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CONCERNING THE SPECIAL TRANS FUNCTION THEORY FOR SOME CLASSES OF NONLINEAR CIRCUITS EQUATIONS

Slavica M Perović, Dejan Tošić*, Sanja Bauk
Faculty of Maritime Studies, University of Montenegro
*Department of Electrical Engineering, University of Belgrade

Abstract - *The genesis of an analytical approach for some classes of nonlinear circuit equations analysis is studied in some detail, and an example is presented theoretically and numerically. Namely, using the Special Trans Function Theory (STFT) the analytical closed-form solution to the nonlinear circuit signals is obtained. The structure of derivation, appropriate proofs and numerical results for a choice example verified the proposed analytical approach and, undoubtedly, confirm the validity of applied STFT. The condition for the existence of the exact solutions is discussed.*

1. INTRODUCTION

The Special Trans Function Theory ([1]-[12]) for obtaining meaningful results from analytical study of the nonlinear circuits is an important one. The present paper researches possibilities for analytical solutions of the nonlinear circuit equations because the former methods (numerical and analytical) cannot express the solutions by an analytical closed form. The subject of the theoretical analysis presented in this Introduction, at the present Special Trans Function Theory understanding level [1], is a nonlinear functional equation of the following general form

$$\sum_{n=0}^{N}\alpha_n Y^n = \sum_{k=1}^{K}\beta_k Y^k e^{-kY} + \delta \qquad (1)$$

The parameters have the following mining: N is the maximum of degree of nonlinearity to the researched functional equation; K is the maximum of exponential terms of nonlinear functional equation; α_n, β_k are functional parameters inherent to the real nonlinear circuit described by transcendental equation (1). Analytical closed form solution to the nonlinear functional equation (1), obtained by application of the STFT, is a new special function, defined as

$$Y = \mathrm{trans}_{\text{"IDprocess"}}(\alpha_n, \beta_k); \quad n = 1,\ldots,N; \quad k - 1,\ldots,K \qquad (2)$$

where “IDprocess” denotes an important characteristic of the studied nonlinear circuit, or a significant parameter to the nonlinear circuit.

The genesis of analytical solution, as new special function in the STFT, is not complicated, and it is based on the fact that the transcendental equation (1) can be identified with a suitable partial differential equation. Consequently, the transcendental equation (1) can be identified with a partial differential equation of type ([1])

$$\sum_{n=0}^{N}\alpha_n \frac{\partial^n \Phi}{\partial x^n} = \sum_{k=1}^{K}\beta_k \frac{\partial^k \Phi}{\partial x^k} + \delta\Phi \qquad (3)$$

This equation for identification (EQID) has the analytical, unique, closed form solution and particular solution which has the appropriate asymptotic nature in comparison with the unique analytical closed form solution. Namely, the equation for identification (3) validity implies the particular (as an asymptotic) unique solution existence in the exponential form:

$$F_{as}(x,\alpha_1,..\alpha_N,\beta_1,...,\beta_K) = F_{ao}\exp(Y(\alpha_1,..\alpha_N,\beta_1,...,\beta_K)x). \qquad (4)$$

By using the unique solution principle in the functional theory we obtain the following equality:

$$\lim_{x\to\infty}\left[\frac{F(x,\alpha_1,...,\alpha_N,\beta_1,...,\beta_K)}{F_{as}(x,\alpha_1,...,\alpha_N,\beta_1,...,\beta_K)}\right] = 1 \qquad (5)$$

After simple modification the equality (5) takes the form

$$\lim_{x\to\infty}\left(\frac{F(x+1,\alpha_1,...,\alpha_N,\beta_1,...,\beta_K)}{F(x,\alpha_1,...,\alpha_N,\beta_1,...,\beta_K)}\right) =$$

$$\left(\frac{F_{as}(x+1,\alpha_1,...,\alpha_N,\beta_1,...,\beta_K)}{F_{as}(x,\alpha_1,...,\alpha_N,\beta_1,...,\beta_K)}\right) = \frac{e^{Y(x+1)}}{e^{Yx}} = \exp(Y) \qquad (6)$$

