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MODIFICATION OF PLANCK BLACKBODY RADIATION BY LAYERED STRUCTURES CONTAINING NEGATIVE REFRACTIVE INDEX METAMATERIALS

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Abstract – *We investigated the power spectrum of thermal radiation in one-dimensional ordered structures consisting of alternating layers of two materials with positive and negative refractive index. We utilized an indirect approach based on the Kirchoff's second law and applied the transfer matrix method to calculate emittance in thermal equilibrium starting from the transmittance and reflectance of the structure. We used the obtained emittance to derive the power spectrum of the multilayer structure under investigation. We analyzed both on-axis and off-axis radiation. We compared the results with those obtained for conventional one dimensional multilayer all-dielectric films.*

1. INTRODUCTION

Negative refractive index metamaterials (NRM) are artificial composite subwavelength structures with effective electromagnetic response functions (permittivity and permeability) artificially tuned to achieve negative values of their real part [1]. These materials were theoretically predicted by Veselago [2], rediscovered by Pendry [3], experimentaly confirmed by Smith [4] and both experimentaly and theoretically investigated by many different teams (see e.g. [1] and references cited therein).

The underlying principle in constructing NRM relies on the appearance of effective permittivity and permeability both lower than zero in the same well defined frequency band. The analyticity of refractive index regarded as a complex function and the causality principle require that the real part of the refractive index also be negative [5]. A consequence is that the product of the electric and magnetic field vectors is antiparallel with the wave vector, i.e. we deal with backward waves whose phase velocity is antiparallel with Poynting vector, while electric, magnetic field and wave vector form a left-oriented set. This is the reason why such structures are sometimes called "left-handed materials" (LHM).

Many interesting phenomena not appearing in natural media were predicted and observed in double negative materials. These include negative refraction (the reversal of the Snell's law), perfect lensing [6], the appearance of subwavelength resonant cavities [7], reversal of Cherenkov radiation etc.

An interesting topic of investigation is the distribution of electromagnetic modes in layered structures incorporating negative index materials. The use of conventional photonic crystal structures to modify thermal radiation was investigated by Cornelius and Dowling [8]. Subsequent theoretical and experimental results on the same topic include [9]-[11].

Until today, no comprehensive analysis of the influence of the NRM media to the thermal radiation distribution appeared in literature. In [12] a modification of Planck law in NRM was derived relying on a simple quantized field description. There are few other papers dealing with the quantum field description of NRM-related phenomena and some interesting phenomena that arise from it, like the modification of spontaneous emission [13] and super-radiance effect [14].

In this paper we investigate the modification of thermal radiation power spectrum by one-dimensional structures incorporating both NRM and conventional materials. We analyze periodic 1D structures and include both normal and oblique wave incidence. We use an indirect method based on the second Kirchoff's law for thermal radiation to investigate the emittance of blackbody when a photonic crystal incorporating NRM is used as a filter on a thick blackbody. In our calculations we use the well known transfer matrix technique [15], [16].

2. THE PLANCK LAW IN NRM

Since metamaterials are structured on a subwavelength scale, it is assumed that their magnetic and electric response can be described by effective permeability μ and permittivity ε. A practice often met in literature when investigating NRM is to analyze the cases with frequency independent ε and μ [17]-[20], but a realizable metamaterial must be dispersive and lossy in order to preserve causality principle [5].

For the sake of simplicity we assume that both effective permittivity and permeability are of the same form

$$\varepsilon_{eff}(\omega) = 1 - \frac{\omega_{pe}^2}{\omega(\omega + i\Gamma_e)}, \quad \mu_{eff}(\omega) = 1 - \frac{\omega_{pm}^2}{\omega(\omega + i\Gamma_m)}, \qquad (1)$$

and that plasma frequencies and dumping constants are equal $\omega_{pe} = \omega_{pm} = \omega_p$, $\Gamma_{pe} = \Gamma_{pm} = \Gamma_p$. This choice simplifies the form of ε and μ without a loss of generality. The refractive index $n_{eff}(\omega) = \sqrt{\varepsilon_{eff}(\omega)\mu_{eff}(\omega)}$ is thus

$$n_{eff}(\omega) = 1 - \frac{\omega_p^2}{\omega(\omega + i\Gamma)}. \qquad (2)$$

In our calculations we assume that dumping is negligible, which is an acceptable approximation even from the experimental point of view [4]. When lossy metamaterial is considered, a common assumption in literature is that the dumping factor is given as a fraction of plasma frequency [21].

