RBFNN Hierarchical Model for DOA Estimation with the Same Number of Radiation Sources and Receiving Antenna Array Elements

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***Abstract*—A new method for determining the direction of arrival (DOA) of received signals from two spatially independent radiation sources, using linear antenna array composed of two elements, is proposed in this paper. The method is based on a hierarchical model composed of the Radial Basis Function Neural Networks (RBFNNs), designed to detect the spatial distribution of the radiation sources and to estimate the DOA of signals, received with the linear antenna array. Analysis of the results shows that it is possible to perform the accurate DOA estimation of received signals, using the linear antenna array composed of the same number of elements as the number of the radiation sources, without affecting the quality of the obtained results, which is not possible using conventional super resolution algorithms.**

***Index Terms*—Detection; DOA estimation; hierarchical model; linear antenna array; RBFNN.**

I. INTRODUCTION

ESTIMATION of the direction of arrival (DOA) of electromagnetic (EM) signals received from the radiation sources, using digital signal processing, represents a technique which is widely applied in such fields as radio detection and ranging, wireless communications, navigation, radio astronomy, moving targets tracking, medicine etc [1]. As a result of increasing complexity of the demands, more advanced methods of angular assessment have been developed. They are characterized by the greater calculation speed, optimization and real time usage, along with increasing the accuracy of the obtained results.

Many conventional algorithms which correspond to different antenna array configurations and characteristics of the received EM signals were developed so far. The main characteristic of the results of DOA estimation using these algorithms is their high angular resolution. However, the major drawback of these algorithms is the usage of cumbersome mathematical procedure, which limits their practical applications. For these reasons, in recent years many researchers are turn to alternative solutions such as Artificial Neural Networks (ANNs), primarily because of their efficiency and high speed of calculation compared to conventional DOA methods [2]-[5].

Despite the fact that the elements of the ANNs called nodes or neurons perform just the basic mathematical operations, thanks to its multi-layered structure and a large number of inter – connections between the neurons, ANNs are capable of solving complex problems. Learning process is what separates them from the conventional techniques of angular assessment. It is a process of optimizing the parameters of the ANN in order to obtain the output values as close as possible to the desired values, wherein the input data used during the training and testing processes of the ANN can be either empirical or simulated. Moreover, ANN models proved to be very suitable for accurate DOA estimation of signals, because during their training, a numerous factors which are impossible to describe using concrete mathematical models can be included. Unlike conventional signal processing algorithms, which are usually based on linear models, ANNs consider the problem of angular assessment as approximation of highly nonlinear multidimensional function [4].

So far, concerning the ANN solutions for DOA problems, the approximation was most often expressed as a mapping between the spatial covariance matrix of the received signals on the antenna elements and the directions of arrival of EM signals. In the ANN hierarchical model presented in this paper, beside the spatial covariance matrix, for the purpose of approximation, the calculated values of the output power for the individual antenna elements were used, as a result of relative position of elements of the antenna array and DOA of signals from the spatially independent radiation sources. The model is developed on the basis of RBFNNs (Radial Basis Function Neural Networks) and implemented in Matlab. The training and testing processes of RBFNNs were conducted using simulated data also created in Matlab.

By using the proposed RBFNN model, in the case where the number of spatially independent radiation sources is equal to the number of elements of the receiving linear antenna array, it is possible to obtain the results of DOA estimation with high accuracy. Moreover, following the fact that ANNs are the networks with parallel access, the aforementioned results of DOA estimation are obtained almost instantaneously. These properties make the proposed model suitable for the practical applications.

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The paper is organized in the following way: Section II describes the model for the signal processing conducted on the receiving antenna array, while Section III describes the method used for the purposes of signal preprocessing in order to optimize the data to a form suitable for the RBFNNs. The description, the operational principles and the performances of the proposed hierarchical model are given in Section IV. Section V provides an analysis of the results of DOA estimation of signals from the two spatially independent radiation sources, using proposed hierarchical model based on RBFNNs, while Section VI provides a brief summary.