where $Y = Y(\alpha_1,...,\alpha_N,\beta_1,...,\beta_K)$. From the equation (6) the formulae

$$Y = \lim_{x\to\infty}\left(\ln\left(\frac{F(x+1,\alpha_1,...,\alpha_N,\beta_1,...,\beta_K)}{F(x,\alpha_1,...,\alpha_N,\beta_1,...,\beta_K)}\right)\right) \qquad (7)$$

directly follows. Now, we have the following definition:

$$Y = \mathrm{trans}_{\text{"IDprocess"}}(x,\alpha_1,...,\alpha_N,\beta_1,...,\beta_K) = \lim_{x\to\infty}\left(\ln\left(\frac{F(x+1,\alpha_1,...,\alpha_N,\beta_1,...,\beta_K)}{F(x,\alpha_1,...,\alpha_N,\beta_1,...,\beta_K)}\right)\right). \qquad (8)$$

The outline of some special trans function derivation is based on the fact that the STFT can be applied for an arbitrary transcendental equation in a straightforward manner ([1-12]):

-Determining the suitable partial differential equation for identification of transcendental equation (EQID). Of course, first of all, we must prove that, in real domain, the transcendental equation has a unique solution;

-Predicting the particular solution, such as asymptotic solution, of the EQID;

-Substituting the asymptotic solution in the EQID. Consequently, after this substitution, we obtain the starting transcendental equation! On the other hand, the partial differential equation must have the unique solution, obtainable by Laplace transform application (according to the Lerch’s theorem validity), for example. *Let us note that STFT paradigm is the fact that EQID has a unique analytical closed form solution*. It is clear that the class of the

functional form (including frequently case of the linear partial differential equation) to the identification of the transcendental equation (EQID), must satisfy two conditions: (a) the existence of the unique exact analytical closed form solution to the EQID and (b) the existence of the unique asymptotic solution in the appropriate form for the EQID;
-Finding the unique analytical closed form solution to the chosen EQID;
-The choice of the optimal equalization between the unique solution and the asymptotic solution. To derive the proofs of validity equalizations (5). Let us note that from the mathematical STFT point of view these proofs are important ones;
-The predicted structures of solutions are examined by the numerical approach and, detailed graphical analysis, for various parameters of the transcendental presentation. The "MATHCAD" and "MATHEMATICA" programs application are suggested.
We must make a point to the formal simplicity of Special Trans Function Theory application on mathematical models to the very different physical phenomena. From the mathematical point of view the problem of unique closed form solution existence and the unique particular solution, such as an asymptotic solution, existence is important one, too. The mathematical model of the EQID that we must use to understand its general behavior involves rigorous mathematical approach, a topic we will study in some detail in [1]. For the present, we will simply accept the following facts about a simplified, but intuitively satisfactory, EQID model is partial differential equation. Let us note that EQID is intuitively constructed by proposal to the particular solution of asymptotic type. It is not difficult for exponential asymptotic solution form, and routinely is applicable. For the other form of the asymptotic solution the problem is more complicated. Remember the Lambert's transcendental equation in [1] and [9].
The Special Trans Functions Theory is a new and consistent theory for the evaluation of the significant parameters or analytical closed form solutions to the mathematical model of the real physical processes. Finally, we must state that proposed STFT comes to be the standard analytical method to the some classes of nonlinear functional equations computation.

2. VOLTAGE DISTRIBUTION NETWORKS FOR DIODES OPERATING IN SERIES

The subject of theoretical analysis presented in this section is the nonlinear circuit from class of the nonlinear circuits described by equation (1) and given in Fig.1.
The base equation for this network (Kirchoff low) takes the form

$$-i_{D1}+i_{R_1}=-i_{D_2}+i_{R_2} \tag{9}$$

where

$$i_{D1}=i_{s_1}\left(e^{-\frac{U_{D_1}}{V_T}}-1\right);\quad i_{D_2}=i_{s_2}\left(e^{-\frac{U_{D_2}}{V_T}}-1\right) \tag{10}$$