An expression for the Planck radiation law in NRM media was obtained in [12] by following a simple quantized-field description for radiation in negative-index material, which was assumed to be isotropic and absorptionless at frequencies of interest. The approach was based on modified Einstein coefficients of spontaneous emission and absorption in the light of a simple electric dipole transition picture. Similar results were obtained in [13], [14].

Here we follow another much simpler formal procedure in deriving the Planck law in NRM and apply the 'photonic picture' approach applicable to any media described by dispersive refractive index [22]. The density of states per volume of a photon set occupying a range of impulses $(\vec{p}, \vec{p} + d\vec{p})$ is $dG(p) = 2(4\pi p^2 dp / h^3)$, where $\vec{p} = \hbar\vec{k}$ and $\vec{k}$ is the photon wavenumber, while the factor 2 is due to the number of polarizations and h^3 stems from Heisenberg relations. We assume the standard free-space boundary conditions. The density of photons can be derived by multiplying the density of states by the mean number of photons occupying the given range. The mean number of photons is described by the Bose-Einstein distribution $\overline{N}(p)=\left[\exp(cp/kT)-1\right]^{-1}$. Thus we arrive to an expression for the density of photons within a given range of impulses

$$dn(p)=\frac{2}{h^3}\frac{4\pi p^2 dp}{\exp(cp/kT)-1} \ . \quad (3)$$

Further we write $p=\hbar\omega n(\omega)/c$, $dp=\hbar\omega\gamma(\omega)n(\omega)/c$, where $n(\omega)$ is the dispersive refractive index and $\gamma(\omega)=n(\omega)+\frac{d(\omega n(\omega))}{d\omega}$. The connection between impulse and frequency leads to the density of photons per unit freq-uency $\frac{dn(\omega)}{d\omega}$. As the last step we derive the spectral energy density as $\rho(\omega)=\frac{dn(\omega)}{d\omega}\hbar\omega$, thus arriving at the expression

$$\rho(\omega)=\frac{\hbar\omega^3}{\pi^2c^3}\frac{n^2(\omega)\gamma(\omega)}{\exp(\hbar\omega/kT)-1} \ . \quad (4)$$

The above expression is valid for NRM without absorp-tion. The same expression was derived in [12] starting from the quantized-field approach for isotropic, dispersive and lossless media. Although our approach is formal and is characterized by a lack of distinction between NRM and ordinary media, it is much simpler and faster in obtaining the result given by the lengthier *ab initio* derivation [12].

As we can see from (4) the Planck's law in NRM media differs from that in the free-space by a factor describing the dispersive properties of the media

$$\rho^{DNG}(\omega)/\rho^{F.S.}(\omega)=n^2(\omega)\gamma(\omega) \ . \quad (5)$$

For the dispersion relations we use the simple form (2) to establish the modification factor of the power spectral density of equilibrium radiation within NRM versus that in vacuum as

$$n^2(\omega)\gamma(\omega)=\left(1-(\omega_p/\omega)^2\right)^2\left(1+(\omega_p/\omega)^2\right) \ . \quad (5)$$

2. THERMAL RADIATION AND PHOTONIC CRYSTAL CONTAINING NEGATIVE INDEX MEDIA

We consider a system in thermal equilibrium at a given temperature, its radiation having a Planck's blackbody (BB) spectrum. From the point of view of prospective practical applications, the blackbody radiation may be modified using a photonic crystal filter and thus altering the spectral emissivity of the BB radiator and/or changing the angular distribution of the radiation. This was done in [8] for the case of purely positive-media structures.

In a most general case the photonic crystal filter may have a full 3D periodicity (or even be quasi-periodic). According to the John and Wang model [8], under certain conditions this can be reduced to a 1D case without a loss o generality.