II. Model of received signal

Determining the DOA of received signals from the two spatially independent radiation sources, using linear antenna array with the same number of element as the number of radiation sources, is almost unsolvable problem for the conventional DOA estimation methods. A limiting factor for the conventional high resolution algorithms, especially for MUSIC (Multiple Signal Classification) algorithm, represents the fact that for their successful practical applications it is necessary that the number of radiation sources is less than the number of the receiving antenna elements [4]. Using the existing solutions for DOA estimation which are based on neural networks, the most widely used nonlinear mapping between the spatial covariance matrix of the received signals on the antenna elements and the DOA of signals does not provide the optimal solution to the problem. In order to overcome this problem, input data used during the training process of the network is expanded with the calculated values of output power for every individual element of the antenna array. In this way, the neural network training is more efficient and interdependence between network inputs and outputs is easily established.

In general, the response function of the *m*-th array element to the signal from the *k*-th radiation source is calculated as

$$a_m(\theta_k) = \exp[-j(m-1)\frac{2\pi d \sin\theta_k}{\lambda}], \tag{1}$$

where $a_m(\theta_k)$ is the response of array element *m* as a function of direction angle θ_k from the radiation source *k*, *d* represents the distance between the antenna elements, while λ corresponds to the wavelength of the received signal [6].

The matrix expression A concerning the response function of the antenna array for all the signals received from *k* spatially independent radiation sources, can be presented as:

$$A = \left[a(\theta_1), a(\theta_2), ..., a(\theta_k)\right]^T = \\ = \begin{bmatrix} 1 & 1 & ... & 1 \\ e^{-j\varphi_1} & e^{-j\varphi_2} & ... & e^{-j\varphi_k} \\ ... & ... & ... & ... \\ e^{-j(m-1)\varphi_1} & e^{-j(m-1)\varphi_2} & ... & e^{-j(m-1)\varphi_k} \end{bmatrix}. \tag{2}$$

Refer to (2), the exponent φ_k is represented as [7]:

$$\varphi_k = \frac{2\pi d \sin\theta_k}{\lambda}. \tag{3}$$

The output signal from each individual antenna array element can be calculated in the following way:

$$x_m(t) = \sum_{k=1}^{D} s_k(t)\exp[-j(m-1)\frac{2\pi d \sin\theta_k}{\lambda}] + n_m(t). \tag{4}$$

In (4), $x_m(t)$ represents the value of the output signal from the antenna element *m*, as a function of time, $s_k(t)$ represents the waveform of the received signal from the signal source *k* (*k*=1,2,…,D), and $n_m(t)$ stands for the additive white Gaussian noise at the output of the antenna element *m* [6].

The output power of array element *m* in any moment of time is calculated as:

$$P_m(t) = \left|x_m(t)\right|^2 = x_m(t)\cdot x_m{}^*(t). \tag{5}$$

Refer to (5), $P_m(t)$ and $x_m(t)$ represent the value of the output power and the value of the output signal of array element *m*, in a moment of time *t*, respectively, while the * represents the complex conjugate value [8].

The output power P_m of array element *m*, which is derived from a total of *k*=1,2,…,D radiation sources, for the *i* snapshots (*i*=1,2,…,N), can be expressed as a combination of aforementioned equations as follows:

$$P_m = \sum_{k=1}^{D}\sum_{i=1}^{N} A(m,k)\cdot s_k(i)\cdot s_k(i)^*\cdot A(m,k) + P_{m(noise)}. \tag{6}$$

Refer to (6), $A(m,k)$ represents the element of the matrix A which stands for the response function of array element *m* to the signal from the radiation source *k*, $s_k(i)$ represents *i*-th snapshot of the received signal from the radiation source *k*, the * represents the complex conjugate value, while $P_{m(noise)}$ stands for the output noise power of array element *m*. The previous equation shows the result of the influence of each received signal from individual radiation source on the output power of the individual antenna element.

Based on the obtained values of the output signal of the antenna array, the spatial covariance matrix is calculated as follows:

$$\tilde{R}_x = \frac{1}{N}\sum_{i=1}^{N} x(i)x^H(i). \tag{7}$$

In (7), $\tilde{R}_x$ represents the maximum likelihood estimation of the covariance matrix, N is the total number of samples of the output signal $x(i)$, while H is a *Hermitian operator* which performs the conjugated transpose matrix of the output signal $x(i)$ [7].