$$i_{D1}=\frac{U_{D_1}}{R_1};\quad i_{D_2}=\frac{U_{D_2}}{R_2} \tag{11}$$

$$U=U_{D_1}+U_{D_2} \tag{12}$$

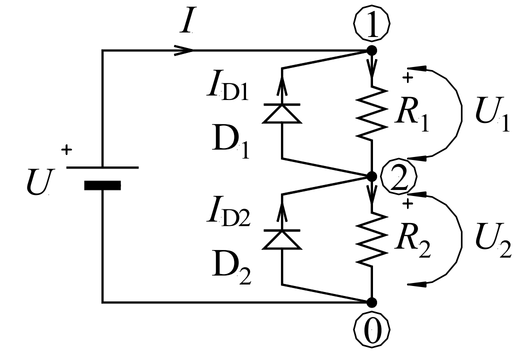

Fig.1. *Voltage distribution networks for diodes operating in series*

i_{s_1}, i_{s_2} are the saturation currents; V_T is the thermal voltage defined as $V_T = kT/e$, where k is the Boltzmann's constant; T is the junction absolute temperature in the degrees Kelvin; e is the electron charge. After substitution equations (10), (11) and (12) in equation (9) we have

$$i_{s_1}\left(1-e^{-\frac{U_{D_1}}{V_T}}\right)+\frac{U_{D_1}}{R_1}=i_{s_2}\left(1-e^{-\frac{(U-U_{D_1})}{V_T}}\right)+\frac{U-U_{D_1}}{R_2}\,. \tag{13}$$

After some modification equation (13) takes the form

$$Z\alpha e^{-Z}=e^{-2Z}+\beta e^{-Z}-\delta \tag{14}$$

where

$$\alpha=\frac{V_T}{R_1 i_{s1}}\left(1+\frac{R_1}{R_2}\right);\quad \beta=\frac{U}{R_2 i_{s1}}+\frac{i_{s2}}{i_{s1}}-1;\quad \delta=\frac{i_{s_2}}{i_{s_1}}e^{-\frac{U}{V_T}};\quad Z=\frac{U_{D_1}}{V_T} \tag{15}$$

now, it is not difficulties to see that equation (14) takes the form

$$e^{-2Y}-aYe^{-Y}-b=0 \tag{16}$$

where

$$a=\alpha e^{\frac{\beta}{\alpha}};\quad b=\delta e^{2\frac{\beta}{\alpha}};\quad Y=Z-\frac{\beta}{\alpha}. \tag{17}$$

Theorem 1. If $a\in R^+$, $b\in R^+$ and b<1 the transcendental equation (16) has the analytical closed form solution

$$Y=\mathrm{trans}_{VD}(a,b) \tag{18}$$

where $\mathrm{trans}_{VD}(a,b)$ is a new special function defined as

$$\mathrm{trans}_{VD}(a,b)=\lim_{x\to\infty}\left[\ln\left(\frac{\varphi_{VD}(x+1,a,b)}{\varphi_{VD}(x,a,b)}\right)\right] \tag{19}$$

where the function $\varphi_{VD}(x,a,b)$ takes the form

$$\varphi_{VD}(x,a,b)=\sum_{n=0}^{\left[\frac{x-1}{2}\right]}\sum_{k=0}^{n}\sum_{m=0}^{k}\sum_{p=0}^{n-k}\frac{(-1)^{k+m+p}n!b^p(x-m-k-1)^{m+1+p}}{(k-m)!m!(n-k-p)!p!(m+1+p)!a^{m+p}} \tag{20}$$

where [(x-1)/2] denotes the greatest integer less or equal to (x-1)/2.

Proof: The transcendental equation (16) can be identified with partial differential equation of type

$$\varphi_{VD}(x-2,a,b)-a\frac{\partial\varphi_{VD}(x-1,a,b)}{\partial x}-b\varphi_{VD}(x,a,b)=0 \tag{21}$$

where $\varphi_{VD}(x,a,b)$ is an arbitrary real function for x>0 and $\varphi_{VD}(x,a,b)=0$ for x<0.

