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PERFORMANCE OF THE PEG LDPC CODES OVER PARTIAL RESPONSE MAGNETIC RECORDING CHANNELS

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Abstract – *Low-density parity-check (LDPC) codes became very interesting for the research community due to their capacity approaching performance and effective and low complexity iterative decoding. This paper examines properties of the irregular high-rate finite-length PEG LDPC codes over magnetic recording channels, modeled with one-track one-head. Also, the PEG LDPC codes are compared with randomly constructed LDPC codes over Dicode, PR4, EPR4 and E^2PR4 ideal partial-response magnetic recording models, in presence of the AWGN noise.*

1. INTRODUCTION

LDPC codes have attracted considerable attention, due to ability to achieve information rates very close to the Shannon limit, when iteratively decoded on AWGN channel [1]. When decoded by the message-passing algorithm [2], these codes can achieve performance comparable to turbo codes, requiring extensively less arithmetic computations than the equivalent turbo decoders [3].

The *Progressive Edge Growth* (PEG) LDPC construction algorithm [4], belongs to a recently discovered class of code design algorithms producing LDPC codes that performs very well over the classical channels, such as binary erasure channel (BEC) or additive white Gaussian noise (AWGN) channel. Good LDPC code performers over the BEC channel are the ones designed to set the size of the minimal stopping set to a maximum possible value [5]. Algorithms such as PEG algorithm and ACE (*Approximate Cycle* EMD (*Extrinsic Message Degree*)) constrained algorithm [6], heuristically produce LDPC code graphs with larger minimal stopping sets on average, compared with the random construction. This is why we expect that their performance can be better on average than the performance of randomly constructed codes.

The PEG algorithm is a heuristic method for design of the LDPC code graphs with as large as possible girth. Since every stopping set contains at least one cycle, girth conditioning removes all stopping sets smaller than girth size. The PEG algorithm is effective for the construction of low-rate LDPC codes, and aims at design of good finite-length performers. Design of a good high-rate code is a challenging problem, and the girth-conditioning that PEG performs tends to be of low effectiveness, due to a relatively small number of the check nodes.

In this paper, we explore the effectiveness of PEG algorithm for a high-rate finite-length code designs. The purpose of our study is to explore if PEG girth conditioning is beneficial at all, comparing to the random design approach, over magnetic recording channels.

Partial-response channels with transfer polynomial of the form $P(D) = (1-D)(1+D)^N$, $N \geq 1$, have been shown to closely match the behavior of the magnetic recording channels for a range of linear recording densities [7].

For the current densities Dicode ($N = 0$) and PR4 ($N = 1$) channels are not suitable equalization target, because they increase high-frequency noise components causing losses in the Viterbi detector performance [8]. These losses are referred as equalization losses. Appropriate choice for the current density is EPR4 ($N = 2$) and at higher densities E^2PR4 ($N = 3$) equalization is better.

The possible PEG LDPC benefits over the one-track one-head partial-response magnetic recording channel in iterative simulation scheme is investigated. The simulation scheme uses soft output Viterbi (SOVA) algorithm for channel detection [9], and message-passing algorithm for the LDPC codeword decoding.

The paper is organized as follows. The PEG LDPC basic definitions with a message-passing decoding algorithm are presented in Section 2. Section 3 gives details about iterative decoding scheme, where different magnetic channel equalization models are applied. Simulation results are presented in Section 4, while concluding remarks on the paper are given in Section 5.

2. PEG LDPC CODES

The PEG is a simple but efficient method for constructing LDPC code graphs, by progressively establishing edges between bit and checks nodes in an edge-by-edge manner. Algorithm proceeds by sequentially adding bit nodes to the code graph G(**H**), and for each bit node by sequentially adding certain amount of edges, according to the *Density Evolution* (DE) [10] optimized degree distribution of bit nodes. For each new bit node first edge is connected randomly to one of the check nodes from set of check nodes with minimal degree. Every following edge is connected to one of the farthest check nodes (there may be more than one such node) from the bit node being appended, under the current graph setting. This heuristically produces a code graphs with largest possible girths.

We use PEG algorithm to design LDPC code by designing its parity check matrix **H**, or equivalently its LDPC code graph G(**H**). Sparse parity-check matrix **H** is MxN matrix and corresponding G(**H**) has M right or parity-check nodes and N left or bit nodes. The designed code rate is equal to:

$$R = \frac{N-M}{N}. \qquad (1)$$

Since the number of degree-2 variable nodes is typically significantly larger than M in DE optimized ensembles of high-rate LDPC codes, we take their number to be equal to M as a DE optimization constraint. This prevents appearance of the cycles among degree-2 variable nodes which are stopping sets. For this reason we have used the parity check matrix **H** of the form:

$$H = [H_1 \mid H_2], \qquad (2)$$

where the matrix $\mathbf{H_2}$ is MxM matrix and has the bi-diagonal structure or zigzag pattern form. Therefore, PEG algorithm is essentially employed in the design of submatrix $\mathbf{H_1}$, since $\mathbf{H_2}$ part is already predetermined. In this sense, obtained codes are at the same time extended *irregular repeat-accumulate* (eIRA) codes, analyzed in [11].

