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A MAGNETIC FIELD TOMOGRAPHY SYSTEM FOR INTERFACE RECONSTRUCTION IN MAGNETIC FLUID DYNAMICS

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Abstract – *There are several applications in magnetic fluid dynamics where it is important to know the behavior of internal fluid surfaces. Magnetic field tomography enables the observation of interface motions if optical methods can not be used. We propose a magnetic field tomography system for the identification and reconstruction of the interface in a cylindrical two-fluid system containing a liquid metal. Stochastic optimization techniques (genetic algorithm, etc.) are applied to solve this inverse field problem.*

1. INTRODUCTION

Magnetohydrodynamics (MHD) describes the behaviour of electrically conducting fluids under the influence of a magnetic field. It applies to a broad range of phenomena from astrophysics (interstellar plasma, stars, accretion disks), geophysics (planetary dynamos), and industrial processes (liquid metal casting, nuclear fusion reactors). MHD flows can behave in very different ways according to how magnetodynamics (governed by the Maxwell equations) and hydrodynamics (Navier-Stokes equation) are coupled.

In Magnetic Fluid Dynamics (MFD) several applications of material processing, metal melting or aluminium electrolysis require deep knowledge of the behaviour of the surface of liquid metals. In the case of aluminium melting in a reduction cell we are interested in how the strong current flow acts on the motion of the surface of the molten aluminium (Fig. 1).

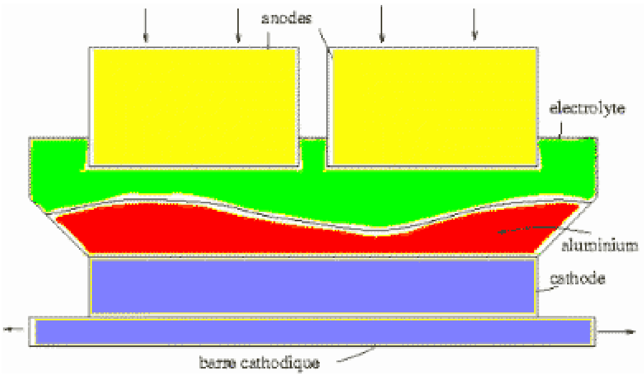

Fig. 1. *Scheme of an aluminium electrolysis cell*

Interfacial MHD waves at gravitational wave frequencies are generally supported by nonlinear magnetohydrodynamic interaction. Depending on physical parameters the wave amplitude can grow, leading to an unstable situation, or waves can dissipate due to turbulent friction and induced electric current. Because the motion can cause undesired high deformation of the surface it is important to have an opportunity to detect this deformation. Due to the limited access and usually hostile environment (high temperature, corrosive chemistry), the use of probes, though necessary for an appropriate control, is difficult or almost impossible. This is the reason why it is helpful to develop appropriate diagnosis methods for such cells.

Recently, we have started a research project to develop a magnetic field tomography (MFT) system which will enable us to reconstruct the interface between two conducting fluids using magnetic field measurements taken in a set of positions close to the surface of the object. In this paper we present the designed magnetic field tomography system, and we describe the technique we use to identify and to reconstruct, respectively, the interface between the two fluids.

2. MAGNETIC FIELD TOMOGRAPHY

A highly simplified model of an aluminum electrolysis cell was assumed to investigate a magnetic field tomography system. A cylindrical conductor contains two fluids, a low conducting compartment consisting of the electrolyte KOH, and a high conducting part representing the molten aluminum (Fig. 2). In our experiments we used Galinstan, a mixture of gallium, indium and tin. This metal is liquid at room temperature. A uniform current density, $\mathbf{J_0}$, is applied to the top of the cylinder.

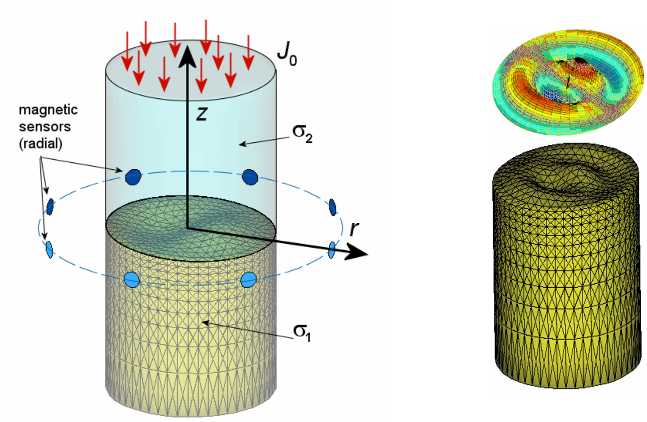

Fig. 2. *Cylindrical two-fluid cell, with a ring of external magnetic sensors (left), and a static, non-axi-symmetric interface as the surface of the lower fluid Galinstan (right).*

Due to the current flow through the cylinder a magnetic field is generated which can be measured outside the cylinder. Any deformation of the interface will change the internal current density distribution and thus the external magnetic field is modified. While the interface deformation in our experiments is established by mechanical vibrations of the cylinder at very low frequencies leading to steady-state oscillations with known interface shapes, we assume a static interface in our simulations.