To calculate the modification of thermal radiation, it is necessary to determine the thermal emmittance E of the photonic crystal. This is done by an "indirect" method based on the Kirchoff's law of detailed balance. According to it a material's emmittance in thermal equilibrium is proportional to its absorbtance, and for a blackbody they are equal [8]. The absorptance is defined by the reflection and transmission coefficient, $A = 1 - R - T$. Once the emittance is obtained, its multiplication by the Planck power spectrum gives the power spectrum of the PBG emitter $\rho^{PBG}(\omega)$ in terms of its emittance $E^{PBG}(\omega)$ and the blackbody spectrum $\rho^{BB}(\omega)$

$$\rho^{PBG}(\omega)=E^{PBG}(\omega)\rho^{BB}(\omega) \ . \quad (6)$$

Fig. 1 represents a photonic band gap (PBG) structure selected to modify the mode density of radiation of the emitting substrate S. In a general case it contains both positive index materials and NRM. The structure inhibits thermal emittance from the substrate at frequencies within the PBG, but enhances it at the band-edge. This result is confirmed for the case of all-dielectric and metal-dielectric positive index PBG materials both theoretically [10] and experimentally [11].

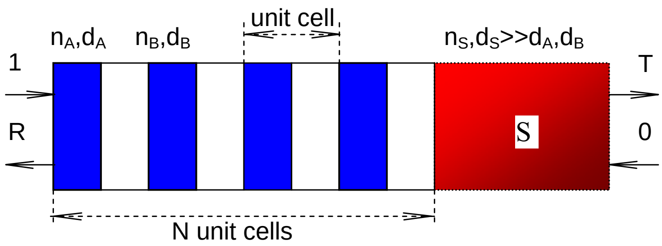

Fig. 1. *Basic 1D PBG structure for emissivity control*

The structure is composed from two media with refractive indices (n_A, n_B) and a geometrical thickness (d_A, d_B), A and B denoting the conventional and the NRM slabs, respectively. The structure is deposited on a thick substrate ($d>>\lambda$) with an index n_S. We chose layers with a quarter-wavelength optical thickness $n_Ad_A=n_Bd_B=\lambda_0/4$. Hence, the phase shifts in the corresponding layers are $\delta_A=(\pi/2)\Omega$ and $\delta_B=(\pi/2)\Omega$ where $\Omega=\lambda_0/\lambda=\omega/\omega_0$ is normalized frequency. The entire structure is surrounded by a medium n_0 (air or vacuum).

The transfer matrix technique which includes material dispersion and absorptive losses [15],[16] can be used to compute transmission and reflection coefficients and the power spectrum. We apply it using interface matrices $M_{\alpha\beta}$ (α,β denote *a* or *b* at the interface) and propagation matrices M_γ (γ denotes *a* or *b* of the given layer).

$$M_{\alpha\beta}=1/2[1+n_\beta/n_\alpha \ 1-n_\beta/n_\alpha; 1-n_\beta/n_\alpha \ 1+n_\beta/n_\alpha] \text{ and} \quad (6)$$

$$M_\gamma=[\exp(-i\delta_\gamma) \ 0; 0 \ \exp(i\delta_\gamma)], \ \delta_\gamma=2\pi n_\gamma L_\gamma/\lambda \ . \quad (7)$$

The product of $M_{\alpha\beta}$ and M_γ uniquely describes wave propagation through the multilayer. In the case of oblique incidence the matrices (6), (7) retain the same form, but the n_α is replaced by $n_\alpha\cos(\theta_\alpha)$ in the propagation matrices for both s- and p-polarization, $n_\alpha\rightarrow n_\alpha\cos(\theta_\alpha)$ for s-polarization and $n_\alpha\rightarrow n_\alpha/\cos(\theta_\alpha)$ for p-polarization in the interface matrices. θ_A, θ_B are the incident angles for the case of oblique incidence. The overall transfer matrix for a chosen structure is the product of the interface and the propagation matrices. The transmittance and the reflectance stem from the overall transfer matrix which has the form $M=[m_{11}\ m_{12}; m_{21}\ m_{22}]$. For a periodic structure composed from two media A and B, surrounded by a medium C the overall transfer matrix is given by $M=M_{CA}T^{(N-1)}(M_A\ M_{AB}\ M_B\ M_{BS}\ M_S\ M_{SC})$, where $T_1=M_A\ M_{AB}\ M_B\ M_{BA}$ is the unit cell transfer matrix and N denoting the number of unit cells.

$$T=|t|^2 \quad R=|r|^2 \text{ where } t=1/m_{11} \text{ and } r=m_{21}/m_{11} \ . \quad (8)$$

In our investigation we consider isotropic media for both non-dispersive and dispersive cases.

