DETERMINATION OF RESONANCE IN LONG RADIAL TRANSMISSION LINES WITH STATIC VAR COMPENSATION USING TRAVELING WAVES

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Abstract – The paper presents an analytical procedure for estimation of resonance frequencies in long distance radial transmission lines with static VAR compensation. The development of the procedure is based on a new approach in solving this problem. It employs the fact that the phenomenon is characterized by an algebraic sum of traveling current waves at the beginning of the line. Relatively simple analytical expressions are obtained by considering the phase angles of only one direct and only one corresponding reflected wave. Besides enabling simpler and faster solution of the problem under various realistic conditions, the obtained analytical expressions also offer more direct analytical insight into the interplay of basic parameters.

1. INTRODUCTION

The capabilities of shunt reactance compensation with static VAR control in improving characteristics of a power system in steady-state stability, in dynamic stability, in damping and in voltage tolerances are considered in detail in [1]. The papers that appeared later considered the adverse effects of subsynchronous resonance on system components (torsional interaction) and the limits to which static VAR compensation can be applied to long distance radial transmission lines [2,3]. It has been concluded that there was a resonance frequency which fell below the frequency of the supply when one attempted to operate the system at power angle δg in excess of 180°. However, bearing in mind the theoretical importance and the practical implications, as well as the fact that the answers came forth mechanically, without the insights either into the origins of the resonance or into the reasons for their multiplicities, a need arose for further research. To this purpose the investigations in [4,5] were performed. Contrary to the belief that the shunt compensations with static VAR control can resonate only at supersynchronous frequency, the results of these investigations demonstrated that the resonance at the supply frequency may even occur at power angles below 180°.

A detailed analysis presented in [4] and [5] offered analytical insights into the interaction of the transmission line modes with the shunt compensation. It explained origins of multiple resonances and offer a guide as to how many resonances there are, where they are located and in what way they are sensitive to changes in the parameters of the shunt compensator. Highly complex mathematical tools were used for this, mostly based on matrix calculus, thus still leaving the whole picture of the influence of particular relevant parameters rather vague. Because of the lack of analytical expressions that would include all the relevant factors [5], it was necessary to analyze the phenomenon in several various idealized models [4] before considering a real physical model.

The work [6] considers the resonance phenomena in uniform ladder circuits and homogeneous transmission lines and proves the following important rule. In a theoretically infinite number of traveling waves which participate in establishing a steady-state in a line where the resonance conditions are to be analyzed it is sufficient to observe the shift of the phase angle of only one direct and only one corresponding reflected wave. Using this fact as a theoretical base, in this paper analytical expressions are derived corresponding to the case of long radial transmission line with static VAR compensation. The simple mathematical form of these expressions permits a more direct insight into the interplay of the relevant factors and more complete physical interpretation of the resonance phenomena in long distance radial transmission lines.

2. MODELS

Basic Assumptions

In this paper it is assumed that the line is balanced. The single-phase representation of the positive sequence elements is used as the working model. The consequence of the fact that the real line is not completely symmetric will be discussed later in the one of the following paper.

Transmission system

A general model of the single phase transmission line with static VAR compensation is presented in Fig. 1. $U_g=U_{b} \angle \delta_g$ and $U_{0}=U_{b} \angle \delta_g$ represent the load and the generator voltages; $Z_g$ and $Z_0$ represent the load and the generator impedances. $U_n \ (n=1, 2, ..., N)$ are the voltages at the nodes and $Y_n \ (n=1, 2, ..., N-1)$ are the shunt admittances of the static VAR compensators. In practical situations each admittance must be such that the system voltages at its location are near 1 P.U.

![Diagram](image-url)
The parameters per unit length of the line: r, l, g and c (series resistance, series inductance, shunt conductance and shunt capacitance, respectively) have the same values in all sections (S₁, S₂,... Sₙ). The lengths of particular sections are different.

**Static VAR Compensator**

In practical circuits, the compensator reactances are usually in the delta arrangement and Fig. 1 shows the line to the neutral equivalent.

The numerical values of admittances \( Y₁, Y₂,... Yₙ₋₁ \) to be used for the verification of the method presented herein are determined by the calculations presented in [5].