The components of the magnetic flux density vector are measured in many positions (sensor ring at different z-positions). This data is then used to reconstruct the interface. The identification of the main interface shape features and the reconstruction of the whole interface, respectively, by means of the magnetic field measurements are called the inverse problem.

3. MFD MODELING

The fluid movement including the gravitational field can be described by the Euler equation [1].

$$[(\mathbf{v} \cdot \text{grad})\, \mathbf{v}] + \frac{\partial \mathbf{v}}{\partial t} = -\frac{1}{\rho}\, \text{grad}\, p + \mathbf{g} \tag{1}$$

Here is $\boldsymbol{v}$ the velocity, ρ the density, p the pressure and g the gravity. If a scalar potential is defined for the incompressible fluids in the two-compartment cylinder, fulfilling the Laplace equation and both boundary conditions

$$\Phi_1 = \Phi_2 = 0, \qquad z = \pm\infty \tag{2a}$$

$$\frac{\partial \Phi_1}{\partial r} = \frac{\partial \Phi_2}{\partial r} = 0, \qquad r = R \tag{2b}$$

we get the solution

$$\Phi_i(r, \alpha, z) = C_i J_m(k_{mn} \cdot r) \cdot e^{-k_{mn}|z| + jm\alpha - j\omega t} \tag{3}$$

where $k_{mn} = x_{mn}/R$ and x_{mn} is the n-th extreme of J_m. The interface deformation can be found from the Euler equation by substituting $\boldsymbol{v}$ = grad Φ. Thus, we finally find for (small) deformations the interface function [2]

$$\eta(r, \alpha) = A \sum_{m=-M}^{M} \sum_{n=1}^{N} \eta_{mn} J_m(k_{mn} r)\, e^{jm\alpha} \tag{4}$$

4. MAGNETIC FIELD MODELING

If the interface between fluids is flat, the current density **J** is homogeneous everywhere. As soon as the interface deviates from its flat shape due to interfacial waves or an external forcing, the current density **J** will become inhomogeneous near the interface. The inhomogeneity of **J** can be represented by the perturbation current density **j**. If the perturbation of the fluid interface is non-axisymmetric, it leads to a perturbation of the radial and axial component of the magnetic field outside the cylinder. This fact is used for the interface reconstructions.

To model the magnetic field, first, we have formulated the problem using the scalar electric potential Φ:

$$\Phi = \Phi_0 + \phi, \; \Phi_0 = J_0 z / \sigma \tag{5}$$

which fulfills the Laplace equations:

$$\begin{aligned} \Delta\phi_1 &= 0, \; -H/2 \le z \le 0 \\ \Delta\phi_2 &= 0, \; 0 \le z \le H/2 \end{aligned} \tag{6}$$

with the following boundary and interface conditions:

$$\frac{\partial \phi}{\partial r} = 0, r = R \quad \phi_1 = \phi_2 = 0, \; z = \pm H/2 \tag{7}$$

$$\mathbf{J}_1 \cdot \mathbf{n} = \mathbf{J}_2 \cdot \mathbf{n}, \qquad \Phi_1 = \Phi_2 \tag{8}$$

along the interface. In the case of analytical approach (applicable only to small perturbations) a linearized interface condition (8) has been used. In that case, the current density distribution can be computed from the electric potential and we get the interface deformation.

The deformation of the interface (at the middle of the cylinder height, see Fig. 2) between two conducting fluids after mechanical excitation can be described by

$$\eta(r, \alpha) = \sum_{m=-M}^{M} \sum_{n=1}^{N} A_{mn} \cdot J_m(k_{mn} r) \cdot e^{jm\alpha} \tag{9}$$

which can be derived from Laplace equation with a modified interface condition. The value n is called the *radial* mode number and the value m the *azimuthal* mode number. Although the quantity of modes is usually unlimited, the highest modes have the smallest amplitudes and can be neglected. The modes η_{mn} are used to describe the interface shapes. The validity of the above interface representation is limited to small amplitudes of the interface oscillations ($\eta << R$). The mode η_{mn} are defined by

