# FROM ROBUST LINEAR CONTROL THEORY TO SIMPLE CONTROLLERS FOR MASTER-SLAVE CURRENT-SHARING DC/DC CONVERTERS

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**Abstract** – *In this paper, the multivariable robust approach is used to obtain simple PI/PID type controllers for the master-slave current-sharing DC/DC converters. Framework for the frequency domain robust design is proposed and all of its elements are defined. The feasibility of the approach is tested with the Matlab and pSpice models of the three buck unit setup. Robust $H_\infty$ reference tracking controller, μ-optimal IMC reference tracking and $H_\infty$ loop-shaping controller are designed and verified for robustness and performance in the master-slave closed-loop control.*

## 1. INTRODUCTION

DC/DC converters [1, 2] are standard components in computer and telecom power supply systems. High power demands, modularity and redundancy reasons bring out the need of several converter units sharing the current to be supplied to the system. From the control point of view, converters in current sharing parallel arrangement make a multivariable plant to be controlled. Classical small-signal analysis and control design in the frequency domain are discussed in [3-9]. Advanced nonlinear techniques in control are presented in [10, 11].

The robustness of the control is always the main point and major problem due to fact that a model of a power supply is always an approximation of the real system. Furthermore, the dynamics of a power system may change during long-term operation, mainly because of the power components' variations. To address the difference between modeled and true systems, various measures of robustness are used [12-15]. The single DC/DC converter robust analysis and control synthesis are conducted in [16-19]. The multivariable $H_\infty$ linear robust analysis of the parallel operating converter units is the subject of the papers [20, 21]. The analysis of the control system itself is discussed in [22]. Robust design for parallel operating DC/DC converters is presented in papers [23-25]. The main drawback of the linear robust approach is the high order of the fully multivariable controllers, often unacceptable for the application both in analog and digital control.

The purpose of this paper is to investigate the possibility to use robust linear theory to obtain simple controllers for master-slave current-sharing DC/DC converters, applicable according to industrial needs. The analysis of the proposed control design will be conducted within *Matlab and Signal Processing Toolbox, The MathWorks Inc, MA-USA*. and *pSpice, Cadence Design Systems Inc, CA-USA*.

The paper is organized in sections. Sect. 2 is the place where the framework for robust control of the parallel operating converters is discussed. The development and verification of the closed-loop design on nonlinear model of three parallel operating buck DC/DC converters is the subject of Sect. 3. The conclusion is presented in Sect. 4.

## 2. FRAMEWORK FOR ROBUST SYNTHESIS IN THE CLOSED LOOP

The simplified block diagram of $n$ paralleled units and control loops is presented in Fig. 1. Each unit $j$ has PWM driver, which applies duty-ratio $d_j$ from the control subsystem to power stage switch or switches. Outer voltage-control loop is managed by the joint voltage controller $K_v(s)$, trying to achieve voltage reference $v_{ref}$ at the voltage output $v_{out}$ of the paralleled converters i.e. at the input of the load. Each converter $j$ provides measurement of its current $i_j$, which is driven by the current controller $K_{ij}(s)$ to attain reference current $i_{ref}$. Choice of weights $\alpha_j$ determines the paralleling scheme: democratic current sharing is established if all of $\alpha_j$ are equal to $1/n$, while master-slave current sharing is obtain with $\alpha_1 = 1$, $\alpha_2 = \alpha_3 = ... = \alpha_n = 0$, and $K_{i1}(s) = 0$.

