Fictitious Shell Method for Stress–Strain Analysis of an Arch Dam: Raslovići Dam Case Study

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Abstract – This paper presents an innovative numerical method for the analysis of arch dams. The dam and the interacting rock mass are approximated using line (beam) finite elements. Unlike the well-known 'single cantilever, multiple arch method' and 'multiple cantilever, multiple arch method,' this approach employs 'extended arches and cantilevers,' whereby the interacting rock mass is modeled with the same beam elements as the dam, but with different cross–sections and corresponding mechanical properties. Furthermore, to distribute hydrostatic load onto the dam's beam elements, a 'fictitious shell' is introduced – an auxiliary numerical element that discretizes surface load through influence areas and transfers equivalent loads to the arches and cantilevers. This approach simplifies the calculation of hydrostatic load distribution on the dam's beam elements and provides a more realistic representation of dam–rock interaction. The application of the Fictitious Shell Method is demonstrated on the case of the Raslovići Dam on the Morača River in Montenegro.

Keywords — arch dam, FEM model, fictitious shell method, line (beam) finite elements.

I. Introduction

The motivation for this paper stems from the inherent complexity of the task of modeling and calculating a dam, and the need to establish a methodology that is both sufficiently accurate and practically applicable. Conventional Finite Element Model (FEM) approaches employing solid elements [1], [2] demand highly complex models and substantial computational resources, which limit their usability in engineering practice [3]. The objective was therefore to develop a methodology that streamlines modeling and calculation, while preserving the ability to evaluate deformations, displacements, and internal forces. In this way, balance was achieved: the method is simpler than standard FEM techniques, yet it incorporates realistic cross-sections and force distributions, thereby ensuring a conservative and reliable assessment of lateral stability.

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II. Methodology

The proposed methodology[1] approximates the dam and the surrounding rock mass using linear (beam) elements with twelve degrees of freedom [1], [2] (arches and cantilevers), combined with a fictitious shell element [4]. This approach simplifies load distribution while achieving a realistic representation of dam–rock interaction, unlike standard methods in the theory of structures [3]. The procedure provides force values at the structure–rock interface, which are subsequently employed as input parameters for abutment stability analyses. This method differs from traditional approximations [5] by integrating realistic cross-sections and emphasizing force transfer into the rock mass, thereby combining simplicity with structural reliability [6]–[8].

III. Geometry and Dam Modeling

The structure is cylindrical, designed as a dome-type arch dam, with a total height $H = 27.00\ m$. Horizontal sections of the dam are arches constructed from a single center, with a constant thickness [1], [2], [3].

Upstream face radius (extrados):

- $R_{EX} = 50.00\ m$ from elevation *110.00 m a.s.l.* to *116.00 m a.s.l.*
- $R_{EX} = 48.80\ m$ from elevation *116.00 m a.s.l.* to *137.00 m a.s.l.*

The downstream face radius (intrados) varies with depth according to the relation:

$$R_I = 47.80 - Z \cdot \tan(8.43^\circ)\ [m], \qquad (1)$$

where depth is defined as $0.00 \le Z \le 27.00\ m$.

[1]Importantly, the methodology is independent of both arch dam type (geometry) and software platform: in this study AutoCAD was used for the geometric modeling and RADIMPEX Tower for the numerical modeling and analysis, but the procedure itself remains applicable with any equivalent software, ensuring broad usability and methodological independence.

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The Raslovići Dam is modeled as a system of 18 arches (horizontal elements) and 75 cantilevers (vertical elements). The arches and cantilevers are rigidly connected to each other (Fig. 1a).

The cross–sections of the arches are trapezoidal, with constant height in the *z*-direction of $h = 1.50\ m$. The thickness of the arches (dimension along the *x*-axis, i.e. upstream-downstream direction) varies linearly with depth (Fig. 1b).

