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LQG OPTIMIZATION APPLIED TO DECENTRALIZED CONTROL OF A PLATOON OF VEHICLES

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Abstract *– In this paper the Stochastic Inclusion Principle is applied to decentralized LQG suboptimal longitudinal control design of a platoon of automotive vehicles. Starting from a stochastic linearized platoon state model, input/state overlapping subsystems are defined and extracted after an adequate expansion. An algorithm for approximate LQG optimization of these subsystems is developed. Vehicle controllers obtained after contraction, provide high performance tracking and noise immunity.*

1. INTRODUCTION

The problem of design of automated highway systems (AHS) has attracted a considerable attention among researchers, e.g. [2,8]. AHS control architecture proposed in [8,2,18] is based on the introduction of a notion of platoons, groups of vehicles following the leading vehicles with small intra-platoon separation. Control of platoons has been studied from different viewpoints [9,7,16]. However, tuning of the local regulator parameters has been based on arguments related to relative stability, without taking into account uncertainties and possibilities to improve the performance by introducing dynamics into the regulator. In [13, 14] a systematic procedure for the design of decentralized overlapping platoon controller on the basis of LQ optimization has been described.

In this paper a generalization of the approach in [13, 14] to the stochastic case is presented. Namely, the Stochastic Inclusion Principle [11, 12] is applied to the design of decentralized LQG suboptimal longitudinal control of a platoon of vehicles, taking into account uncertainty resulting from the influence of the environment and measuring devices. The first part of the paper contains the results related to platoon modeling [8, 18, 2, 9, 14], taking into account stochastic disturbances and measurement noise. An optimization technique resembling to the methodology for deriving LQ suboptimal control for systems with the hierarchical LBT structure proposed in [6, 10, 14], is developed and presented. Contraction to the original space provides a decentralized controller for the whole platoon. Experimental results are given in order to illustrate main properties of the proposed methodology.

2. MODEL FORMULATION

It will be adopted in this paper that i-th automotive vehicle in a close formation platoon consisting of n vehicles can be represented by the following dynamic model:

$$\begin{aligned} &\dot{d}_i = v_{i-1} - v_i, \quad \dot{v}_i = a_i \\ &a_i = f_a^i(y_i - k_1^i v_i^2 - k_2^i - e_i), \quad \dot{y}_i = f_j^i(\alpha_i(u_i - y_i)), \end{aligned} \tag{1}$$

where $d_i = x_{i-1} - x_i$ is the distance between two consecutive vehicles, x_{i-1} and x_i represent their positions, v_i, a_i and $\dot{y}$ are the velocity, acceleration and jerk, respectively, $f_a^i(.)$ and $f_j^i(.)$ are static nonlinearities of saturation type, α_i represents the inverse time-constant of the basic vehicle dynamics, k_1^i and k_2^i constants defining rolling resistance, u_i is the corresponding control input, while e_i represents the white random noise force input with variance r_i^e, resulting from wind gusts and road roughness. Supposing for the sake of simplicity that n=3 and that all the vehicles have the same models, we obtain the model

$$\begin{aligned} \begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \end{bmatrix} &= \begin{bmatrix} A_v & 0 & 0 \\ A_d & A_v & 0 \\ 0 & A_d & A_v \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} + \\ &\begin{bmatrix} B_v & 0 & 0 \\ 0 & B_v & 0 \\ 0 & 0 & B_v \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} + \begin{bmatrix} G_e & 0 & 0 \\ 0 & G_e & 0 \\ 0 & 0 & G_e \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}, \end{aligned} \tag{2}$$

where $X_i^T = \begin{bmatrix} d_i & v_i & a_i \end{bmatrix}$ ($x_0 = 0$ in d_1) and

$$A_v = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & +1 \\ 0 & 0 & -\alpha \end{bmatrix}, \quad A_d = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$B_v^T = \begin{bmatrix} 0 & 0 & \alpha \end{bmatrix}, \quad G_e^T = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}.$$

Permissible control strategies should essentially be decentralized, having in mind the supposed information structure [9], i.e. the local control u_i is to be calculated on basis of the noise measurements of the local vehicle state variables $\{d_i \quad v_i \quad a_i\}$, together with the noisy information about the velocity and acceleration of the preceding vehicle $\{v_{i-1} \quad a_{i-1}\}$, which is assumed to be transmitted by appropriate communication channels. Each vehicle is also supplied with the information about the spacing, velocity and acceleration reference command $\{d_r \quad v_r \quad a_r\}$. The theory of large scale systems abounds with methodologies for both decentralized design of complex control structures and decentralized design of completely decentralized control structures, e.g. [17, 13, 14]. One of elegant and powerful methodologies is based on the Stochastic Inclusion Principle [12, 11].