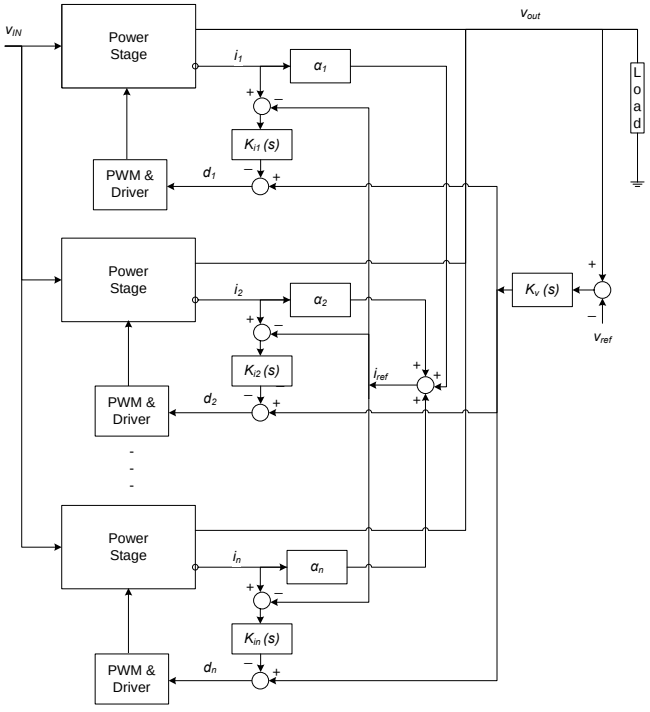


Fig. 1. *Block diagram of n paralleled dc/dc converters with current sharing control loops*

In the paper [20] multivariable model of the parallel operating converters is obtained as:

$$v_{out}(s) = \frac{1}{\frac{1}{R} + \sum_{i=1}^{n} \frac{1}{Z_{out\,i}(s)}} \sum_{i=1}^{n} \frac{P_{vi}(s)}{Z_{out\,i}(s)} d_i(s) = \sum_{i=1}^{n} P'_{vi}(s) d_i(s)\,, \quad (1)$$

$$i_{Li}(s) = P_{ii}(s) d_i(s), \quad i = 1, 2, ... n\,, \quad (2)$$

where $R$ is the nominal load, $P_{v\,i}$ is the $i$-th unit's transfer function from control to the output voltage, $P_{i\,i}$ is the $i$-th unit's transfer function from control to the unit's current and $Z_{out\,i}$. is the output impedance of $i$-th unit.

In the form of transfer functions matrix, model of the complete power stage of parallel operating converters is given by:

$$\mathbf{y}=\begin{bmatrix} v_{out} \\ i_{L1} \\ i_{L2} \\ \vdots \\ i_{Ln} \end{bmatrix}=\begin{bmatrix} P_{v1}' & P_{v2}' & P_{v3}' & \cdots & P_{vn}' \\ P_{i1} & 0 & 0 & \cdots & 0 \\ 0 & P_{i2} & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & P_{in} \end{bmatrix}\begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ \vdots \\ d_n \end{bmatrix}=\mathbf{P}(s)\mathbf{u}\,. \tag{3}$$

Since the output vector **y** is of dimension $n$+1 and there are only $n$ independent input switch control signals, the transfer function matrix **P** is not square. One way to make it square, in order to obtain a closed-loop control, is to redefine the outputs to represent the output voltage and the current distribution between the units [20]:

$$\mathbf{y}=\begin{bmatrix} v_{out} & \Delta i_{L1} & \Delta i_{L2} & \cdots & \Delta i_{Ln-1} \end{bmatrix}^T,\ \Delta i_{Li}=i_{Li}-\sum_{j=1}^{n}\alpha_{ij}i_{Lj}\,. \tag{4}$$

The transformation matrix

$$\mathbf{S}=\begin{bmatrix} 1 & 0 & 0 & 0 & \cdots & 0 \\ 0 & -1 & 1 & 0 & \cdots & 0 \\ 0 & -1 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 \end{bmatrix}. \tag{5}$$

introduces the current difference between the $i$-th unit and the reference (master) unit 0, making the redefined outputs of the squared plant **P' = SP** fit into the *master-slave* (M-S) control configuration.

Multiplicative input uncertainty of the robust modeling is presented by the matrix expression:

$$\tilde{\mathbf{P}}(s)=\mathbf{SP}(s)(\mathbf{I}+\mathbf{W_i}(s)\mathbf{\Delta}(s))\,, \tag{6}$$

where $\tilde{\mathbf{P}}(s)$ is the perturbed plant, **I** is the unity matrix, $\mathbf{\Delta}(s)$ is an unknown but unity-normed diagonal transfer function matrix that represents multiplicative uncertainty of the modeling and $\mathbf{W_i}(s)$ is the diagonal multiplicative input uncertainty bound matrix:

$$\mathbf{W_i}(s)=\mathrm{diag}(l_{MI1}^*(s),l_{MI2}^*(s),...l_{MIn}^*(s))\,. \tag{7}$$

Transfer function $l^*_{MIi}(s)$ is the uncertainty bound for $i$-th channel of the control. Uncertainty associated matrices are diagonal because parametric uncertainty of every consisting unit is not dependent on the uncertainty of the others.