III. Data preprocessing

In order to obtain the results of DOA estimation of received signals from two spatially independent radiation sources using linear antenna array composed of two elements, the pairs of angles of arrival of signals from the two radiation sources were created in Matlab. The angular pairs were generated in three groups. The first group comprises the angular pairs which correspond to the received signal pairs from the radiation sources spatially distributed between 0° and 90°, relative to the centerline of the antenna array. The second group is defined for the situation when the radiation sources are spatially distributed between - 90° and 0°, while the third group is defined for the case when one source is in the range between - 90° and 0°, and another between 0° and 90°, relative to the centerline of the antenna array. Taking into account the integers of the angles in the range of 0° to 90°, angular pairs of the first group were formed as a combination of the second class without repetition, which gives the matrix of angular pairs with dimensions 2x4095. The second group was formed in the same manner, but for the range of angles between - 90° and 0°, so the matrix has the same dimensions. The third group was formed as a set of all possible combinations of angular pairs, wherein one element of the angular pair is in the range from - 90° to 0°, and the other between 0° and 90°, which gives the matrix of angular pairs with dimensions 2x8100. The groups of angular pairs formed in this way were used during the training process of RBFNNs from the second level of decision-making (responsible for the DOA estimation of signals from the radiation sources) of the proposed hierarchical model. In order to conduct the training of RBFNNs from the first level of decision-making (responsible for the detection of the spatial distribution of individual radiation sources) of proposed hierarchical model, a new group of angular pairs were created, which includes all the combinations of the second class without repetition for the integers of the angles in the range of - 90° to 90°, randomly arranged. In this way, a matrix with dimensions 2x16290 was formed.

Once trained, the network should be tested by checking its ability of generalization, which represents the ability to predict the response values to those inputs that have not been presented during the training process. Well-trained network provides at its output the results close to target values, or in other words, the approximation of the target values with minimal errors [5]. For that reason, test sets for all the RBFNNs from the hierarchical model were formed by adding or subtracting the pseudo-random values of angle (in the range between 0° and 1°) from the elements that make the training sets. The dimensions of the test sets and their corresponding training sets are the same.

Pseudorandom values of phase were generated for each individual angle, previously created, while the pseudorandom values of frequency were generated in the range between 100 MHz and 150 MHz. In order to achieve the best results of DOA estimation of signals, the distance between the elements of the antenna array were equal to $\lambda/4$, corresponding to the highest receiving frequency of 150 MHz. Amplitude of signals for the both radiation sources, in all possible cases were the same. The duration of the signals were 0.1 s, while signal to noise ratio (SNR) were 20 dB, for each individual signal, respectively. Based on previously defined elements, program creates the pairs of received signals. Also, based on (1) and (4), the response functions and output signal pairs of two element antenna array were created. Furthermore, refer to (6) and (7), program calculates the output power for each of the two elements of the antenna array (as a function of DOA referred to all simultaneously received signal pairs), and the spatial covariance matrix based on the obtained values of output signals, respectively.

Information about the difference between the calculated output power of the two elements of the antenna array, in the case where the number of spatially independent radiation sources is equal to the number of elements of the receiving linear antenna array, represents is essential information for the proper network training. Without this information it is very difficult to achieve high accuracy of the results of DOA estimation, and acceptable calculation speed.

The number of neurons in the input layer of each RBFNN depends on the dimensions of the input vector. By reducing the dimensions of the input vector and using the optimization process, the training of ANNs becomes more efficient. Since the receiving antenna array consists of two elements, the spatial covariance matrix is 2x2. Keeping in mind that ANNs cannot operate with complex numbers, as well as using the spatial symmetry of the covariance matrix, the program creates vector b composed of three elements of the covariance matrix $r_{i,j}$ which are located above the main diagonal of the matrix as follows [2]:

$$b = [r_{1,1} \quad \mathrm{Re}\{r_{1,2}\} \quad \mathrm{Im}\{r_{1,2}\}]. \tag{8}$$

After that, vector b should be normalized by its norm $z = b/\|b\|$ [2]. The process of normalization has been also applied on a vector of output power pairs, calculated from individual antenna elements. After that, the elements of the vector z along with the elements which represents the normalized output power pairs, form the input vector for the proposed hierarchical model, with dimension 6x1 (one vector for each simultaneously received signal pairs). Input vector created in this manner, represents the form suitable for the operations conducted by the RBFNNs from the proposed hierarchical model.