In our setting, the PEG is implemented for the design of $\mathbf{H_1}$ submatrix of parity-check matrix **H**. As an input of the PEG algorithm a DE optimized left-degree distribution polynomial λ(*x*) is provided. This polynomial is designed for a rate R = 0.89 and with the number of degree-2 bit nodes equal to M [12]. All of M degree-2 nodes are arranged in the cycle-free submatrix $\mathbf{H_2}$, so the PEG proceedes with appending degree 3 or more columns as $\mathbf{H_1}$ part. Finally, what we obtain is the **H** structured PEG LDPC codes.

LDPC codeword iterative decoding is based on the message-passing algorithm which computes codeword bit *a posteriori* probability in the bipartite graph assigned to the parity check matrix **H**.

Using the log-likelihood ratio of the binary variable *x*:

$$LLR(x) = \log\frac{P(x=1)}{P(x=0)}, \quad (3)$$

easily can be shown for every graph nodes the equation

$$LLR^{a\ posteriori}(x) = LLR^{a\ priori}(x) + LLR^{extrinsic}(x), \quad (4)$$

is satisfied [2].

In the message-passing algorithm steps the *extrinsic* information $LLR^{extrinsic}$, exchanges between connected nodes. This information is collected over all node branches except the branch through which are sent. Extrinsic information of bit node is output information, and for the check node represents input *a priori* information $LLR^{a\ priori}$.

In the final step, the hard decision of the codeword bit is made, based on the node *a posteriori* information $LLR^{a\ posteriori}$.

3. ONE-TRACK ONE-HEAD SIMULATION SCHEME

Simulation of the PEG and randomly constructed LDPC codes is done over one-track one-head partial-response magnetic recording channels. Different models are applied in the encoding scheme shown in Fig. 1.

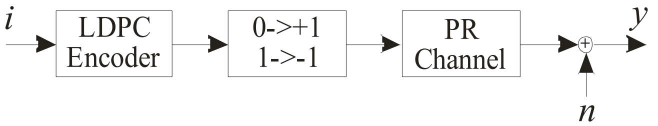

Fig. 1 *LDPC encoding of partial-response channel*

It is assumed that read back signal ***y*** is distorted with additive, white, zero-mean Gaussian noise ***n*** and that the signal-to-noise ratio (SNR) is defined as:

$$SNR = 10\log\left(\frac{E_b}{N_o}\right) = 10\log\left(\frac{E_b}{2\sigma^2}\right) = 10\log\left(\frac{E_c}{2R\sigma^2}\right), (5)$$

where E_c=RE_b is the symbol bit energy at the channel output, N_o is the one-sided power spectral density and σ^2 is noise variance.

Iterative LDPC decoding over partial-response channels is performed in simulation scheme shown in Fig. 2.

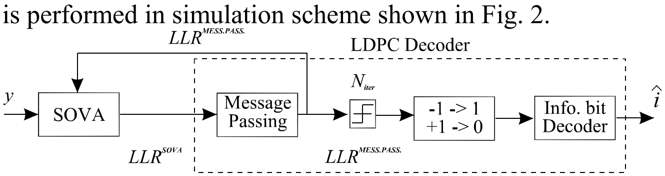

Fig. 2 *LDPC iterative decoding over AWGN channel*

The partial-response channel output sequences detection is performed using optimum soft output Viterbi (SOVA) detector with 20 symbols detection window, while LDPC decoding employs standard message-passing algorithm.

4. SIMULATION RESULTS

For the PEG LDPC code an irregular finite-length code consisting of sequence of N = 1024 bits in which (N – M) message bits satisfy a set of M = 113 parity checks, is constructed. The code rate is R = 0.89.

Also, a randomly constructed short finite-length LDPC code of the same code rate and same parameters (M, N) has been taken for comparison.

In all simulations the N_{iter} = 10 iterations was performed between SOVA and message-passing. Simulation results for the PEG and randomly constructed LDCP coding over partial-response magnetic recording channels are presented in Fig. 3, 4, 5 and 6.

