beta^2}\frac{\mathrm{d}f_i(s_k)}{\mathrm{d}s_k}\frac{\mathrm{d}f_j(s_l)}{\mathrm{d}s_l}\Big\} g(R)\mathrm{d}s_k\mathrm{d}p\mathrm{d}s_l \quad ,(2)$$

where s_k, s_l and $p = p_k - p_l$ are local coordinates for the k^{th} and l^{th} element, $\beta = \omega\sqrt{\mu_0\varepsilon_0}$ is the phase coefficient , $Z = \sqrt{\mu_0/\varepsilon_0}$ is the intrinsic impedance, $R = |\mathbf{r}_l - \mathbf{r}_k|$ is the distance between the field point $\mathbf{r}_l$ and the source point $\mathbf{r}_k$, and $g(R) = e^{-j\beta R}/(4\pi R)$ is the Green's function.

If modified bases are used then impedance integrals (2) can be represented as a linear combination of impedance integrals due to power functions

$$P_{ljki} = \int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} s_l^j s_k^i g(R)\cos^{\alpha}(\pi p)\mathrm{d}s_k\mathrm{d}p\mathrm{d}s_l, \quad \alpha = 0,1 \,. \quad (3)$$

Using modified bases maximal expansion order that can be applied is $n = 10$ (20) in single (double) precision. If max-

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ortho bases are used it is expected to obtain stable results up extremely high orders. They can be expressed in terms of power functions, in which case max-ortho impedance integrals can again be obtained as a linear combination of integrals (3). Also, max-ortho bases can be expressed in terms of Legendre polynomials, as

$$f_i(s)=\begin{cases} L_0 \mp L_1 + \sum_{k=2}^{n} D_{ik} f_k(s), & i=0,1 \\ L_2(s)-L_0(s), & i=2 \\ L_3(s)-L_1(s), & i=3 \\ L_i(s)-L_{i-2}(s)+D_i f_{i-2}(s), & i=4,..n \end{cases}, \quad (4a)$$

$$D_i=\frac{2}{(2i-3)\langle f_{i-2},f_{i-2}\rangle},\quad D_{ik}=\frac{\langle L_0\mp L_1, f_k\rangle}{\langle f_k, f_k\rangle}, \quad (4b)$$

in which case max-ortho impedance integrals can be represented as a linear combination of impedance integrals due to Legendre polynomials and their derivatives, given with

$$Q_{ljki}=\int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} L_j(s_l)L_i(s_k)\cos^{\alpha}(\pi p)g(R)\mathrm{d}s_k\mathrm{d}p\mathrm{d}s_l \ ,(5a)$$

$$Q'_{ljki}=\int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1}\frac{\mathrm{d}L_j(s_l)}{\mathrm{d}s_l}\frac{\mathrm{d}L_i(s_k)}{\mathrm{d}s_k}g(R)\mathrm{d}s_k\mathrm{d}p\mathrm{d}s_l \ , \quad (5b)$$

where $\alpha=0$, 1. To evaluate these integrals recurrent formulas for Legendre polynomials and their derivatives are applied, which are given with

$$L_i(s)=\frac{1}{i}[(2i-1)sL_{i-1}(s)-(i-1)L_{i-2}(s)] \quad (6a)$$

$$L'_i(s)=L'_{i-2}(s)+(2i-1)L_{i-1}(s) \quad (6b)$$

where $L_0(s)=1$, $L_1(s)=s$, and $L'_i(s)=\mathrm{d}L_i(s)/\mathrm{d}s$. Since it is expected that evaluation of max-ortho impedance integrals in terms of *P*-integrals (3) results in insufficiently precise results, they are obtained as a linear combination of *Q*-integrals (5). In order to enable evaluation of system matrix elements with desired accuracy formulas for predicting the number of integration points developed in [4] are modified so that the number of points is increased.

III. Numerical Results

In what follows analysis time requirements for max-ortho and modified bases are considered on the example of a thick dipole antenna shown in inset of Fig. 1. Antenna dimensions from the inset are $h_{1,3}=a=0.15\,\mathrm{m}$ and $h_2=0.15\sqrt{2}\,\mathrm{m}$. Frequency of analysis is $f=300$ MHz, number of unknowns is $N=767$, and used expansion orders are $n=\{1, 2, 4, 8, 16, 32, 64, 128\}$. It consists of $n_e=6$ initial elements that are further subdivided into $n_s=128/n$ sub-elements. Processor Intel(R) Core(TM) i7-4700MQ CPU @ 2.40GHz 2.39GHz with 8 cores is used for analysis, while the simulations are parallelized on 7 cores.