The cross–sections of cantilevers are rectangular, with a constant width in the *y*-direction of $b = 1.00\ m$. The height of the cross–section (dimension along the *x*-axis) also varies with depth. It has a trapezoidal shape in the vertical sectional planes (Fig. 1c).

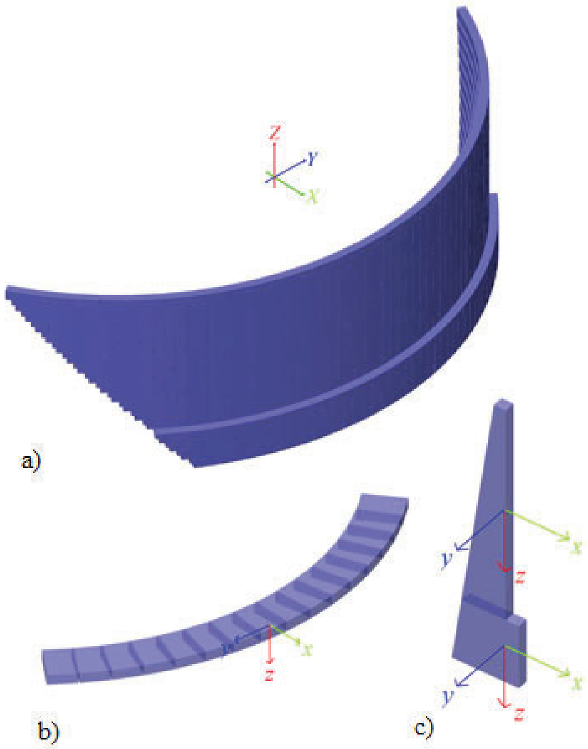


Fig. 1 The dam structure (a), horizontal section/element-arch (b), vertical section/element-cantilever (c)

The surrounding rock mass was modeled using the same type of elements as the dam–linear (beam) elements with twelve degrees of freedom. The dam elements were extended in both the horizontal and vertical directions by half of the total structural height, $H/2 = 13.50\ m$.

A total of 18 arches were assigned to the left abutment (Fig. 2a), 18 arches to the right abutment (Fig. 2b), while 9 arches were assigned beneath foundation joint, representing the basic rock mass (Fig. 2c). In addition, 2 arches were used to represent alluvial soil beneath the foundation joint (Fig. 2d).

Similarly, 28 cantilevers were assigned to the left abutment (Fig. 2a), 33 cantilevers to the right abutment (Fig. 2b), 39 cantilevers to the basic rock mass beneath the foundation joint (Fig. 2c), and 21 cantilevers for the alluvial soil beneath the foundation joint (Fig. 2d).

The geometric properties of the finite elements representing the rock mass were defined in the same manner as those of the dam elements, but with increased cross–sectional dimensions to account for the greater stiffness of the rock mass.

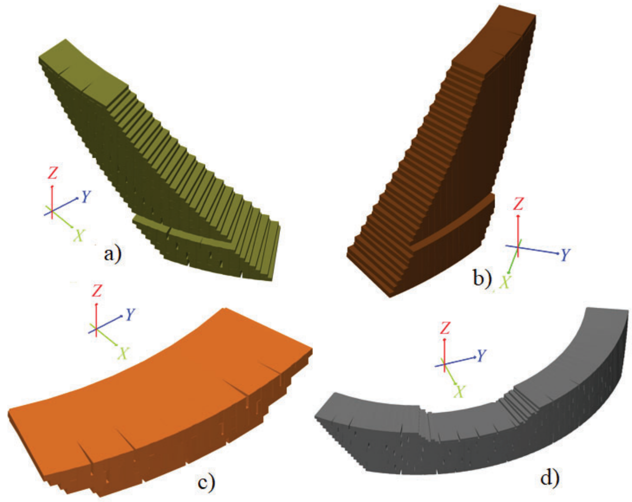


Fig. 2 The surrounding rock mass: left abutment (a), right abutment (b), alluvial soil (c), basic rock mass (d)

Enlargement of the cross–sections follows the geometry of the dam. At the junction between the dam cantilever and the rock mass, the increase amounts to approximately *60%*, while in the crown region, upon transition to the rock, it reaches up to *500%*. The rock mass was modeled by extending the domain radially by *2.00 m* upstream and downstream, thereby providing a reinforced representation of the cross-section in the dam–rock contact zone.

