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PHASE CONJUGATION IN A MESOSCOPIC ELECTRON INTERFEROMETER

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Invited Paper

Abstract- *We discuss the possibility of applying the techniques of nonlinear wave mixing from nonlinear optics to mesoscopic electron devices. Specifically, we focus on phase-conjugation of electrons via degenerate four wave mixing of de Broglie waves in a superconductor. It is shown that for a beam of spin polarized electrons incident on a normal-superconductor junction, a time reversed backward propagating beam is generated similar to the case in nonlinear optics. By adopting an operational definition of the phase, we show that it is possible to infer the presence of the phase-conjugate field by the loss of the interference pattern in a mesoscopic electron interferometer.*

1. Introduction

In contrast with the situation for classical fields where one can readily define the phase of the field at each point, there are considerable difficulties associated with the definition of a quantum mechanical phase operator $\hat{\phi}$ for bosonic fields. This problem has been discussed at great length in quantum optics [1], where the measurement of the phase of the electromagnetic field is of considerable importance for interferometry. This work has recently been extended to the context of Bose-Einstein condensation [2]. With none of the numerous attempts at formally introducing a phase operator for the electromagnetic field being fully satisfactory, Noh, Fougères and Mandel adopted instead an operational approach based on an analysis of what is actually measured in an experiment, namely the relative phase between interfering fields. It can be represented by a combination of photon counting operators that depends on the particular experimental scheme [3]. A similar approach has been adopted for Bose-Einstein condensates [4].

For fermions the question of the phase of the field is even more difficult than it is for bosons. Unfortunately, one cannot define a phase operator or phase eigenstates for an eigenmode of the Fermi field because of the Pauli exclusion principle: since number and phase are canonically conjugate variables, a well-defined phase requires a large uncertainty in particle number. Indeed, a straightforward generalization to fermions of the various phase operators and phase states discussed for the eigenmodes of bosonic fields leads to mathematically ill-defined results. Therefore, it would appear that if phase is to have any meaning in Fermi systems, it must be associated with a multimode collective effect.

Our goal is to show that despite the apparent difficulties associated with the concept of phase for the eigenmodes of fermionic fields, it is possible to introduce it in an operational way. As a previous indication that this might be possible, we recall that it is theoretically possible to operate interferometers with quantum-degenerate fermionic beams, thereby measuring the relative phase of the partial beams [5, 6].

Mesoscopic electron interferometers have been fabricated over the last two decades that exhibit coherent electron transport in structures that are smaller than the electron coherence length, $\ell_\phi = (D\tau_\phi)^{1/2}$ where D is the diffusion coefficient and τ_ϕ the time between phase randomizing scatterig events[7]. These interferometers exhibit an interference pattern as a function of the magnetic flux through the center of the interferometer due to the Aharonov-Bohm effect where the electron phase acquired through the interferometer is $\phi = 2\pi(e/h)\int \mathbf{A}\cdot d\ell$ [8]. Similar interferometers have been used to study the coherent transmission through a quantum dot imbedded in one arm of the interferometer[9]. More recently an electronic version of the optical Mach-Zehnder interferometer [10] has been constructed from a two dimensional electron gas [11]. In this interferometer, quantum point contacts acted as beam splitters for the incident electrons and ohmic contacts count the electrons from the two output ports of the interferometer.

In polarizable media with nonlinear susceptibilities [12], $\chi^{(n)}$, it is possible to produce a light field that is the phase-conjugate state of some incident signal light beam. This process is known as optical phase conjugation and occurs when the signal combines with a strong classical laser field known as the pump inside a nonlinear medium to generate an idler field that is the time-reversed state of the signal field. This process can occur via three-wave mixing in a $\chi^{(2)}$ medium or by four-wave mixing in a $\chi^{(3)}$ medium [13]. In classical optics, phase conjugation can be used to correct the phase aberrations incurred by the signal field while in quantum optics, phase conjugation via four-wave mixing can lead to the generation of squeezed states [14]. Here, we show that it is possible to phase conjugate a beam of phase coherent fermions, so that its evolution is "time reversed". This is clear evidence that from an operational point-of-view, that the phase of a fermionic beam is a perfectly appropriate concept.

Since matter waves interact via two-body collisions there is no matter-wave analog of three-wave mixing.

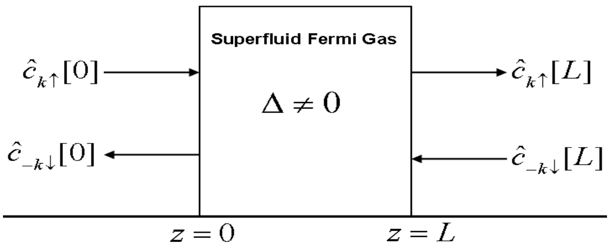

FIG. 1: Schematic diagram of input-output relations for fields incident on superfluid Fermi gas.