This partial differential equation for identification (EQID) is analytically solvable using a Laplace Transform. Thus, after a Laplace Transform the equation (21) takes the following form:

$$\Phi(s,a,b)e^{-2s} - as\Phi(s,a,b)e^{-s} - b\Phi(s,a,b) = -a\varphi_o e^{-s} \quad (22)$$

where $\Phi(s,a,b) = L\{\varphi_{VD}(x,a,b)\}$. By using the elementary modification of equation (22) it is also easily to verified that

$$\Phi(s,a,b) = -\frac{a\varphi_o e^{-s}}{e^{-2s} - ase^{-s} - b} = \frac{\varphi_o e^{-s}}{s\left(1-\left(\frac{e^{-2s}}{as} - \frac{b}{as} - e^{-s} + 1\right)\right)}. \quad (23)$$

Using the sum infinite series method, Laplace Transform (23) takes the form

$$\Phi(s,a,b) = \frac{\varphi_o e^{-s}}{s}\sum_{n=0}^{\infty}\left(1 + \frac{e^{-2s}}{as} - e^{-s} - \frac{b}{as}\right)^n.$$

Now, using the well known binomial theorem approach the series expansion becomes

$$\Phi(s,a,b) = \frac{\varphi_o e^{-s}}{s}\sum_{n=0}^{\infty}\sum_{k=0}^{n}\frac{n!}{(n-k)!k!}\left(1-\frac{b}{as}\right)^{n-k}(-1)^k\left(1-\frac{e^{-s}}{as}\right)^k e^{-ks}$$

$$\varphi_o\sum_{n=0}^{\infty}\sum_{k=0}^{n}\sum_{m=0}^{k}\sum_{p=0}^{n-k}\left[\frac{n!}{(k-m)!m!}\cdot\frac{(-1)^{k+m+p}e^{-(k+m+1)s}b^p}{(n-k-p)!p!a^{m+p}s^{m+1+p}}\right]. \quad (24)$$

Then, we can invert term by term to obtain in the original domain x

$$\varphi_{VD}(x,a,b) = \sum_{n=0}^{\infty}\sum_{k=0}^{n}\sum_{m=0}^{k}\sum_{p=0}^{n-k}\frac{(-1)^{k+m+p}n!b^p(x-m-k-1)^{m+1+p}h(x-m-k-1)}{(k-m)!m!(n-k-p)!p!(m+1+p)!a^{m+p}}$$

where h(x-m-k-1) is the Heaviside's unit function.

Finally, applying the Laplace Transform, the analytical solution for the partial differential equation (21) can be written in the closed form representation

$$\varphi_{VD}(x,a,b) = \sum_{n=0}^{\left[\frac{x-1}{2}\right]}\sum_{k=0}^{n}\sum_{m=0}^{k}\sum_{p=0}^{n-k}\frac{(-1)^{k+m+p}n!b^p(x-m-k-1)^{m+1+p}}{(k-m)!m!(n-k-p)!p!(m+1+p)!a^{m+p}}. \quad (25)$$

3. THE UNIQUE SOLUTION PRINCIPLE

Equation (25) is analytical closed-form solution to the partial differential equation (21). On the other hand, differential equation (21) has an asymptotic solution of the form

$$\varphi_{VDA}(x,a,b) = \varphi_o \exp(Yx). \quad (26)$$

Applying the STFT ([1]-[12]), we obtain the following model of equalization in the form

$$\lim_{x\to\infty}\left(\frac{\varphi_{VD}(x,a,b)}{\varphi_{VDA}(x,a,b)}\right) = 1 \quad (27)$$

$$\lim_{x\to\infty}\left(\frac{\varphi_{VD}(x+1,a,b)}{\varphi_{VD}(x,a,b)}\right) = \left(\frac{\varphi_{VDA}(x+1,a,b)}{\varphi_{VDA}(x,a,b)}\right) = \exp(Y) \Rightarrow Y = \lim_{x\to\infty}\left[\ln\left(\frac{\varphi_{VD}(x+1,a,b)}{\varphi_{VD}(x,a,b)}\right)\right] \quad (28)$$