This simplified geometric model preserves the essential stiffness characteristics of both the dam and the surrounding rock mass, ensuring that the interaction between the structure and its foundation is realistically represented (Fig. 3).

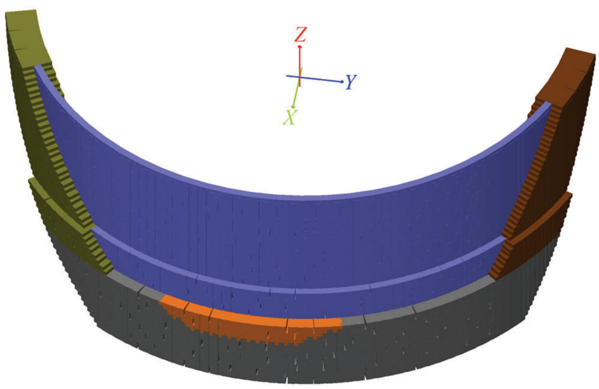


Fig. 3 Model of the dam and the surrounding rock mass

IV. Material Properties

The numerical model of the dam and its foundation system was developed using representative material parameters derived from geotechnical investigations conducted in accordance with the *JUS U.B1* standards, valid in Yugoslavia during 1987.

The adopted properties ensure realistic simulation of structural behavior and interaction between the dam body, the surrounding rock mass, and the underlying alluvial deposit (Table I) [9].

TABLE I. MATERIAL PROPERTIES

Section	ρ [kg/m^3]	γ [kN/m^3]	E [GPa]	ν
Left abutment	2200	22.00	31.00	0.20
Right abutment	2200	22.00	30.00	0.20
Basic rock mass	2400	24.00	33.00	0.20
Alluvial soil	2000	20.00	1.57	0.33
Dam – concrete (C25/30)	2400	24.00	21.00	0.20

The material properties reflect the different mechanical characteristics of the dam body and its foundation environment. The base rock mass provides the primary stiffness and stability, while the alluvial soil beneath the foundation joint is characterized by significantly lower strength and deformability. Concrete of class *C25/30*, exposure classes *XC4* and *XA2* [10]–[12], with the elastic modulus reduced to $Ec = 21.00\ GPa$ to account for rheological effects [13] and ambient pressure, represents the structural material of the dam. Together, these parameters ensure a realistic representation of the interaction between the dam and its foundation in the numerical model.

V. FEM MODEL AND BOUNDARY CONDITIONS

The FEM (numerical) model of the dam and its foundation was established under a set of assumptions and boundary conditions that ensure consistency with the adopted material properties and the geometry of the system.

Assumptions:

- The dam material (hydrotechnical concrete) is assumed to be homogeneous, isotropic, and elastic, with all assumptions of linear elasticity theory applied.
- The dam is elastically embedded into the rock mass, which is treated as a quasi–continuous elastic medium. The rock material is also assumed to be homogeneous, isotropic, and elastic.
- Kirchhoff's theorem for deformable bodies is assumed to be valid.
- A plane stress–strain state is assumed within the dam body, consistent with the applied finite elements.
- Displacements and rotations of the dam due to reservoir loading are neglected.
- Vertical displacements caused by self–weight, shrinkage, and temperature variations prior to the grouting of contraction joints are absorbed by the cantilever elements before the arch action of the dam is established.
- Cracking occurs when the tensile strength of concrete is exceeded, after which loads are carried only by the uncracked portion of the dam section.
- The elastic properties of concrete and the rock mass are expressed through Young's modulus and Poisson's ratio.