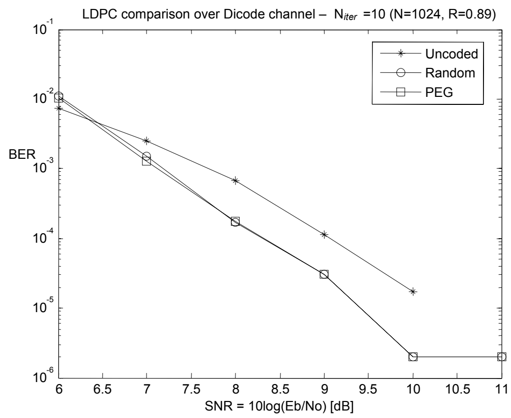

Fig. 3 *LDPC codes over Dicode channel*

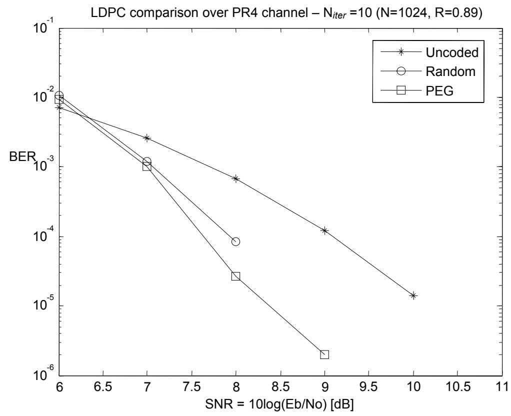

Fig. 4 *LDPC codes over PR4 channel*

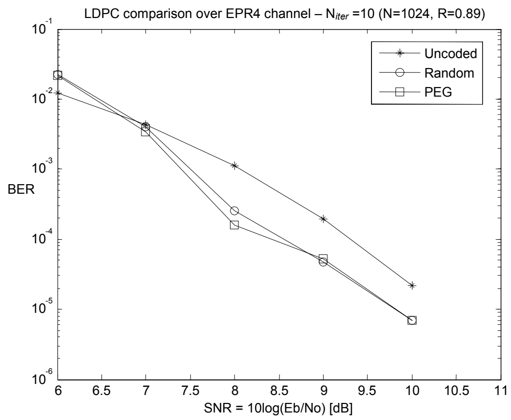

Fig. 5 *LDPC codes over EPR4 channel*

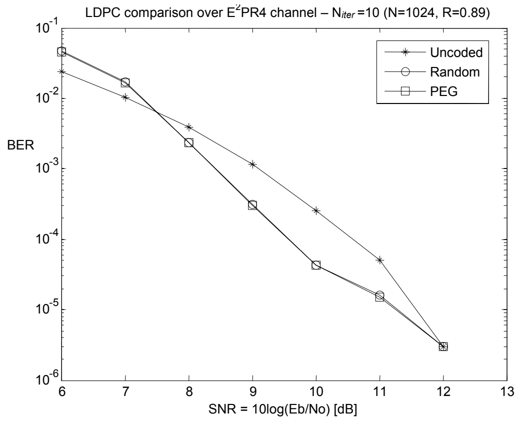

Fig. 6 *LDPC codes over E^2PR4 channel*

The finite-length PEG and randomly constructed LDPC codes have nearly the same characteristics over Dicode and E^2PR4 channels. For the PR4 channel model, the PEG constructed LDPC codes outperform randomly constructed ones for about 0.3 dB at higher values of SNR (BER ~ 10^{-4}). In the case of EPR4 channel the PEG constructed LDPC codes are slightly better than randomly constructed ones. These results are in a way expected, since girth-conditioning becomes increasingly harder for shorter, high-rate codes. PEG constructions larger length codes and code rate of ½ or below has shown very good results. Unfortunately, PEG constructed high rate codes generate girths of similar lengths compared with random constructions.

5. CONCLUSION

Construction of irregular finite-length and high-rate PEG LDPC code over ideal magnetic recording channel was considered. Simulations results have shown that this kind of the LDPC codes and randomly constructed ones have the similar behavior at the high code rates.

The coding gain, compared to the uncoded case, is in the range from 0.8-1.3 dB at a BER of 10^{-4}, for all equalization models, using LDPC code with $R = 0.89$.

This paper presents the starting point for the PEG LDPC codes analysis, over the magnetic recording channels. Some future works will exemine the behavior of longer high-rate PEG codes, possible matching to the partial-response magnetic recording models and possibilities for improvement of PEG construction method.

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Sadržaj – U ovom radu ispitivane su performanse neregularnog PEG LDPC koda, sa kodnom brzinom većom od ½, u kanalima za magnetski zapis, sa jednom stazom za zapisivanje i jednom glavom za čitanje. Pefromanse ovog koda upoređene su sa performansama LDPC koda iste kodne brzine, koji je konstruisan na slučajan način. Rezultati su dati za idealni Dikod, PR4, EPR4 i E^2PR4 model magnetskog kanala.

PERFORMANSE PEG LDPC KODA NA KANALIMA ZA MAGNETSKO ZAPISIVANJE SA PARCIJALNIM ODZIVOM

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