3. RESULTS AND DISCUSSION

Figs. 2a-2c show the calculated emissivity of NRM-containing 1D PBG structures for different polarizations. For a comparison, we also calculated the emissivity of an all-dielectric PBG material. Fig. 2d shows this emissivity for the unpolarized case. We obtained similar dependencies for s- and p-polarizations (not shown here). The dependence in Fig. 2d llustrates a problem pertinent to all positive-material PBG filters: such structures can either have an optimum performance in a very narrow wavelength range for all incident angles, or for a larger wavelength range, but for a very limited spatial angle. This is not the case with the NRM-containing filters. Fig. 2a-c shows that the angular dependence in emittance spectrum is much less prominent than in ordinary PBG structures. This points out to the possibility of designing efficient NRM filters almost insensitive on the radiation propagation angle. Structures containing NRM influence differently the thermal radiation spectrum in comparison to ordinary media PBG structure. The suppressed region of thermal radiation is wider, and the spectral characteristics more flat, i.e. without sharp oscillation typical for positive index materials. The influence of the angle of incidence is less noticeable than for the corresponding ordinary structures. The emittance shows no ripples and no sharp frequency shifts between polarizations. Such behavior is a result of phase compensation [23], [24].

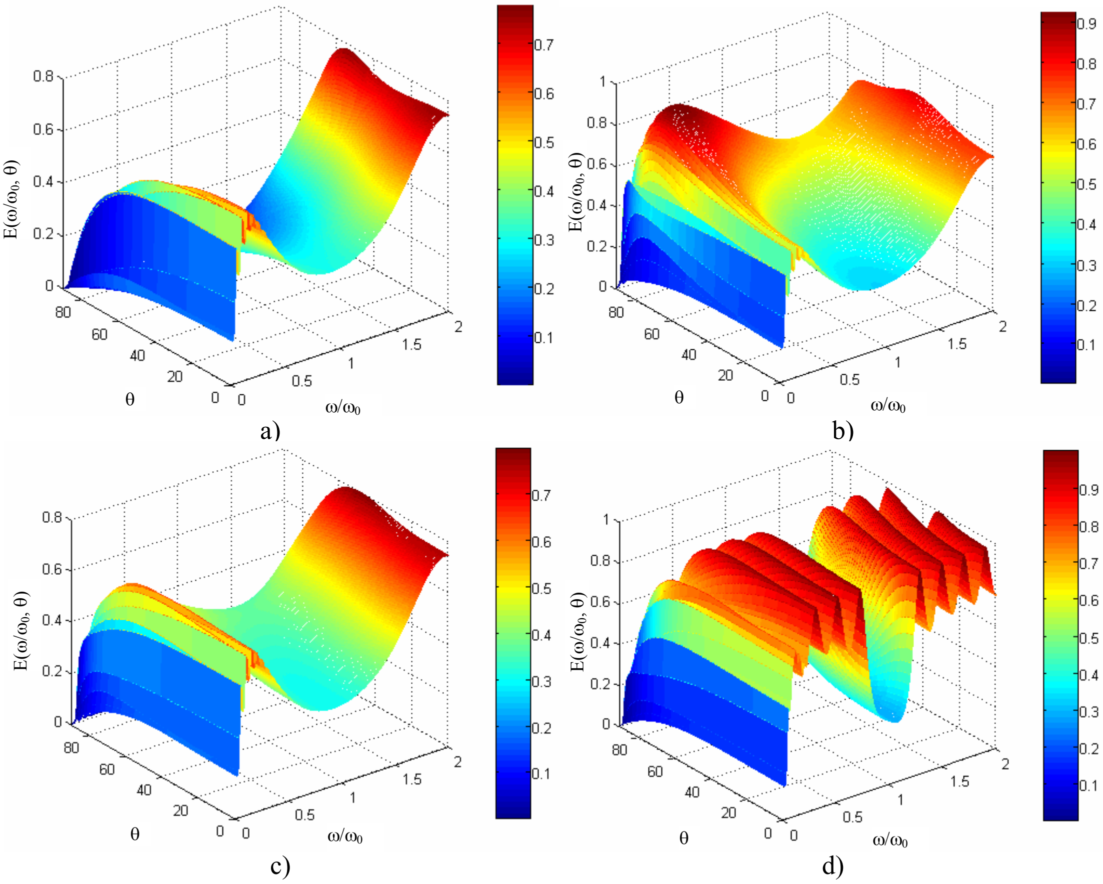

Fig. 2. *Emittance as a function of incident angle θ and normalized frequency ω/ω_0 for NRM-containing 1D PBG, 5 periods, $n_A=1.41$ $n_B=-2$; a) TE-mode polarized emittance E_{TE}; b) TM-mode polarized emittance E_{TM}; c) unpolarized $E=1/2\,(E_{TE}+E_{TM})$; d) unpolarized case, positive index material, $n_A=1.41$ $n_B=2$. In all cases $n_S=3+i0.3$ for the substrate.*