IV. Hierarchical model based on RBFNNs

The concept of the hierarchical model for DOA estimation of received signals, based on RBFNNs is shown in Fig. 1.

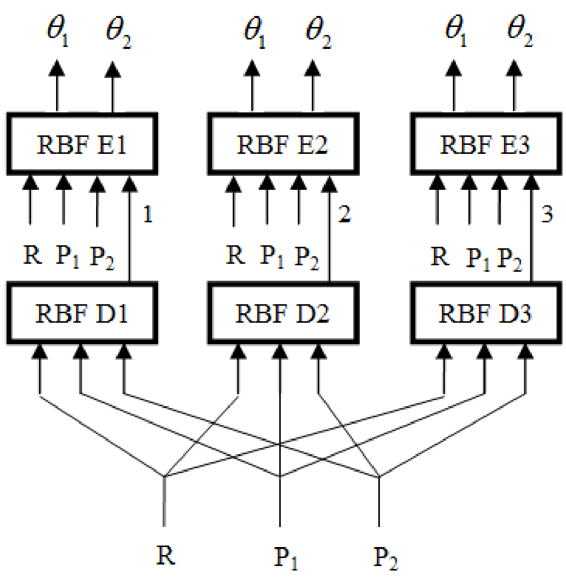

Fig. 1. The concept of the hierarchical model based on RBFNNs.

The first level of decision-making consists of RBFNNs responsible for the detection of the spatial distribution of individual radiation sources (*RBF D1,.., RBF D3*), while the second level of decision-making consists of RBFNNs responsible for the accurate DOA estimation of the received signals from two radiation sources using two element linear antenna array (*RBF E1,…,RBF E3*). The spatial covariance matrix *R* and the values of the output power from each individual elements of the antenna array P_1 and P_2 represent the input data to the RBFNNs of the first level of hierarchical model. *RBF D1* consists of 23 RBFNNs created in order to detect the spatial distribution of pairs of independent radiation sources, which are deployed in the range of angles between 0° and 90°, relative to the centerline of the antenna array. *RBF D2* and *RBF D3* are also composed of 23 RBFNNs designed for the detection of the spatial distribution of pairs of independent radiation sources, which are deployed in the range of angles between - 90° and 0° (for the *RBF D2*) or between the - 90° and 0°corresponding to one radiation source and between the 0° and 90° corresponding to another one (for the *RBF D3*). In other words, the first level of decision-making contains a total of 69 independently trained RBFNNs.

The large number of neural networks in the first level of decision-making is a result of the fact that for the range of angles between - 90° and 90° there is a total of 16290 different combinations of angular pairs, corresponding to signal pairs (using only integers) which can be received on the antenna array. Besides that, received signals do not represent the signal pairs which correspond to the angles of arrival distributed in linear ascending or descending order. In order to increase the ability of generalization of decision-making, angles of arrival corresponding to signal pairs are distributed pseudo randomly. By dividing the 16290 combinations of pairs in several sub bands, the training process for each individual neural network is significantly shortened. In order to reduce the estimated error in detection of the spatial distribution of individual radiation sources to a minimum, neural networks are specially trained for the borderline cases (around - 90° and 90°).

Opposed to the large number of neural networks from the first level of decision-making, the second level is composed of a single RBFNN for each of the three above mentioned groups of angles. The signal pairs used for the training and testing purposes of these three RBFNNs correspond to the angles of arrival distributed in linear ascending order.

Neural networks which belong to the first level of decision-making are designed to give number 1 on its output in the situation when the spatial distribution of radiation sources belongs to the first group of angles, number 2 when the spatial distribution of radiation sources belongs to the second group of angles, and number 3 for the third group of angles. Activation of the specific RBFNN from the second level depends exclusively on the output value from the first level (1, 2 or 3). Activated RBFNN of the second level receives the input data from the network which caused its activation.