In [5] duration of matrix fill and average number of integration points per system matrix element versus expansion order for max-ortho bases are considered for several cases of desired number of significant digits χ. Here, in Figs. 1 and 2 results for max-ortho bases for $\chi=6$, 15 are compared with the case where modified bases are used. Fig. 1. shows average number of integration points per system matrix element versus expansion order, and it is seen that the number of integration points is larger in case of max-ortho bases. It is also seen that, in case of modified bases, number of integration points N_p decreases until $n=64$ where $N_p=90$ in single precision and afterwards remains practically constant. In contrast to that in case of max-ortho bases, the number of integration points for single precision decreases until $n=16$ where $N_p=150$ and afterwards increases.

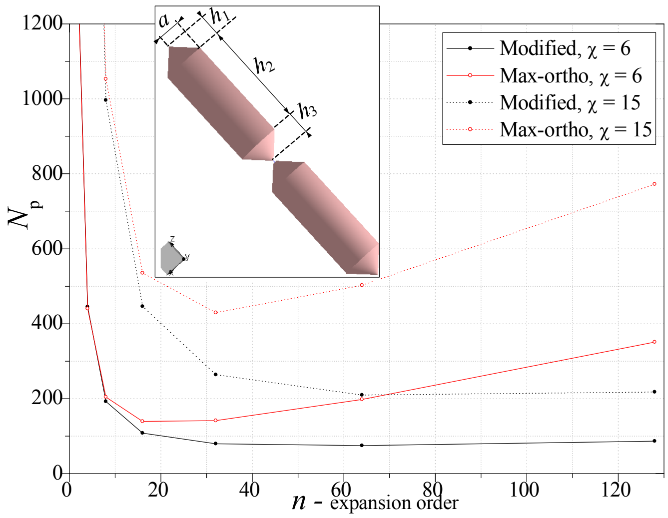

Fig. 1. Average number of integration points per system matrix element N_p versus expansion order n for thick dipole antenna in case of modified and max-ortho bases for number of significant digits $\chi=6$, 15.

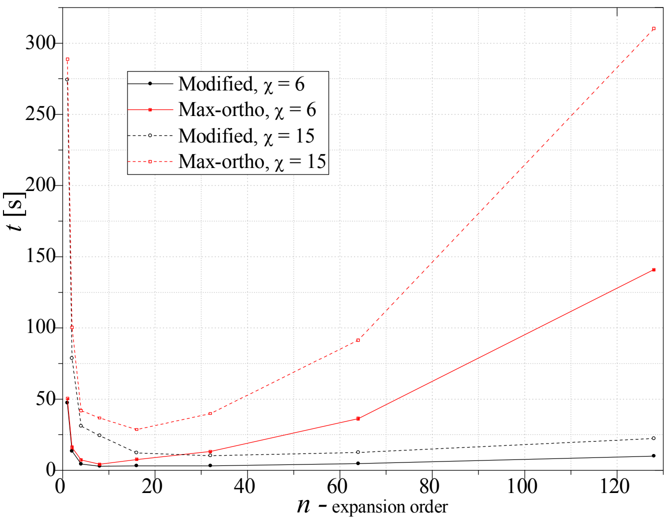

Fig. 2. Matrix fill-in time t versus expansion order n for thick dipole antenna in case of modified and max-ortho bases for number of significant digits $\chi=6$, 15.

Fig. 2 shows matrix fill-in time versus expansion order, and it can be seen that higher order bases up to $n=128$ require about the same time or less than lower order bases (i.e. for $n=1$, 2). It is also seen that for higher expansion orders matrix fill-in time is dramatically longer for max-ortho bases than for modified. In case of modified bases minimum matrix fill-in time of about $t = 2.5$ s is achieved with expansion order $n = 8$, and with further increasing the expansion order time increases very slowly. However stable results are obtained with maximum of 20th order. In case of max-ortho bases the minimum time achieved is about $t = 4.3$ s for $n = 8$, and with further increasing the approximation order time increases relatively quickly. This difference in matrix fill-in time is due to the greater number of integration points, and also because of the time consuming recurrent formulas (6) that are used for precise evaluation of max-ortho impedance integrals.

When trying to improve the efficiency of the method in case of max-ortho bases it was noticed that large amount of time goes to division of $1/i$ (i is the order of observed basis function f_i) within the 1st integration of (5), in the loop for integration points where Legendre polynomials and their derivatives are evaluated. Therefore, it was chosen to form in advance (i.e. outside of the loop) an array $g_i = 1.0/i$ which will be afterwards used at every integration point in the loop.

Fig. 3 shows matrix fill-in time in case of max-ortho bases with and without this modification. From the results it is seen that analysis lasts around 90 s less for $n = 128$ and $\chi = 15$, when this modification is implemented which is around 30% of initial time.