### 3. ANALYTICAL EXPRESSIONS

The analytical procedure developed in [8] follows successively all the phases of a real physical process which preceded establishing the steady state in the line and are generated with each new reflection of the traveling wave at the ends of the line. It is concluded that the steady state in the line is the result of superposition of an infinite number of direct and reflected waves. The resonance phenomena occur when all of the direct and reflected waves represented by the corresponding phasors appear at the beginning of the line with the same phase angle, and consequently can be summed algebraically [6, 8]. However, according to the consideration presented in [6], to consider the resonance phenomena it is sufficient to observe the phase of only one current wave at the beginning of the line and the phase of the same wave, but after it has passed to the end of the line and returned as reflected to the line beginning. If the phase angles of the direct and reflected current wave at the line beginning are such that the waves are summed algebraically, then the resonance oscillations will appear in the line. On the basis of the condition defined in such a manner, for the particular line shown in Fig. 1, we can write the following equation:

\[
L_{SN} \lambda(\omega_r) - \varphi_{N-1}(\omega_r) + L_{S(N-1)} \lambda(\omega_r) - \ldots - +
+ L_{S2} \lambda(\omega_r) - \varphi_{1}(\omega_r) + L_{S1} \lambda(\omega_r) - \varphi_{0}(\omega_r) +
+ L_{S1} \lambda(\omega_r) - \varphi_{1}(\omega_r) + L_{S2} \lambda(\omega_r) - \ldots - +
+ L_{S(N-1)} \lambda(\omega_r) - \varphi_{N-1}(\omega_r) +
L_{SN} \lambda(\omega_r) - \psi_{N}(\omega_r) = 2m\pi,
\]

\[m=1, 2, 3, \ldots\]

or, in a more compact form:

\[
\lambda(\omega_r) \sum_{1}^{N} L_{Sn} - \sum_{1}^{N-1} \psi_{i}(\omega_r) + \frac{\psi_{N}(\omega_r)}{2} = m\pi,
\]

\[m=1, 2, 3, \ldots\]

The imaginary part of the propagation coefficient is determined by the well-known relation:

\[
\gamma = \sqrt{(r + j\omega_l)(g + j\omega_c)}
\]

According to [7], the refraction coefficient of a traveling wave in the n-th node, which is obviously a point of discontinuity, is determined by

\[
\eta_n = \frac{2Y_{n-1}}{Z_c + 2Y_n^2}
\]

where \( Z_c \) denoted the characteristic impedance of the line determined by the well-known expression:

\[
Z_c = \sqrt{\frac{r + j\omega_l}{g + j\omega_c}}
\]

The reflection coefficients at the ends of the line are determined by the following well-known expressions

\[
\beta_0 = \frac{Z_c - Z_b}{Z_c + Z_b}
\]

\[
\beta_N = \frac{Z_c - Z_g}{Z_c + Z_g}
\]

The addends in the first and the second row of (1) are written in the sequence of the changing phase angle of the direct wave, while the addends in the third and the fourth row are written in the sequence in which the phase angle of the reflected wave is shifted.

In practical cases parameters r and g can be neglected (r=g=0), so that the parameters \( \lambda \) and \( Z_c \) can be presented by the following expressions:

\[
\lambda(r = g = 0) = \omega_c \sqrt{\frac{1}{c}}
\]

\[
Z_c(r = g = 0) = \sqrt{\frac{1}{c}}
\]

Also, under practical conditions the admittances \( Y_1, Y_2,... Y_{N-1} \) contain only the imaginary part, so that according to eq. (4) we obtain the following relations for the phase angle of the refraction coefficient:

\[
\varphi_n(\omega_r) = \arctg Z_c(r = g = 0)
\]

\[
\varphi_n(\omega_r) = \arctg -\frac{\omega_c C_n Z_c(r = g = 0)}{2}
\]

Finally, under real conditions the impedances \( Z_b \) and \( Z_g \) also contain only the imaginary part: \( Z_b = j\omega_l L_b \) and \( Z_g = j\omega_c L_g \), so that according to eqs. (6) and (7) the following relations can be written for the phase angles of the reflection coefficients:

\[
\psi_0(\omega_r) = \arctg \left( -\frac{\omega_c C_n Z_c(r = g = 0)}{Z_c^2(r = g = 0) - \omega_c^2 L_c^2} \right)
\]
\[ \psi_N(\omega_r) = \arctg \left( \frac{-2\omega_r L_g Z_r (r = g = 0)}{Z_c (r = g = 0) - \omega^2 L_g^2} \right) \]  

Eq. (2) can also take the following more simple form:

\[ \omega_r \sqrt{\frac{1}{c} \sum_{1}^{N} L_{sn}} - \sum_{1}^{N-1} \phi_n^{(\omega_r)} + \frac{\psi_0(\omega_r) + \psi_N(\omega_r)}{2} = m\pi, \]  

\( m=1, 2, 3, \ldots \) 

All the resonance frequencies of the circuit which presents a combination of the elements with distributed parameters and elements with lumped parameters (Fig. 1) are not determined by the transcendental equation (2). The reason is that the direct wave is not reflected only at the line end. Thus the lowest resonance frequency is obtained for \( m=1 \), but the first higher resonance frequency is not the one obtained from eq. (2) for \( m=2 \). Based on the principle used to determine eq. (1), but taking into account that a partial reflection of the direct wave appears on each one of the (N-1) nodes, it is possible to write (N-1) more equations which define the resonance frequencies of the circuit shown in Fig. 1. The lowest of these frequencies is obtained when the wave reflected in the node 1 returns to the beginning of the line with its phase angle shifted to such a degree that it adds itself algebraically to the direct wave. Accordingly, this resonance frequency is in a general case determined by the following equation:

\[ \lambda(\omega_r) \sum_{1}^{N} L_{sn} - \frac{1}{2} \sum_{1}^{N-1} \phi_n^{(\omega_r)} + \frac{\psi_1(\omega_r) + \psi_N(\omega_r)}{2} = m\pi, \]  

\( m=1, 2, 3, \ldots \) 

where \( \psi_1(\omega_r) \) is the phase angle of the reflection coefficient in the node 1.