The performances of the trained network can be assessed using statistical parameters, which define the quality of the prediction, as Worst Case Error (WCE), Average Case Error (ACE) and Pearson Product - Moment correlation coefficient (usually denoted by „*r*"). Table I provides a performance overview concerning the trained RBFNNs from the second level of decision-making of hierarchical model, expressed through its statistical parameters.

TABLE I
PERFORMANCE OVERVIEW OF TRAINED RBFNNS FROM THE SECOND LEVEL OF HIERARCHICAL MODEL

RBFNN	goal/ spread	WCE	ACE	r
RBF_1050	0.1/0.1	0.4670	0.0028	0.9982
RBF_1244	0.1/0.1	0.5028	0.0026	0.9967
RBF_723	0.1/0.2	0.7564	0.0013	0.9991

Model with 1050 neurons in the hidden layer represents the neural network selected for the accurate DOA estimation of signals from the radiation sources that corresponds to the first group of angles of arrival (RBF E1), model with 1244 neurons in the hidden layer represents the neural network selected for the accurate DOA estimation of signals from the radiation sources that corresponds to the second group of angles of arrival (RBF E2) and model with 723 neurons corresponds to the third group. Analyzing the performances of RBFNNs from Table I, it can be concluded that the proposed hierarchical model is not inferior in any way comparing to the conventional methods of DOA estimation, as well as comparing to models bases on neural networks used so far.

V. ANALYSIS OF THE RESULTS

In order to verify the quality and accuracy of estimated

DOA of signals from two independent radiation sources, a group of 30 pairs of signals received on two element antenna array was created in Matlab. Angles of arrival corresponding to these 30 pairs of received signals were randomly generated in the range between - 90° and 90°, relative to the centerline of antenna array. Generated angular values are different from the angular values used for the training and test sets. All the procedures corresponding to data processing explained in Section III and Section IV were carried out on above mentioned group.

Fig. 2 shows the response of the first level of decision-making of proposed hierarchical model on 30 generated signal pairs which correspond to two independent radiation sources. The positions of the radiation sources are randomly selected. Blue circles represent the target values corresponding to the possible spatial distribution of radiation sources (labeled as 1, 2 or 3), and red crosses represent their estimated values or the output values of the first level of the hierarchical model. Despite the fact that input data represent the signal pairs from two radiation sources spatially distributed in a random law in the range of angles between - 90° and 90° relative to the centerline of the two element antenna array, the first level of decision-making of the proposed hierarchical model properly performs its function.

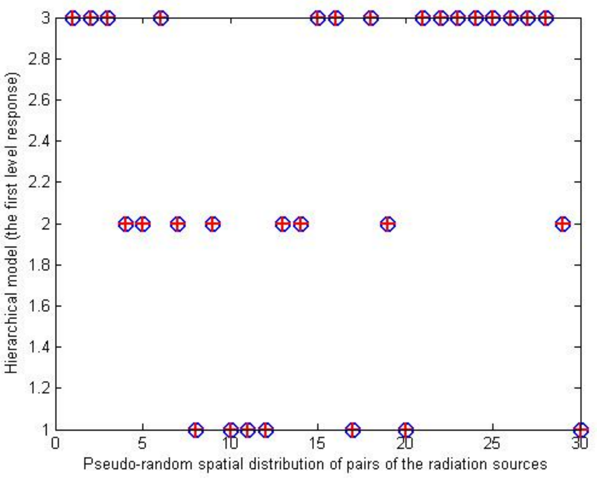

Fig. 2. The response of the first level of decision-making of the proposed hierarchical model.

Fig. 3 shows the response of the second level of decision-making of proposed hierarchical model, which represents the result of the accurate DOA estimation of 30 pairs of received signals from two independent radiation sources.

Green circles represent the target values of DOA corresponding to each individual signal from 30 pairs of received signals, while red crosses represent the final results of estimated values of DOA for each individual signal.

The quality of angular assessment of the received signal pairs, using proposed hierarchical model, does not change, even in cases when the angular position of the radiation sources has its boundary values (around - 90°, 0° and 90°), or in cases when the two radiation sources are spatially very close to each other, which represents a very difficult situation for all conventional high resolution methods.