3. DECENTRALIZED LQG SUBOPTIMAL PLATOON CONTROL

Following the Stochastic Inclusion Principle, a decentralized LQG suboptimal control strategy will be developed by considering a platoon of vehicles as a concatenation of overlapping "subsystems". The i-th subsystems is defined by the following state model (see [13, 14] for the deterministic case)

$$\dot{\xi}_i = \begin{bmatrix} A_L & 0 \\ \overline{A}_d & A_v \end{bmatrix} \xi_i + \begin{bmatrix} B_L & 0 \\ 0 & B_v \end{bmatrix} \begin{bmatrix} u_{i-1} \\ u_i \end{bmatrix} + \begin{bmatrix} G_L & 0 \\ 0 & G_e \end{bmatrix} \begin{bmatrix} e_{i-1} \\ e_i \end{bmatrix} \qquad (3)$$

where

$$A_L = \begin{bmatrix} 0 & -1 \\ 0 & -\alpha \end{bmatrix}, \quad \overline{A}_d^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$B_L^T = [0 \quad \alpha], \quad G_L^T = [1 \quad 0]$$

and $\xi_i^T = [v_{i-1}\; a_{i-1}\; d_i\; v_i\; a_i]$. According to (2), the overlapping part in the state matrix is, obviously, A_L with both the preceding and the following subsystems. The subsystems are not only state overlapping, but also input overlapping (they have one input in common), so that the input expansion is needed, as well. After expansion, the subsystems in the platoon model appear as disjoint. Application of the LQG methodology based on the definition of local performance indices leads to local state feedback control. Contraction to the original space provides a physically implementable control law.

3.1 Leading Vehicle control

If the leading vehicle model is represented by

$$\dot{X}_L = A_L X_L + B_L u_1 + G_L e_1 \qquad (4)$$

where $X_L^T = [v_1 \quad a_1]$, then the optimal feedback control law using noisy measurements $Y_L^T = [v_1 + n_1^v \quad a_1 + n_1^a]$ (where n_1^v and n_1^a are mutually independent white noise with variances r_1^v and r_1^a, respectively) should be found from the condition for the minimum of the performance index

$$J_L = E\left\{\int_{t_0}^{\infty} \left[(X_L - X_{1r})^T Q_L (X_L - X_{1r}) + R_L u_1^2\right] dt\right\} \qquad (5)$$

where $X_{1r}^T = [v_r \quad a_r]$ is a time-varying reference supplied to the first vehicle, known entirely in advance, and $Q_L \geq 0$ end $R_L > 0$ are corresponding weights. This is, in fact, an LQG optimal tracking problem, which can be solved in the following way [1]:

$$\begin{aligned} &u_1 = -K_1 \hat{X}_L - M_1 X_{1r}, \quad K_1 = R_L^{-1} B_L^T P_L \\ &M_1 = R_L^{-1} B_L^T (A_L - B_L K_1)^{-T} Q_L \\ &P_L A_L + A_L^T P_L - P_L B_L R_L^{-1} B_L^T P_L + Q_L = 0 \end{aligned} \qquad (6)$$

where $\hat{X}_L$ is obtained by the locally optimal Kalman filter obtained from (4).

3.2 General Subsystem Control

Control of the second vehicle assumes that the leading vehicle control is appropriately designed. Control design can be decomposed into two parts: first, u_{i-1} is found and the corresponding regulator is implemented and, second, u_i is found for the resulting system by using the complete feedback starting from the noisy state measurements. Accordingly, we have

$$u_{i-1} = -K_1 [\hat{v}_{i-1} \quad \hat{a}_{i-1}]^T - M_1 X_{1r}. \qquad (7)$$