Consequently, we have $Y = \mathrm{trans}_{VD}(a,b)$ where $\mathrm{trans}_{VD}(a,b)$ is a new special function defined in (19). For practical calculation we have

$$\langle Y\rangle_{[G]} = \ln\left(\frac{\varphi_{VD}(x+1,a,b)}{\varphi_{VD}(x,a,b)}\right), \quad \text{for} \quad x > x_o \quad (29)$$

or, more explicitly

$$\langle Y\rangle_{|G|} = \langle \mathrm{trans}_{VD}(a,b)\rangle_{|G|} = \left\langle \ln\left(\frac{\sum_{n=0}^{\left[\frac{x}{2}\right]}\sum_{k=0}^{n}\sum_{m=0}^{k}\sum_{p=0}^{n-k}\frac{(-1)^{k+m+p}n!b^p(x-m-k)^{m+1+p}}{(k-m)!m!(n-k-p)!p!(m+1+p)!a^{m+p}}}{\sum_{n=0}^{\left[\frac{x-1}{2}\right]}\sum_{k=0}^{n}\sum_{m=0}^{k}\sum_{p=0}^{n-k}\frac{(-1)^{k+m+p}n!b^p(x-m-k-1)^{m+1+p}}{(k-m)!m!(n-k-p)!p!(m+1+p)!a^{m+p}}}\right)\right\rangle_{|G|} \quad (30)$$

where, G is the error function defined as: $G = e^{-2Y} - aYe^{-Y} - b$, x_o is value of x, when the error function G satisfies the inequality $|G| \le g_o$ for $x \ge x_o$ where g_o is an arbitrary small positive real value. $\langle Y\rangle_{[G]}$ is value of Y given with [G] accurate digits ([1], [3]).

4. THE FINAL FORM OF THE ANALYTICAL SIGNAL ANALYSIS TO THE NONLINEAR CIRCUITS OF DIODES OPERATING IN SERIES

Applying the STFT for nonlinear circuit, from equations (15), (17) and (28) we find the following expression

$$\langle U_{D_1}\rangle_{|G|} = \left\langle V_T\left(\mathrm{trans}_{VD}(a,b) + \frac{\beta}{\alpha}\right)\right\rangle_{|G|} \quad (31)$$

$$\langle i_{D_1}\rangle_{|G|} = \left\langle i_{s_1}\left(e^{-\mathrm{trans}_{VD}(a,b)-\frac{\beta}{\alpha}} - 1\right)\right\rangle_{|G|} \quad (32)$$

where $\langle \mathrm{trans}_{VD}(a,b)\rangle_{|G|}$ is defined in equation (30).

Table 1. *Numerical example for nonlinear circuit on Fig.1.*

$i_{s_1} = 10\mu A;\ i_{s_2} = 6\mu A;\ R_1 = 1k\Omega;\ R_2 = 2R_1;\ U = 0.5mV;\ V_T = 25mV$

$\alpha = 3.75;\ \beta = -0.375;\ \delta = 0.588119;\ a = 3.39314;\ b = 0.481511$

x	$U_{D_1} x10^4$ [V]	Number of accurate digits
111	**1.727**870984	4
121	**1.727**754702	4
131	**1.727**714781	4
141	**1.727**701076	4
151	**1.727696**370	6
161	**1.727694**755	6
171	**1.727694**200	6
181	**1.727694**010	6
191	**1.727693**945	7
201	**1.727693**922	7

5. THE RESULTS

To verify the theory a numerical example was carried out using the Mathematica program for equation (30). The obtained numerical results given in Table 1. show a good agreement between analytical closed form solution calculation of equation (30), by Mathematica program and numerical results obtained by solver application.