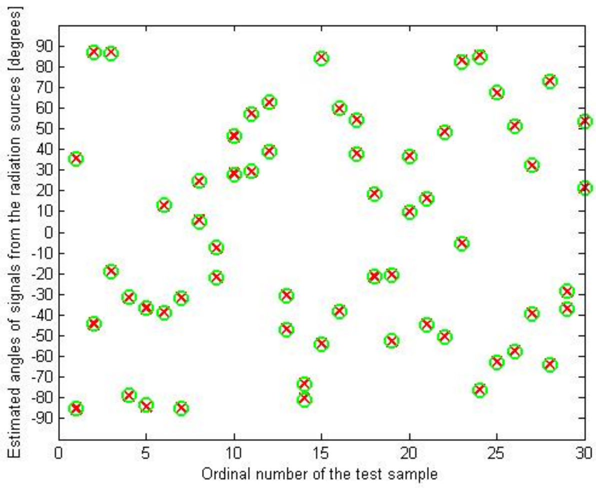

Fig. 3. The response of the second level of decision-making of the proposed hierarchical model.

In order to verify the accuracy of the results obtained using proposed hierarchical model, a comparison between the estimated and actual values of DOA of generated 30 pairs of received signals was carried out, as shown in Fig. 4.

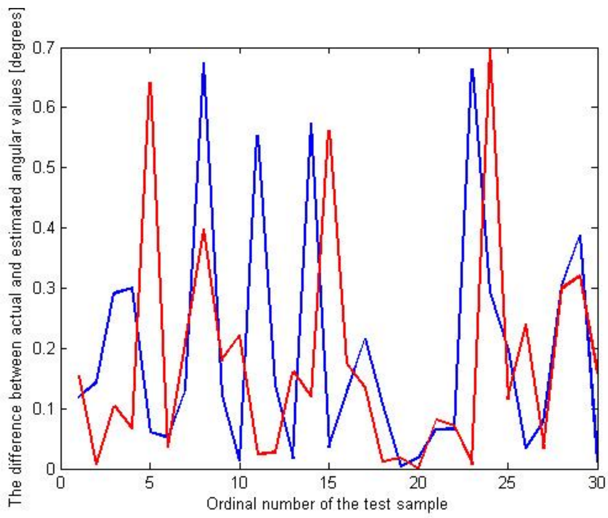

Fig. 4. The interpolated values of difference between the actual and estimated values of DOA of generated 30 pairs of received signal.

Blue and red line represents the interpolated values of difference between the actual and estimated value of DOA of received signals, corresponding to position of the first and of the second radiation source, for all 30 pairs, respectively. The mean value of difference between the actual and estimated value of DOA, corresponding to the first radiation source from each pair (blue line) is 0.1934°, and for the second source (red line) is 0.1759°, while its maximum value of difference is 0.6740° and 0.6984°, respectively.

The average working time of the completed hierarchical model, from the moment of generating signal pairs, to the moment of obtaining the angular assessment, for all 30 pairs of received signal together, is 0.85 s.

Considering the fact that the duration of each individual signal was 0.1 s, and also the fact that the results of angular assessment were obtained almost instantaneously, the results of estimated DOA of the received signals, provided by the proposed hierarchical model, in situation when the number of radiation sources is equal to the number of elements of the receiving antenna array, are very good, which is practically impossible using conventional methods of DOA estimation.

VI. Conclusion

This paper presents a very efficient two-level hierarchical model for DOA estimation of signals, based on RBFNNs. The essential advantage of the proposed model, compared to conventional super-resolution algorithms, represents the possibility of detection and DOA estimation of received signals with high accuracy, in case when the number of independent radiation sources is equal to the number of elements of the receiving antenna array. Signals with short duration as well as radiation sources spatially distributed very close to each other do not have significant impact on the quality of angular assessment provided by this hierarchical model. Following the fact that ANNs are the networks with parallel access, the aforementioned results of DOA estimation are obtained almost instantaneously, which makes the proposed model suitable for the practical applications.

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