However two-body collisions are analogous to a polarization of the electric field $\mathbf{P} = \epsilon_0 \chi^{(3)} \mathbf{E}^3$, which leads to four-wave mixing of optical fields. Therefore one can extend the discussions of four-wave mixing in optics to matter-waves [15–18]. In order to achieve phase-conjugation via four-wave mixing of fermionic fields, one needs a c-number order parameter that is analogous to the two counterpropagating pump lasers used in optical experiments. This type of c-number field can be achieved if the wave mixing occurs inside a superconducting material where the BCS order parameter serves the same role as the optical pump lasers.

2. Model

Specifically, we consider two counterpropagating beams of fermions interacting with a phase-conjugate mirror (PCM) see Fig. 1. This "mirror" is formed by a superconductor at zero temperature confined in the region $0 \leq z \leq L$, with a normal conductor in the region $z < 0$ and $z > L$. The extent of the gas in the x and y directions is taken to be infinite so that we can treat the gas as being spatially homogeneous with density $n_F = N_F/V$ and volume V.

The phase-conjugate mirror is therefore described by the linearized BCS Hamiltonian in the region $0 < z < L$ [19],

$$H = \sum_{\mathbf{k}} \left[\hbar\omega_{\mathbf{k}} \left(\hat{c}^\dagger_{\mathbf{k}\uparrow} \hat{c}_{\mathbf{k}\uparrow} + \hat{c}^\dagger_{-\mathbf{k}\downarrow} \hat{c}_{-\mathbf{k}\downarrow} \right) + \hbar\Delta \left(\hat{c}^\dagger_{\mathbf{k}\uparrow} \hat{c}^\dagger_{-\mathbf{k}\downarrow} + \hat{c}_{-\mathbf{k}\downarrow} \hat{c}_{\mathbf{k}\uparrow} \right) \right] \quad (1)$$

where $\omega_{\mathbf{k}} = \hbar k^2/2m - \hbar k_F^2/2m$ and $k_F = (6\pi^2 n_F)^{1/3} \gg L^{-1}$ is the Fermi wave number of the gas. $\Delta = -|g| \sum_{\mathbf{k}} \langle \hat{c}_{-\mathbf{k}\downarrow} \hat{c}_{\mathbf{k}\uparrow} \rangle$ is the BCS order parameter [19] where $g < 0$ represents the strength of the attractive interactions and $\hat{c}_{\mathbf{k}\sigma}$ is the annihilation operator for an electron of momentum $\hbar\mathbf{k}$ and spin σ.

The electrons incident on the PCM at $z = 0$ are spin polarized with momenta $\hbar\mathbf{k} = \hbar k\hat{\mathbf{z}}$, $k > 0$. Similarly, at $z = L$ spin-down electrons are incident on the gas with momenta $\hbar\mathbf{k} = -\hbar k\hat{\mathbf{z}}$, $k > 0$. The number of atoms in these beams, N_B, satisfy $N_B \ll N_F$ so that the superfluid electrons forming the PCM can be treated as undepleted.

Now consider a fermion initially located at $z < 0$ and described as a wave-packet with average momentum $\hbar k_0$, $\psi(z,t) \sim \psi(z - v_{k_0}(t - t_0), 0)$ where $v_{k_0} = \hbar k_0/m_n$ is the group velocity and m_n the effective mass in the normal state conductor. When the atom enters the superconductor it propagates with the new group velocity $\bar{v}_{k_0} = \hbar\bar{k}(k_0)/m_s$ where

$$\bar{k}(k_0) = \sqrt{(m_s/m_n)k_0^2 + 2m_s g n_F/\hbar^2},$$

where m_s is the effective mass in the superconductor and gn_F is an attractive Hartree-Fock potential. The wave-packet therefore takes a time $\tau_{k_0} = L/\bar{v}_{k_0}$ to reach the other end of the mirror. Similarly, a wave-packet at $z = L$ with mean momentum $-\hbar k_0$ takes a time τ_{k_0} to reach $z = 0$. The *input* fields are then related to the initial conditions, $\hat{c}_{k\uparrow}[z = 0] = \hat{c}_{\bar{k}(k)\uparrow}(t = 0)$ and $\hat{c}_{-k\downarrow}[z = L] = \hat{c}_{-\bar{k}(k)\downarrow}(t = 0)$. (To simplify the notation we define $\hat{c}_{k\hat{\mathbf{z}}\sigma} = \hat{c}_{k\sigma}$.) The output states, $\hat{c}_{k\uparrow}[L]$ and $\hat{c}_{-k\downarrow}[0]$, are then obtained by integrating the equations of motion from $t = 0$ to τ_k.