After implementing (7), one comes to the following subsystem model

$$\begin{aligned} &\dot{\xi}_i = \begin{bmatrix} A_L - B_L K_1 & 0 \\ \overline{A}_d & A_v \end{bmatrix} \xi_i + \begin{bmatrix} 0 \\ B_v \end{bmatrix} u_i + \\ &\begin{bmatrix} G_L & 0 \\ 0 & G_e \end{bmatrix} \begin{bmatrix} e_{i-1} \\ e_i \end{bmatrix} + \begin{bmatrix} K_1 \\ 0 \end{bmatrix} \varepsilon_{i-1} + \begin{bmatrix} -M_1 \\ 0 \end{bmatrix} X_{1r} \end{aligned} \qquad (8)$$

where ε_{i-1} is the estimation error for $[\hat{v}_{i-1} \quad \hat{a}_{i-1}]^T$ obtained by Kalman filter belonging to leading vehicle control law. Now, u_i is found by minimizing

$$J_i = E\left\{\int_{t_0}^{\infty} \left[(\xi_i - X_{2r})^T Q_i (\xi_i - X_{2r}) + R_i u_i^2\right] dt\right\}. \qquad (9)$$

where $Q_i \geq 0$ and $R_i > 0$, while $X_{2r}^T = [d_r \; v_r \; a_r]$ is complete set of reference commands. The state weighting matrix is assumed to have the following specific form, coming out basically from the regulator structure adopted in [9]:

$$Q_i = \begin{bmatrix} p_1 & 0 & 0 & -p_1 & 0 \\ 0 & p_2 & 0 & 0 & -p_2 \\ 0 & 0 & q_{33} & 0 & 0 \\ -p_1 & 0 & 0 & q_{44} + p_1 & 0 \\ 0 & -p_2 & 0 & 0 & q_{55} + p_2 \end{bmatrix}. \qquad (10)$$

In (10), q_{33} influences the spacing reference tracking, p_1 and p_2 tracking of the velocity and acceleration of the preceding vehicle, respectively, while q_{44} and q_{55} influence velocity and acceleration reference tracking. An approximately optimal solution, in the sense that all the gains are assumed to be constant, is given by

$$\begin{aligned} &u_i = -K_2\hat{X}_i - M_2X_{2r} - M_3X_{1r}; K_2 = R_i^{-1}B_i^TP_2 \\ &M_2 = R_i^{-1}B_i^T(A_i - B_iK_2)^{-T}Q_i \\ &M_3 = R_i^{-1}B_i^T(A_i - B_iK_2)^{-T}P_2B_M \\ &P_2A_i + A_i^TP_2 - P_2B_iR_i^{-1}B_i^TP_2 + Q_i = 0 \end{aligned} \qquad (11)$$

where

$$A_i = \begin{bmatrix} A_L - B_LK_1 & 0 \\ \overline{A}_d & A_v \end{bmatrix}, B_i^T = \begin{bmatrix} 0 & B_v \end{bmatrix}, B_M^T = \begin{bmatrix} -M_1 & 0 \end{bmatrix}$$

$\hat{X}_i$ represent the estimate of the subsystem state obtained by using the corresponding Kalman filter, taking into account specific properties of the input disturbance Consequently, M_2 represent the feedforward gain for the complete reference X_{2r}, while M_3 compensates the effects of the disturbance.

The state feedback gain $K_i^T = [K_1^T \quad K_2^T]$ has the LBT structure, in accordance with the information supposed to be locally available.

3.3 Platoon Control

Local regulators formulated for the subsystems are to be contracted to the original space before implementation. The feedforward gains multiplying the reference signals are not contracted in accordance with the Inclusion Principle, since they are out of the feedback loop. The estimator gains are not contracted, as well, having in mind that all the local subsystem estimators remain uncontracted in the original system state space. The main additional requirement is here to keep the steady-state error at zero. It can be easily shown that the structure of M_2 and M_3 in (23) is such that the predefined information structure is preserved and a correct steady-state regime is preserved.