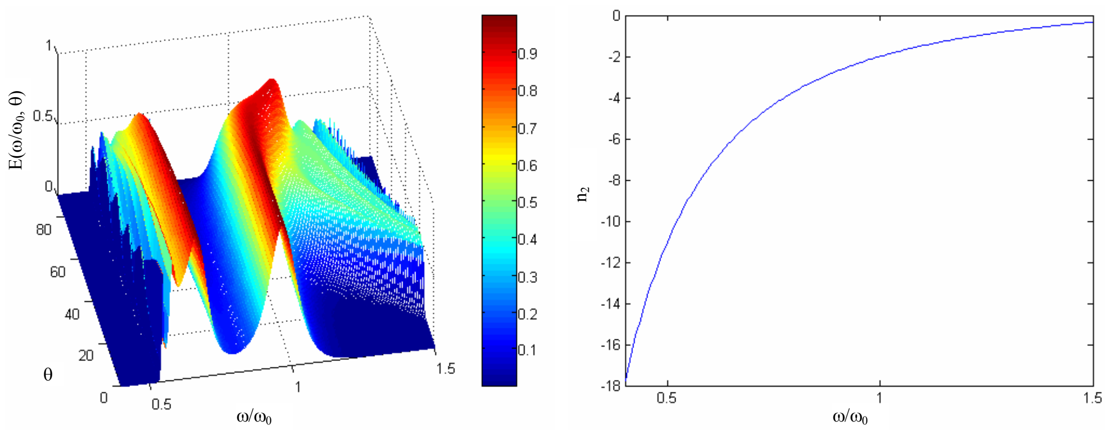

Fig. 3. *Left: emittance $E(\omega,\theta)$ as a function of incident angle θ and normalized frequency ω/ω_0 for unpolarized case, where $n_2(\omega/\omega_0)=1-3\,(\omega_0/\omega)^2$ and $\omega_0=2\pi c/\lambda_0$-quarter-wavelength frequency (at this frequency $n_2=-2$) $n_1=2, L_1=0.25\,\lambda_0/n_1$, $L_2=0.25\,\lambda_0/n_2$, $L_3=10\,\lambda_0/Re(n_3)$; right: refractive index dispersion for the NRM part*

Fig. 3 depicts a realistic case of NRM including dispersion and clearly shows the existence of full phase compensation in the emittance spectrum (the peak values in the $E(\omega,\theta)$ dependence).

Similar to zero-n photonic band gap [18], also obtained by stacking alternating layers of positive index materials and NRM but furnishing transmission minimum, this phase compensated situation arises when the averaged effective refractive index of the structure equals zero. The resulting narow transmission peak is invariant with respect to a length scale change and insensitive to angular dependence. There is a shift toward higher frequencies in spectral emittance for phase compensated situation in Fig. 3 for larger angles. The same feature can be observed both in periodic and in quasi-periodic spectra.

The quasi-periodic filter geometries are another situation of interest. Their band structure complexity is increased and sharp resonances appear [24]. Similar to periodic structures, quasi-periodic NRM multilayers also exhibit a strong influence of phase compensation to transmission [24]. Spectral self-similarity and narrow resonance spectral peaks occur, which has an applicative potential itself.

4. CONCLUSIONS

We analyzed modification of Planck's blackbody spectra by periodic structures incorporating NRM. Similar to positive-index photonic crystals to which such structures are related, they can be used to enhance, suppress or attenuate spontaneous emission in all or certain directions by changing the density of modes. The paper handles the case of finite structures and does not consider the less realistic case of infinite crystals. Our results show that structures containing NRM show larger influence to the thermal radiation spectrum than all-dielectric PBGs. The suppressed region of thermal radiation is wider, and the spectral characteristics more flat, i.e. without sharp oscillation typical for the all-dielectric case. It can be also seen that the NRM -containing 1D structures are less dependent on the angle of incident radiation. The procedure presented here is of interest in designing special types of quasi-periodic layers for emittance tailoring and will be published elsewhere. The described approach can be generalized to 2D and 3D photonic crystals incorporating NRM media.

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