The Hamiltonian H may be diagonalized by the Bogoliubov transformation

$$\hat{\alpha}_{\mathbf{k}\uparrow} = \cos(\theta_{\mathbf{k}}/2)\hat{c}_{\mathbf{k}\uparrow} - \sin(\theta_{\mathbf{k}}/2)\hat{c}^\dagger_{-\mathbf{k}\downarrow}, \quad (2)$$

$$\hat{\alpha}^\dagger_{-\mathbf{k}\downarrow} = \cos(\theta_{\mathbf{k}}/2)\hat{c}^\dagger_{-\mathbf{k}\downarrow} + \sin(\theta_{\mathbf{k}}/2)\hat{c}_{\mathbf{k}\uparrow}, \quad (3)$$

where $\alpha_{\mathbf{k}\sigma}$ is an annihilation operator for a quasiparticle in the gas with energy $\hbar\zeta_{\mathbf{k}} = \hbar\sqrt{\omega_k^2 + \Delta^2}$ and $\tan\theta_{\mathbf{k}} = |\Delta|/\omega_k$. By using the Bogoliubov transformation (2-3) and the solution of the equations of motion for the quasiparticles, $\hat{\alpha}_{k\sigma} = \hat{\alpha}_{k\sigma}(0)\exp[-i\zeta_k t]$, one readily obtains the output states in terms of the input states as

$$\hat{c}_{k\uparrow}[L] = T_{\bar{k}(k)}\hat{c}_{k\uparrow}[0] + R_{\bar{k}(k)}\hat{c}^\dagger_{-k\downarrow}[L] \quad (4)$$

$$\hat{c}_{-k\downarrow}[0] = T_{\bar{k}(k)}\hat{c}_{-k\downarrow}[L] + R^*_{\bar{k}(k)}\hat{c}^\dagger_{k\uparrow}[0] \quad (5)$$

where

$$T_k = \cos(\zeta_k mL/\hbar k) - i\cos\theta_k \sin(\zeta_k mL/\hbar k), \quad (6)$$

$$R_k = i\sin\theta_k \sin(\zeta_k mL/\hbar k). \quad (7)$$

Notice that phase conjugation only occurs when $R_k \neq 0$, the output states being then a superposition of the transmitted input state plus its time-reversed state, $\hat{c}^\dagger_{-k\downarrow}[0] = \mathcal{T}\hat{c}_{k\uparrow}[0]\mathcal{T}^{-1}$ where $\mathcal{T}$ is the time-reversal operator. Note that $R_k = 0$ in the absence of a superfluid state, $\Delta = 0$, so that the existence of this state is essential for the operation of the PCM. Just as is normally the case in optics, the phase-conjugate mirror has a finite bandwidth, since $\sin\theta_k = |\Delta|/\zeta_k$ is only different from zero in the interval $\delta k \approx |\Delta|m/\hbar k_F$ around k_F. The phase-conjugate signal is therefore optimized by using an input state with average momentum $\hbar k_0$ and bandwidth Δk such that $\bar{k}(k_0) = k_F$ and $|\bar{k}(k_0 + \Delta k) - \bar{k}(k_0 - \Delta k)| < \delta k$. In this case one can make $|R_{k_F}| = 1$ for $L = (2j + 1)\pi\hbar k_F/2m|\Delta|$ for $j = 0, 1, 2, \ldots$.

3. Discussion

Since $|T_k|^2 + |R_k|^2 = 1$, there is no amplification of the individual modes of the fermion beams, which is a direct consequence of the Pauli exclusion principle. This is in stark contrast to the case of bosons where one has instead $|T_k|^2 - |R_k|^2 = 1$ [13]. The transmitted field is therefore always amplified for bosons since $|T_k| \geq 1$. Note, however, that the total number of fermions in the output beams can be amplified. To see this, we take for definiteness the input state of the fermion beams to be

$$|\Psi\rangle = \prod_{|k-k_0|\leq\Delta k} \hat{c}^\dagger_{k\uparrow}[0]|0\rangle \tag{8}$$