4. EXPERIMENTAL RESULTS

Numerous simulations have been undertaken; the platoon has been assumed to obey the nonlinear model and control has been generated according to the described algorithm. Figures 1 and 2 give time histories for a platoon of eight vehicles, containing velocities and inter–vehicle spacings; the first velocity and spacing plots correspond to a direct application of LQ feedback (not containing the estimators, [14]), while the second plots are obtained by using the whole proposed LQG suboptimal algorithm, including the local Kalman filters. The remaining design parameters have been $Q_L = diag\{200, 10\}$, $R_L = 10$, $p_1 = 100$, $p_2 = 50$, $q_{33} = 500$, $q_{44} = 300$ $q_{55} = 10$, $R_i = 10$, so that we obtained the following feedback and feedforward gains:
$K_1 = [4.472 \quad 0.710]$, $K_2 = [-4.061 - 1.258 - 7.071 \; 6.728 \; 1.291]$,
$M_1^{31} = 44.721$, $M_2^{51} = 26..672$, $M_3^{51} = -70.711$, (for $\alpha = 10$).
Tracking capabilities and noise immunity of the proposed algorithm are obvious. Comparison with the results presented in [9] shows a substantial advantage of the proposed. The authors are grateful to the staff of the PATH Program, University of Berkeley, for providing real experimental data.

5. CONCLUSION

In this paper the Stochastic Inclusion Principle has been applied to LQG suboptimal control of a platoon of automotive vehicles. Identification of input/state overlapping stochastic subsystems and their extraction by an appropriate expansion have lead to approximate LQG optimization, adapted to the LBT structure of the subsystem model. Simulation results show a high efficiency of the proposed algorithm, from the point of view of both tracking precision and noise immunity. One of the main problems for further investigations is the tracking precision in the case of long platoons.

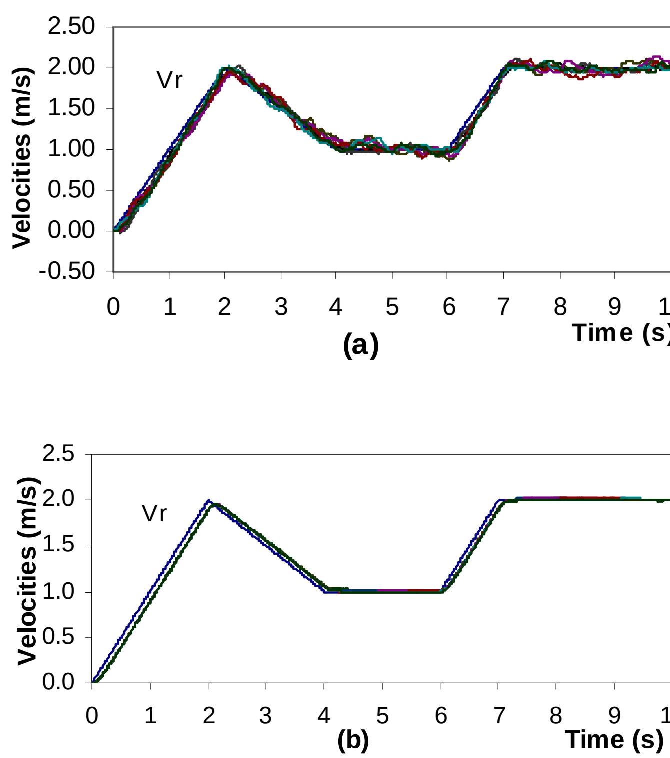

Fig. 1 *Velocities: (a) LQ and (b) LQG*

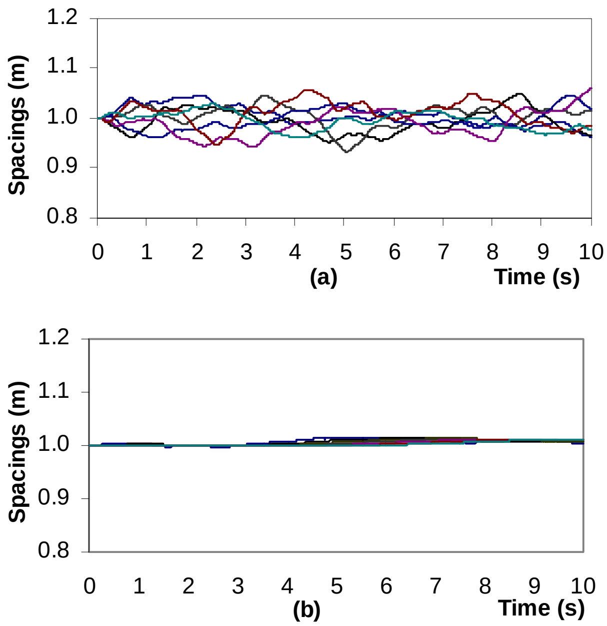

Fig. 2. *Spacing: (a) LQ and (b) LQG*

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