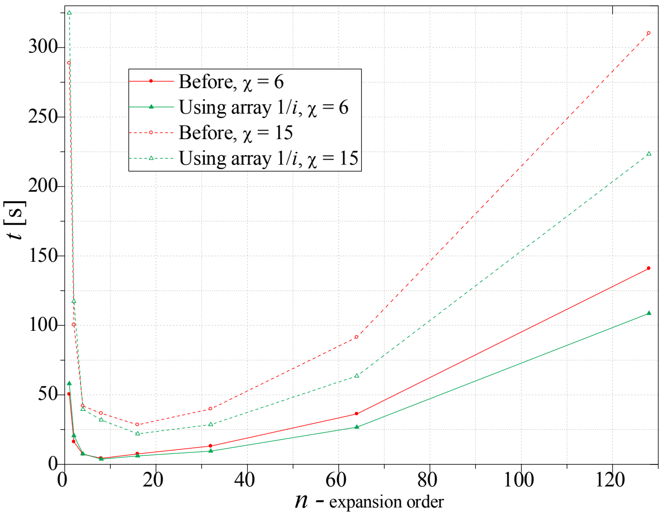

Fig. 3. Matrix fill-in time t versus expansion order n in case of max-ortho bases for number of significant digits $\chi = 6$, 15, 1) when modifications regarding array $1/i$ are implemented and 2) before (i.e. without these modifications).

Results from Figs. 2 and 3 are obtained in the case when the 1st integration of impedance integrals due to Legendre polynomials and their derivatives is evaluated simultaneously, in the same loop. The 1st integration of impedance integrals due to Legendre derivatives (5b) can also be evaluated in terms of impedance integrals due to Legendre polynomials (5a) using (6b), which would enable their evaluation outside of the loop.

Fig. 4 shows duration of matrix fill in case of max-ortho bases when the 1st integration of impedance integrals due to Legendre derivatives is evaluated 1) simultaneously, in the same loop with Legendre impedance integrals and 2) outside of the loop using (6b). From Fig. 4 it is seen that in the second case simulations last less time so that for $\chi = 15$ and $n = 128$ it is saved around 33 s for matrix fill which is around 15% of total time.

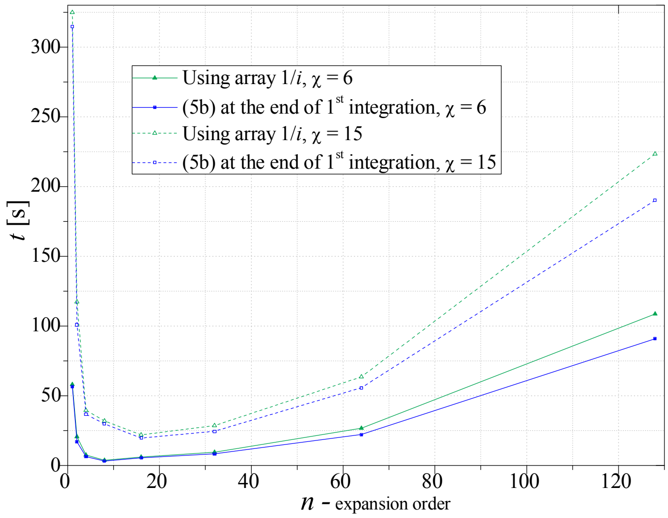

Fig. 4. Duration of matrix fill versus expansion order n in case of max-ortho bases and $\chi = 6$, 15, when the 1st integration of impedance integrals due to Legendre derivatives are evaluated 1) simultaneously, in the same loop with Legendre impedance integrals and 2) outside of the loop using (6b).

Another possibility is to evaluate impedance integrals due to Legendre derivatives after the 2nd integration as a linear combination of corresponding impedance integrals due to Legendre polynomials using (6b). Accordingly, Fig. 5 shows matrix fill-in time when impedance integrals due to Legendre derivatives (5b) are evaluated as a linear combination (6b) of impedance integrals (5a) 1) after their 1st integration, and 2) after their 2nd integration. Results from Fig. 5 show that if 2nd modification is applied instead of the 1st one, matrix fill for $n = 128$ and $\chi = 6$, 15 lasts additional 4 s less. Note that the 2nd integration takes around 8% of total matrix fill-in time, whereas the 1st integration lasts around 90% of this time.