$$\eta_{mn} = A_{mn} \cdot J_m\left(q_{mn} \frac{r}{R}\right) \cdot \cos(m\alpha) \tag{10}$$

where q_{mn} is the n-th solution of $J'_m\, (q_{mn} \cdot r/R) = 0$. If we impress a uniform current density $\mathbf{J_0}$ at the top of the cylinder, the interface deformation causes a distortion of the potential Φ_0 leading to the total potential $\Phi = \Phi_0 + \varphi$. Consequently, we get for the current density $\mathbf{J} = \mathbf{J_0} + \mathbf{j}$ where **j** is caused by the interface deformation. The current density can be calculated either analytically or numerically by FEM. Then, the corresponding magnetic field **b**(**r**) in the sensor positions close to the cylinder can be found using Biot-Savart law.

$$\mathbf{b}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int_V \frac{\mathbf{j} \times (\mathbf{r} - \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^3}\, dV' \tag{11}$$

5. INVERSE PROBLEM

A uniform current flow disturbed by a given non-axisymmetric interface, separating two electrical conducting fluids, in an isolating cylindrical container generates magnetic field perturbations which can be measured (using a multi-sensor system) and computed numerically using FEM. A given interface shape function can be approximated by superimposed Bessel functions and trigonometric functions. The goal of the solution of the corresponding inverse problem is the estimation of these interface shape functions based on the measured magnetic field data. This can be done by means of Genetic Algorithms (GA) where the search space is defined by the magnitudes of those interface modes. If we are able to estimate the relevant mode numbers and the mode amplitudes, we can find an approximation of the interface shape function causing the measured magnetic field distribution around the cylinder.

The inverse problem is formulated as follows: having the magnetic field flux density distribution (b_r, b_z components) in the sensors positions around the cylinder we would like to reconstruct the interface shape described by (1). First, for the chosen mode, e.g. mode η_{13}, the magnetic flux density distribution at the sensors positions is calculated using the described forward method. To simulate more realistically the problem, the magnetic field is modified by adding some white noise to the calculated values and reducing the number of sensors. The identification of interface modes is based on the use of a genetic optimizer [3]. To avoid the usual scan of the whole parameter space for all possible solutions we first analyze the spectra of power and amplitudes of the magnetic field data by means of FFT. This information is used to select the most probable modes (or mode combinations) before the GA is started. This procedure gives us in a very short time the azimuthal modes with a high probability, and it is applied to the estimation of the mode amplitudes in the same way, too. Further improvements were achieved due to some modifications of the simple GA [4].

6. INTERFACE MODE IDENTIFICATION

With respect to first experimental results we have designed a general technique for mode identification as well as interface reconstruction based on the configuration shown in Fig. 2. The flowchart of the technique we apply to the solution of the inverse problem, i.e. the mode identification and interface reconstruction, respectively, is given in Fig. 3.

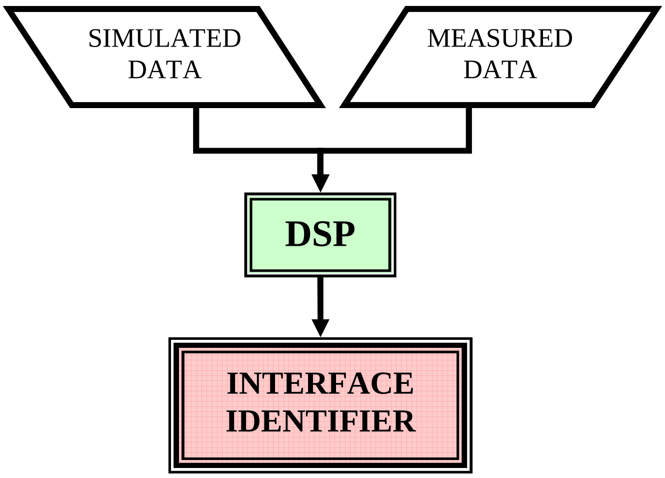

Fig. 3. *Flowchart of the interface mode identifier system.*

The simulated data which are given to the digital signal processing (DSP) module can be either static or time-dependent, and can be either ideal or noisy. Noise can be assumed either for the field values or for the geometry parameters of the interface. In the DSP module the correct time instants are selected, the data are filtered and the computation of power and amplitude spectra is done.