with $k_0 > \Delta k > 0$. This corresponds to a beam of spin-up fermions with momenta centered around $\hbar k_0$ incident from $z < 0$ with no particles incident from $z > L$. The occupation numbers for this state are $n_k = \langle \hat{c}^\dagger_{k\uparrow}[0]\hat{c}_{k\uparrow}[0]\rangle$. Defining the total number operators for the input and output fields as $\hat{N}^{(\text{in/out})}_\uparrow = \sum_{k>0} \hat{c}^\dagger_{k\uparrow}[0/L]\hat{c}_{k\uparrow}[0/L]$ and $\hat{N}^{(\text{in/out})}_\downarrow = \sum_{k>0} \hat{c}^\dagger_{-k\downarrow}[L/0]\hat{c}_{-k\downarrow}[L/0]$, one finds that their expectation values are,

$$\langle \hat{N}^{\text{out}}_\uparrow\rangle = \langle \hat{N}^{\text{in}}_\uparrow\rangle + \langle \hat{N}^{\text{out}}_\downarrow\rangle \tag{9}$$

$$\langle \hat{N}^{\text{out}}_\downarrow\rangle = \sum_k |R_{\bar{k}(k)}|^2(1 - n_k). \tag{10}$$

Eqs. (9) and (10) show that the number of atoms in both beams increase after having passed through the gas. However, Eq. (10) shows that the increase results from the scattering of electrons out of the superfluid into those modes that are not occupied in the incident beam. Consequently, only incoherent amplification of the vacuum fluctuations occurs.

The identical increase in both beams reflects the underlying pair creation process given by the $\hat{c}^\dagger_{\mathbf{k}\uparrow}\hat{c}^\dagger_{-\mathbf{k}\downarrow}$ term in the Hamiltonian, as well as the fact that the number difference operator, $\hat{c}^\dagger_{\mathbf{k}\uparrow}\hat{c}_{\mathbf{k}\uparrow} - \hat{c}^\dagger_{-\mathbf{k}\downarrow}\hat{c}_{-\mathbf{k}\downarrow}$ commutes with the Hamiltonian. Defining the covariance matrix as $\text{Cov}[\hat{N}^{\text{out}}_{\sigma_1}\hat{N}^{\text{out}}_{\sigma_2}] = \langle \hat{N}^{\text{out}}_{\sigma_1}\hat{N}^{\text{out}}_{\sigma_2}\rangle - \langle \hat{N}^{\text{out}}_{\sigma_1}\rangle\langle \hat{N}^{\text{out}}_{\sigma_2}\rangle$ we find that

$$\text{Cov}[\hat{N}^{\text{out}}_{\sigma_1}\hat{N}^{\text{out}}_{\sigma_2}] = \sum_k |T_{\bar{k}(k)}|^2|R_{\bar{k}(k)}|^2(1 - n_k) \tag{11}$$

which shows that the number fluctuations in each of the spin polarized beams that have passed through the PCM as well as the correlations between these beams are the same. Again, this reflects the fact that an electron with definite spin created in one beam coincides with the creation of an electron of opposite spin in the other beam so that the fluctuations in both beams must be the same.

Eqs. (10) and (11) show that for $|R_{\bar{k}(k)}| = 1$ one can have amplification with no fluctuations. Thus by using an input state in which all the k states are fully occupied *except* for a narrow window of width Δk centered at k_0, one can have amplified output beams with negligible number fluctuations provided $\bar{k}(k_0) = k_F$, $|\bar{k}(k_0 + \Delta k) - \bar{k}(k_0 - \Delta k)| < \delta k$, and $L = (2j + 1)\pi\hbar k_F/2m|\Delta|$. However, since these are the same conditions required for a finite phase conjugate signal, one cannot simultaneously have a phase conjugate signal *and* an amplified output with reduced fluctuations.

These results show that despite the lack of a well-defined phase for fermionic fields, the phase-conjugation and time-reversal of these fields is readily possible in principle. In the present example, the phase-conjugate signal provides a direct signature of the BCS state since for a Fermi gas in the normal state, an incident beam of fermions would not generate a backward propagating reflected beam.