As for the single unit, bound of the multiplicative input channel uncertainties for the multivariable model should be developed to asymptotically describe the *parametric uncertainty* of the linear part of the circuitry: on low frequencies they should have the value of the maximal relative error of the model DC gain, then to rise with 20 dB per decade slope, reach 0 dB level before the half of the switching frequency and remain on the constant level in high frequencies.

If the output impedance is the same for all consisting units and it is negligible comparing to the load resistance, eq. (1) simplifies to:

$$v_{out}(s)\approx\frac{1}{\frac{n}{Z_{out}(s)}}\sum_{i=1}^{n}\frac{P_{vi}(s)}{Z_{out}(s)}d_i(s)=\frac{1}{n}\sum_{i=1}^{n}P_{vi}(s)d_i(s)\,, \tag{8}$$

holding for most of the current-sharing applications having the same topology converters with the same parameters. Channel uncertainty is therefore the $n$-th part of $n$ transfer functions uncertainties i.e.

$$l_{MIi}^*(s)=l_{MI}^{PARAM}(s),\quad i=1,2,...n\,. \tag{9}$$

The block diagram of control structure is presented in Fig. 2, where $r$ denotes reference signals, $e = r - y$ is error in reference tracking, $e'$ is the performance weighted error, $d$ – plant output disturbance signal, and **K** is the closed-loop controller to be designed.

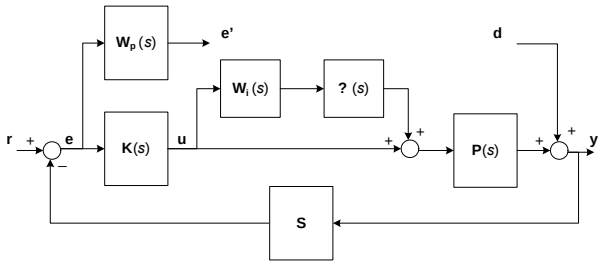


Fig. 2. *Block diagram of control setup*

With the choice of the performance weighting function $\mathbf{W_p}(s)$, dependant on the specific control design, multivariable setup depicted in fig. 2 is fully defined and ready for any robust control design procedure.

Once when control design is obtained, zero-pole cancellation should be applied and dynamics higher then the half of the switching frequency should be neglected. Further, full-order controllers are to be diagonalized to fit into the M-S control framework.

## 3. CONTROL OF THREE PARALLEL OPERATING BUCK DC/DC CONVERTERS

Parameters of the general power supply setup is: $V_{IN}$ = 10V, $V_{OUT}$ = 5V, $I_{OUT}$ = 30A. Each of the three buck DC/DC unit has the parameters: $f_{sw}$ = 50kHz, $L$ = 50μH, $R_L \approx$ 46mΩ, $C$ = 4700μF, $R_C \approx$ 24mΩ.

Maximum allowed uncertainty of the modeling is proposed [18, 21]:

$$l_{MI}^*(s)=0.5\cdot\frac{\frac{s}{\omega_0}+1}{\frac{s}{4\cdot\omega_0}+1},\quad \omega_0=9000\mathrm{rad/s}\,, \tag{10}$$

so the control in the closed loop will be tested for robustness according to proposed measures.