In the main part, the interface identifier, first can be decided whether an algorithm for fast identification of the azimuthal mode applying a FFT module should be performed or not. Then, there are three different procedures which can be applied:

- Principle component analysis (PCA)
- Amplitude estimation
- Full interface reconstruction

Whereas in the first case only the main components (azimuthal and radial mode numbers) of the signals are calculated, in the second case additionally the amplitudes of the interface modes are estimated. The PCA is usually done by means of the FFT module. To realize the amplitude detection stochastic optimization methods are used, Genetic Algorithms (GA), Simulated Annealing (SA) or Evolution Strategies (ES) can be applied. In the present version we study different GA´s or the SA algorithms. The most complex task, the full interface reconstruction, is not realized yet.

In Fig. 4 are shown some results we have got with this interface detection system. In this study it was analyzed how the interface mode η_{13} (left figure) can be reconstructed if different kinds of GA are applied. In the presentation more details will be given. Further details of the whole system will be presented in another paper during this conference.

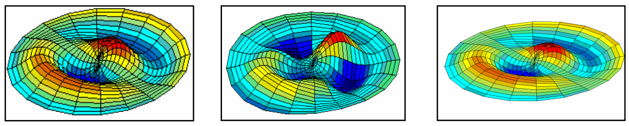

Fig. 4. *Some results of reconstruction of mode η_{13}*
left – original mode,
middle – reconstruction with simple GA (deviation 31%),
right – reconstruction with modified GA (deviation 5%).

7. EXPERIMENTS

Because the main goal of our research project is the development of magnetic field tomography system which function has to be demonstrated for the above mention cylindrical two-fluid-cell an experimental setup was design as well. The general layout is shown in Fig. 5.

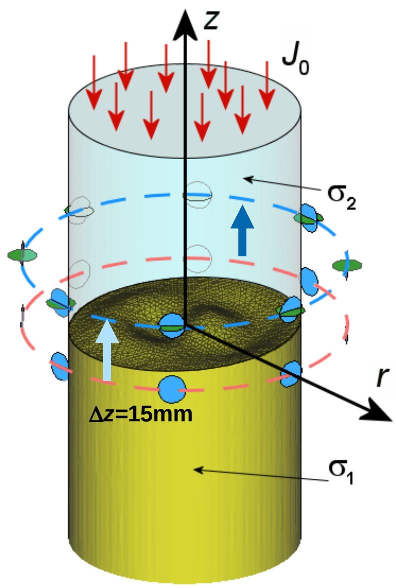

Fig. 5. *The two-fluid-cell with magnetic sensors mounted on a ring, which can be moved to different z-positions*

There were fixed eight 2D-fluxgate sensors on a ring which can be shifted in azimuthal and z-direction, respectively. Thus, we can record the magnetic field values in much more positions than we have sensors. Because in the experiment the interface is an stable oscillating surface we selected from the measured data always the time instants with the maximum interface deformations.

By means of the control of the pneumatic shaker which is moving the whole cylinder we are able to generate different stable interface modes [5]. The interaction between experiments and simulations will enable us to optimize the measuring configuration. On the other hand, we can verify the numerical simulations with the measuring data.

8. CONCLUSIONS

Interface shape between the two conducting fluids is identified with sufficient accuracy by means of simulated magnetic fields and using a genetic algorithm optimizer. Accuracy of reconstruction depends on the number and distribution of sensor points. The usual genetic optimization is enough sensitive for solving the magnetic fluid dynamics inverse field problem but is much too slowly. The efficiency of the optimization process has been improved by applying the FFT to reduce the GA search space. The proposed technique enables the detection of the possible information content of the magnetic field data (i.e. measured magnetic field components, b_r and b_z) depending on number and positions of sensors. Additionally, to solve the inverse problem using GA in reasonable time a highly sophisticated forward solver is required. For this purpose a highly optimized FEM-solver was developed.

REFERENCES

[1] Davidson P.A., "Magnetohydrodynamics in material processing," *Annu. Rev. Fluid Mech.*, vol. 31, pp. 273-300, 1999.

[2] Miles J., and D. Henderson, "Parametrically forced surface waves," *Annu. Rev. Fluid Mech.*,vol. 22, pp. 143-165, 1990.

[3] Brauer H., Ziolkowski M., Dannemann M., Kuilekov M., Alexeevski D.: "Forward simulations for free boundary reconstruction in magnetic fluid dynamics", *COMPEL*, Vol 22, No 3, 2003, pp.674-688.

[4] Ziolkowski M., Brauer H., Kuilekov M.: "Interface reconstruction between two conducting fluids applying genetic algorithms", *IEEE Transaction on Magnetics*, Vol 40, No 3, 2004 (in press)

[5] Men S., Resagk C., Ziolkowski M., Kuilekov M., Brauer H.: "Measurement of magnetic flux on a rotating distorted electrolyte-metal interface", *Measurement Science & Technology*, 2004 (accepted)