By applying an analogous procedure we can obtain the equations defining the resonance frequencies due to the reflection of the direct wave in subsequent nodes. However, only the lowest resonance frequencies are of the practical importance for us [5], and therefore we will focus our attention only to them. The previous considerations offer a basis for the following observations.

The algebraic summation of the direct current wave and the one reflected from the end of the line (i.e. the algebraic subtraction of the voltage waves [6]) at the beginning of the line does not obligatory mean the algebraic summation with all other waves reflected along the line in (N-1) nodes. It happens only in the case when the frequency in the line is such that it satisfies simultaneously all the N equations which, according to the previous explanation, may be written for the resonance frequencies of the line along which there are (N-1) points of discontinuity. The conclusion from this is that the input impedance and the voltage at the beginning of the line do not have to be equal to zero as in the case of the line without points of discontinuity (the case considered in [6]). Because of this, the current application according to which the resonance frequencies in compensated transmission lines are defined by their input impedance [2] or by the voltage at the beginning of the line [5] should be critically re-examined.

The consideration of eq. (2) brings us to the following observations. The value of the lowest resonance frequency (\( m=1 \)) depends only on the total length of the line, and not on its distribution in particular sections. When we also take into account the expressions (10) and (11) it can be seen that under the real conditions, when the admittances \( Y_i, Y_2, \ldots, Y_N \) differ among themselves, the influence of changes in the parameters of the shunt compensator must be considered for each of the compensators separately.

4. METHODOLOGY VALIDATION

To verify the accuracy of eqs. (2) and (14) we used the numerical examples and calculation results given in [4] and [5].

First numerical example

The line consists of 4 equal sections, each 386.2 km long. The line parameters are \( r=0.01, l=0.907 \) mH/km and \( c=12.55 \) nF/km. The sending and receiving ends are short-circuited, and admittances \( Y_1, Y_2, \) and \( Y_3 \) are all equal and are assumed to be pure capacitance, C. The results of calculation using (14) are shown in Fig. 2.

Second numerical example

The total length of the transmission line is 1635.1 km. There are three reactances dividing the system into four sections of lengths: \( L_{s1}=415.2 \) km, \( L_{s2}=434.5 \) km, \( L_{s3}=342.8 \) km and \( L_{s4}=442.6 \) km. The line parameters are \( r=0.0112 \) \( \Omega/km \), \( l=0.9050 \) mH/km and \( c=12.542 \) nF/km. The load bus impedance, \( Z_n \), is assumed to be zero. The generator impedance is assumed to be \( Z_g=|Z_g| \). Because of the wide range of values which \( X_g \) can take, we will consider separately five different values of \( X_g \) given in function of the output power (P.U.), as shown in Fig. 3.
The values of the shunt admittances are calculated in [5] and are determined as a function of power angle $\delta_g$ such that the moduli of the voltages of the nodes along the line are held constant, i.e. $|U_n| = U_n$ (n=1,...,N). The results of the calculation performed in [5] are presented in Fig. 4.

The results of the calculation obtained applying eq. (2) and using the presented numerical data are shown in Fig. 6.

A comparison of the results given in Fig. 4 and the results presented in the paper [5] (Fig. 5) shows that no significant discrepancies are present. The subsynchronous resonance frequencies appear when the angle $\delta_g$ is in the range from 73 to 186°, while, according to the calculations in [5] the limits of this range are 74 and 182°. The demonstrated differences may be explained by the fact that the resonance frequencies in [5] are determined under the condition that the voltage at the line beginning is equal to zero, while here from the condition that the traveling waves reflected both at the beginning and at the end of the line are summed algebraically at the beginning of the line. Which of these results is more accurate obviously depends on which of the above conditions reflects the phenomenon itself more accurately. According to the presented considerations, an advantage might well be on the side of the condition defined by the traveling waves.

5. CONCLUSIONS

The paper presents a relatively simple procedure for the identification of the causes of electrical subsynchronous oscillations in long distance radial transmission lines with static VAR compensation. The presented analytical expressions are determined from the condition that the direct and reflected traveling waves are collinear at the beginning of the line.

REFERENCES