Nominal (M-S squared) model of the three parallel operating converters is given by:

$$\mathbf{P}'(s)=\frac{2.97}{D(s)}\begin{bmatrix} 6.15\left(\frac{s}{8865}+1\right) & 6.15\left(\frac{s}{8865}+1\right) & 6.15\left(\frac{s}{8865}+1\right) \\ -\left(\frac{s}{405}+1\right) & \frac{s}{405}+1 & 0 \\ -\left(\frac{s}{405}+1\right) & 0 & \frac{s}{405}+1 \end{bmatrix}$$

$$D(s)=\frac{s^2}{4.43\cdot 10^6}+\frac{s}{1783}+1 \qquad (11)$$

Robust performance specification $\mathbf{W_p}$ is selected to ensure zero steady-state in reference tracking, +20 dB per decade growth of sensitivity operator in the low frequency range, regulation bandwidth of 2000 rad/s and worst case robust sensitivity peak value of 1.4. Appropriate transfer function for such description is:

$$\mathbf{W_p}(s)=\frac{1}{1.4}\cdot\frac{\frac{s}{2000}+1}{s}\mathbf{I}_3, \qquad (12)$$

Three robust controllers are obtained: $H_\infty$ reference tracking controller ($H_\infty$ *rt*) [13, 14], μ-optimal IMC reference tracking (*IMC rt*) [12, 13] and $H_\infty$ loop-shaping controller (*lsdp*) [13, 15]. After zero-pole cancellation and diagonalization, voltage-loop and current loop controllers for master-slave control are obtained and enlisted in Table 1.

Table 1. *Robust controllers for three buck setup*

| Robust Design | Voltage and current-loop controllers |
|---|---|
| *lsdp* | $K_v(s)=\frac{5.26(s^2+7328s+1.43\cdot 10^7)}{s(s+8406)}$<br>$K_i(s)=\frac{0.5(s+3000)}{s}$ |
| $H_\infty$ *rt* | $K_v(s)=\frac{692.4\left(\frac{s^2}{5.87\cdot 10^6}+\frac{s}{3730}+1\right)}{s\left(\frac{s}{6.2\cdot 10^4}+1\right)\left(\frac{s^2}{1.02\cdot 10^8}+\frac{s}{1.91\cdot 10^4}+1\right)}$<br>$K_i(s)=\frac{0.33(s+2711)}{s}$ |
| *IMC rt* | $K_v(s)=\frac{0.31\left(s^2+1903s+4.5\cdot 10^6\right)}{s(s+8865)}$<br>$K_i(s)=\frac{0.0047\left(s^2+1783s+4.4\cdot 10^6\right)}{s\left(s+405\right)}$ |

Final robust controllers are of simple PI/PID type.

For multivariable plants, as the ones proposed, performance and robust stability objectives are written as requirements on the singular values of particular transfer functions [12-14].

Projected uncertainty bound along with the permitted uncertainty achieved with originally designed and diagonalized controllers are presented in Fig. 3.

It can be observed that the uncertainty level provided by the $H_\infty$ *rt* is not preserved after diagonalization and it is far bellow satisfactory. *IMC rt* has the best robustness properties, while *lsdp* control in the closed loop violates the proposed bound (10). However, this happens in the frequency region where more than 100% uncertainty is demanded, so it doesn't influence the robustness of the designed *lsdp* solution.

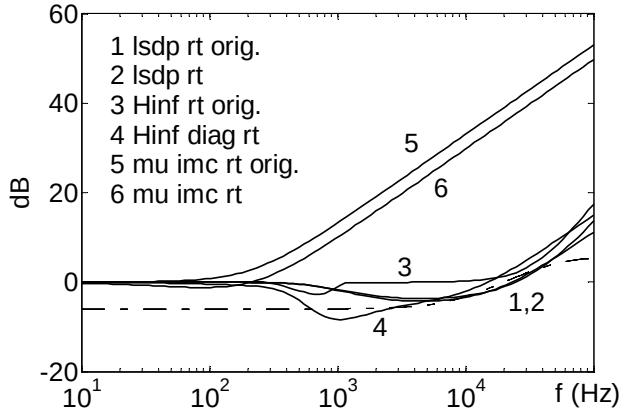


Fig. 3. *Projected and achieved uncertainty for designed controllers*

In order to verify control design performance and robustness in the time-domain, simulation is performed on nonlinear *Simulink* model of the converters with PWM and M-S control scheme fully implemented. The parameters of the second slave converter are changed to $L'=0.75L$, $C'=0.75C$, $V_{in}'=0.9V_{in}$. Load current consumption change of 33% is given (from 30A to 40A) and dynamic behavior of the perturbed slave current is presented in Fig. 4.