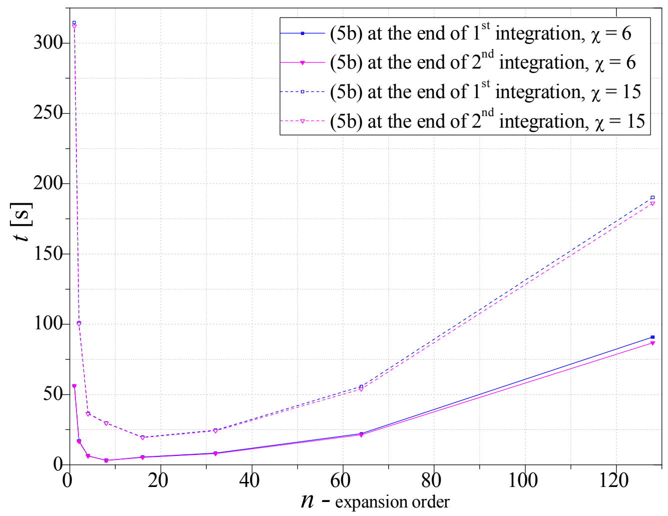

Fig. 5. Duration t of matrix fill versus expansion order n in case of max-ortho bases and $\chi = 6$, 15, when impedance integrals due to Legendre derivatives (5b) are evaluated as a linear combination (6b) of integrals (5a) 1) after their 1st integration and 2) after their 2nd integration.

Although the improvements made to the algorithm (when using max-ortho bases) reduce analysis time dramatically, when comparing results from Fig. 5 with results for modified bases from Fig. 2 it is seen that max-ortho bases still require more time especially for ultra high orders (i.e. for $n > 32$). The reason for this is larger number of integration points needed for precise evaluation of max-ortho impedance integrals than for evaluation of impedance integrals due to modified bases.

Fig. 6 shows results for matrix fill-in time in case of 1) modified bases and 2) max-ortho bases when the same number of integration points are used as for the modified bases. Results show that duration of matrix fill in case of max-ortho basis functions is now comparable with the time for modified. For example in case of $n = 128$ and $\chi = 15$ time for max-ortho basis functions is ~32 s, and for modified is ~22 s.

From these results it can be concluded that, after implementing above mentioned modifications, time for matrix fill in case of max-ortho bases is longer for ultra high expansion orders compared to the case of modified bases primarily because of the larger number of integration points needed for precise evaluation of max-ortho impedance integrals.

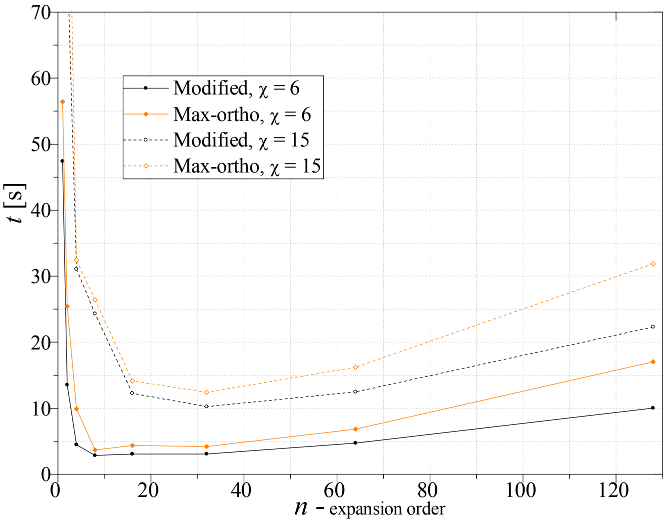

Fig. 6. Matrix fill-in time t versus expansion order n in case of thick dipole antenna using 1) modified bases, and 2) max-ortho bases when the same number of integration points are used as for the modified bases.

IV. Conclusion

The work inspects analysis-time requirements of a new method for analysis of axially symmetric metallic antennas [3], [4] when using 1) modified and 2) max-ortho basis functions.

From the results for matrix fill-in time requirements and average number of integration points needed per system matrix element it is seen that there is a trade-off between used expansion order and analysis time requirements in case of max-ortho bases, which does not exist in case of modified bases. However in case of modified basis functions, stable results can be obtained with maximum of 20th order.

Several improvements in implementation of max-ortho basis functions are inspected in order to reduce matrix fill-in time. After implementing the array $1/i$ before the loop for the 1st integration of (5), and after evaluating impedance integrals due to Legendre derivatives (5b) as a linear combination (6b) of corresponding impedance integrals due to Legendre polynomials (5a) after their 2nd integration (outside of the loops for the 1st and 2nd integration) around 40% of total time for matrix fill is saved.

After these improvements are made to the algorithm, the difference in time for matrix fill between max-ortho and modified bases is primarily because of the larger number of integration pointes needed for precise evaluation of max-ortho impedance integrals.

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