6. CONCLUSIONS

A novel analytical approach has been proposed and signals for nonlinear circuit in Fig.1. are obtained exactly in analytical closed form ((31), (32)). An attractive feature of these formulae is possibility to obtain the gradients of the form:

$$\frac{\partial S_k}{\partial v_k};\quad S_k = \{U_{D_1}, U_{D_2}, i_{D_1}, i_{D_2}\}$$

$$v_k = \{i_{s_1}, i_{s_2}, R_1, R_2, U, T\} \tag{33}$$

Namely, if we abstract that Special Trans Function genesis is complicated, we have the transcendental equation solutions obtainable in the simple forms ((18), (19) and (30)). For available data base of trans functions it is possible to obtain directly solutions. Consequently, this is an easy approach to the nonlinearity. On the other hand, from equation (30) we have possibility to obtain, for example, parameters described in (33). It is clear that this analytical parameter's analysis is impossible in the numerical approach to the nonlinear problem analysis. Let us note that the analytical parameters analysis possibility in STFT is the essential advantage in the comparison with more conventional methods.

REFERENCES

[1] Perovich S.M., The Special Trans Function Theory, Research Monograph, University of Montenegro, (2004).

[2] Perovich S.M., The transcendental method in the theory of neutron slowing down, Journal of Physics A: Mathematical and General, 25 (1992), 2969-2988.

[3] Perovich S.M., Reply to the comment on the transcendental method, Journal of Physics A: Mathematical and General, Vol. 28 (1995), 4191-4200.

[4] Perovich, S.M.; Tosic, D.V.: Transcendental method in nonlinear circuit theory. Electronics Letters, 32 (16), Vol. 32, (1996), 1433.

[5] Perovich, S.M.; Stankovic, LJ. Analytical model to the signal analysis in the N-components R diode network, Proc. IEEE Workshop on Intelligent Signal Processing, Budapest (1999), 200 - 204.

[6] Perovich, S.M.; Djurovic, I.; Tosic, D.V.: Some Comments Concerning the Discrete Eigenvalue, Nuclear Science and Engineering, 131 (3), (1999), 439.

[7] Perovich, S.M.: "Concerning the analytical solution of the depressive equation in the linear transport theory", Transport Theory and Statistical Physics, 26 (6) (1997), 705.

[8] Perovich, SM.: Concerning a Contribution to the Special Trans Function Theory, ETF, Journals of Electrical Engineering, University of Montenegro, 5 (3) (2001), 27.

[9] Perovich S.M., Vujanovic N., et all. Concerning the High Quality Special Trans Function Calculations, Seventh ISSAT, Intern. Conf. on Reliability and Quality in Design, Washington, USA, Proc. ISSAT'01, pp 242, (2001).

[10] Perovich S.M., Djurovic I., The Special Trans Function Theory to the Conductive Fluid Level Estimation, Proc. Sicon'02 ISA/IEEE Sensors for Industry Conference, 19-21 November 2002, Houston, TX, 77027 USA, pp. 291, (2002)

[11] Perovich S.M., Bauk S., Kulic R., The Special Trans Function Theory to the AM-detector transfer factor analytical analysis, 9 th IEEE, ICECS'2002, September 15-18, Dubrovnik, Croatia, Proc. ICECS'2002, pp.991, (2002).

[12] Perovich S.M. and Bauk S.I., The STFT Time Signal Analyzes to the Nonlinear Greatz's Circuits, Seventh International Symposium on Signal Processing and its Application, ISSPA/IEEE 2003, 1-4 July, Paris, France, (2003).

Sadržaj – U radu je detaljno razmotrena geneza analitičkog pristupa u analizi nekih klasa jednačina koje opisuju nelinearna kola. Prezentiran je odgovarajući primjer, teorijski i numerički. Naime, korišćenjem Teorije specijalnih trans funkcija (STFT) dobijaju se analitička rješenja nelinearnih jednačina, odnosno, vrijednosti signala u nekim nelinearnim kolima. Struktura izvođenja, odgovarajući dokazi i numerički rezultati, za odabrani primjer, verifikuju predloženi analitički pristup i nesumnjivo potvrđuju validnost primijenjene STFT. Diskutovan je i uslov egzistencije egzaktnih rješenja.

O TEORIJI SPECIJALNIH TRANS FUNKCIJA ZA NEKE KLASE JEDNAČINA KOJE OPISUJU NELINEARNA KOLA

Slavica M Perović, Dejan Tošić, Sanja Bauk