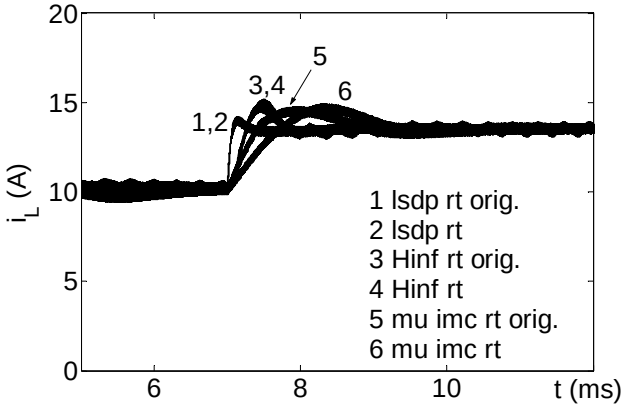


Fig. 4. *Current dynamics of the perturbed plant (Simulink model) on load consumption step-change ($\Delta I_{OUT}$=33%)*

The dynamic response quality is mainly kept with all the controllers after diagonalization. The best response is with *lsdp*, while $H_\infty$ *rt* and *IMC rt* exhibit current overshoots and are of lower bandwidth.

As *lsdp* has shown the best properties in the closed loop, it was implemented in analog technique and tested in the same setup within the *pSpice* circuit-oriented environment. A detail of the consisting converter unit model in *pSpice* is given in Fig. 5.

Dynamic response of the perturbed unit's current with the *pSpice* model is presented in Fig. 6.

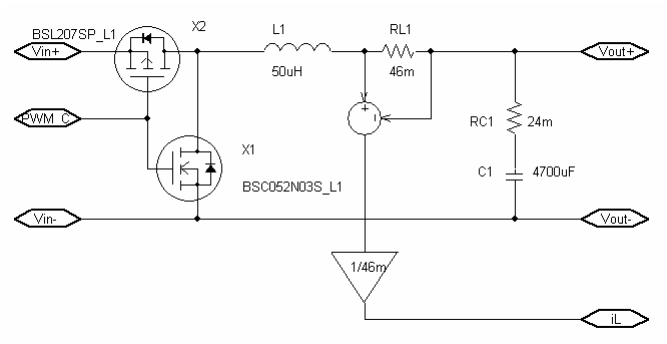


Fig. 5. *Detail of the pSpice buck converter model*

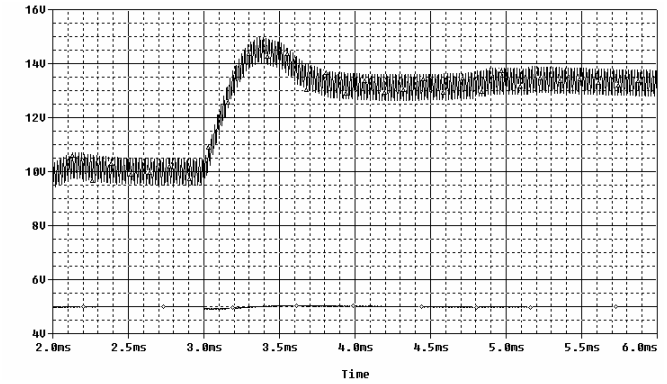


Fig. 6. *Current dynamics of the lsdp controlled perturbed plant (pSpice model) on load consumption step-change ($\Delta I_{OUT}$=33%)*

The *pSpice* simulation validates the results of *ldsp* linear design both in performance and robustness aspect.

## 4. CONCLUSION

Complex robust control theory was used to obtain simple PI/PID type controllers for the master-slave current sharing DC/DC converters. Framework for the frequency domain robust design is proposed and all of its elements are defined. Three robust controllers are designed for three buck converter setup: $H_\infty$ reference tracking controller, μ-optimal IMC reference tracking and $H_\infty$ loop-shaping controller. Designs are verified both in *Matlab/Simulink* and in *pSpice*.

Further research will be directed towards investigation in the robust design of simple controllers for current-sharing random switching DC/DC converters, which will in efficient way eliminate disturbances and relax the EMI problems in the closed loop.

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