As an example to illustrate the effect of the PCM on electron transport, we consider an electronic version of the optical Michelson interferometer shown in Fig. 2. We will show how phase-conjugate beams of fermions can compensate the relative phase accumulated between the two paths of the interferometer when one of the mirrors, M_1 or M_2, is replaced by a phase-conjugate mirror. Here, BS labels a beam splitter with transmission and reflection amplitudes t and r, respectively, with $|r|^2 + |t|^2 = 1$ and $rt^* + r^*t = 0$ while D represents an electron counter. ℓ_1 and ℓ_2 are the one way path lengths in the two arms of the interferometer, which are assumed to be much less than ℓ_ϕ. The operators $\hat{a}_k$ and $\hat{b}_k$ denote annihilation operators for electrons with momentum $\hbar k$ that are incident on the two input ports of the interferometer. The input states only involve a single spin state so we drop the spin label for convenience. The outputs of the beam splitter are $\hat{g}_k = r\hat{a}_k + t\hat{b}_k$ and $\hat{f}_k = t\hat{a}_k + r\hat{b}_k$. After propagation through the arms and recombination at the beam splitter, the fermion annihilation operator at the atom counter is $\hat{d}_k = rt(e^{-i2k\ell_2} + e^{-i2k\ell_1})\hat{a}_k + (r^2e^{-i2k\ell_2} + t^2e^{-i2k\ell_1})\hat{b}_k$. Using an input state of the same form as Eq. (8), $|\Psi'\rangle = \prod_{|k-k_0|\leq\Delta k} \hat{a}^\dagger_k|0\rangle$, we find that the number of atoms at D, $\langle \hat{N}_D\rangle = \sum_k \langle \hat{d}^\dagger_k\hat{d}_k\rangle$, is

$$\langle \hat{N}_D\rangle = 2|r|^2|t|^2N_B\left(1 + \cos(\Delta\ell k_0)\frac{\sin(\Delta\ell\Delta k)}{\Delta\ell\Delta k}\right) \tag{12}$$

where $\Delta\ell = 2(\ell_1 - \ell_2)$ and N_B is the number of incident atoms. There is a discernible interference pattern in the "white light interferometry" regime [20], $\Delta\ell\Delta k \ll 1$. However, the broadband nature of the Fermi beam leads to a loss of contrast due to *dephasing* between different k for large path differences, $\Delta\ell\Delta k \gg 1$. Even though the incident fermions are in Fock states, which for the sake of argument can be regarded as a classical field with random phases ϕ_k for each of the occupied k-states, this phase is the same in both arms of the interferometer so that the phase difference between the arms is independent of ϕ_k.

We now replace the mirror M_2 by a phase-conjugate mirror. The reflected output of that mirror is $\hat{f}'_k = R^*_{\bar{k}(k)}\hat{f}^\dagger_k e^{+ik\ell_2} + T_{\bar{k}(k)}\hat{c}_k$ where $\hat{c}_k$ is an annihilation operator for the fermions incident on the other side of the mirror in the opposite spin state, see Eqs. (4-5). The annihilation operator for fermions at the counter is

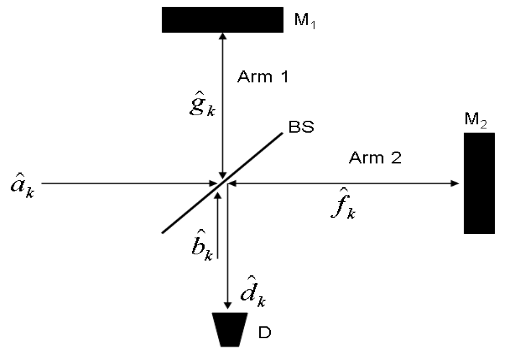

FIG. 2: Schematic diagram of a Michelson interferometer.

now $\hat{d}_k = te^{-i2k\ell_1}(r\hat{a}_k + t\hat{b}_k) + R^*_{\bar{k}(k)}r(t^*\hat{a}^\dagger_{\bar{k}} + r^*\hat{b}^\dagger_{\bar{k}}) + rT_{\bar{k}(k)}e^{-ik\ell_2}\hat{c}_k$, which, again using $|\Psi'\rangle$, gives $\langle\hat{N}_D\rangle = \sum_k \left(|rtT_{\bar{k}(k)}|^2\Theta(\Delta k - |k-k_0|) + |rR_{\bar{k}(k)}|^2\right)$ where Θ is the unit step function. In this case, there is no detectable interference pattern. To understand this we note that the phase in arm 1 is $2k\ell_1 + \phi_k$ while the accumulated phase after round trip propagation in arm 2 is $-\phi_k$ due to the phase-conjugate mirror. Since ϕ_k is a random variable, there is no interference pattern since the average phase difference between the arms is zero. A similar effect has been predicted for chaotic bosonic fields [21]. Based on our semi-classical analogy, we see that if $\phi_k \equiv 0$ then there would still be an interference pattern of the form (12) but with $\Delta\ell = 2\ell_1$ since the path length of arm 2 is zero due to the time reversal of the PCM. Therefore even in the limit of a monochromatic beam, the absence of the ensemble average interference pattern is attributable to the random phases, ϕ_k, associated with each mode.

4. Conclusion

In conclusion we have shown that one can directly probe the phase of individual electron states via a process analogous to optical phase-conjugation. We have shown that this phase, although random, can have observable effects in an electron interferometer. Future work will examine the role of environmental dephasing on the phase conjugate signal [22].

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