Boundary Conditions – The dam elements are rigidly connected to the rock mass elements, so that at nodes where the dam and the rock elements meet, the following conditions apply:

- Radial displacements of the arch and cantilever elements of the dam and rock are equal and non-zero: $\Delta r_x^c = \Delta r_x^r \neq 0$.
- Axial displacements of the arch and cantilever elements of the dam and rock are equal and non-zero: $\Delta s_{y,a}^c = \Delta s_{y,a}^r \neq 0$, $\Delta s_{z,c}^c = \Delta s_{z,c}^r \neq 0$
- Vertical displacements of the arch elements of the dam and rock are equal and non-zero: $\Delta\zeta_{z,a}^c = \Delta\zeta_{z,a}^r \neq 0$.
- Rotations of sections in the arch and cantilever elements of the dam and rock are equal and non-zero:
  $\Delta\theta_x^c = \Delta\theta_x^r \neq 0$, $\Delta\theta_y^c = \Delta\theta_y^r \neq 0$, $\Delta\theta_z^c = \Delta\theta_z^r \neq 0$.

Accordingly, in the computational model, the dam is elastically embedded into the rock mass.

At the supports of the rock mass elements, total fixation is applied, i.e., all displacements and rotations are equal to zero: $\Delta r_x^r = \Delta s_y^r = \Delta\zeta_z^r = 0$, $\Delta\theta_x^r = \Delta\theta_y^r = \Delta\theta_z^r = 0$.

These supports represent the ultimate boundary of the model, corresponding to deep rock layers where both translations and rotations are prevented [1]–[3].

The adopted assumptions and boundary conditions define a consistent computational framework for the numerical model of the dam. By idealizing the dam and rock mass as homogeneous, isotropic, and elastic materials, and by applying plane stress–strain conditions, the model captures the essential mechanical behavior while remaining tractable for analysis. Elastic embedding of the dam into the rock mass ensures realistic interaction at the contact, whereas the ultimate boundary conditions at deep rock layers prevent both translations and rotations, representing the final limits of the model domain. Together, these simplifications provide a rational basis for evaluating the structural response of the dam within the finite element scheme [3].

VI. LOAD MODELING

Hydrostatic pressure (w) is represented as a surface load whose intensity increases linearly with depth [3]:

$$p_w = \gamma_w \cdot Z;\quad 0.00 \leq p_w \leq 270.00\ kN/m^2, \tag{2}$$

where $\gamma_w = 10.00\ kN/m^3$ is the unit weight of water, and Z is depth ($0.00 \leq Z \leq 27.00\ m$).

In order to realistically represent the action of hydrostatic pressure within the numerical model, a load modeling procedure is introduced. Hydrostatic pressure, defined as a surface load varying with depth, is transferred onto the structural system through a fictitious shell element [4].

This auxiliary numerical element has no physical role in the structure (Fig. 4).

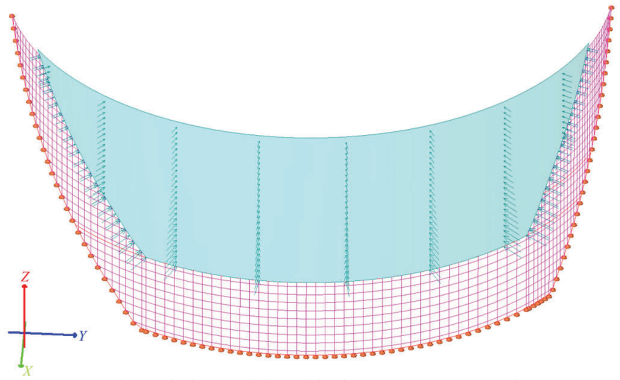


Fig. 4 Fictitious shell and hydrostatic pressure load

The shell receives a hydrostatic load as a surface action and discretizes it at a step of *25.0 cm* in this example, thereby producing equivalent loads on the beam elements of the arches and cantilevers (Fig. 5).

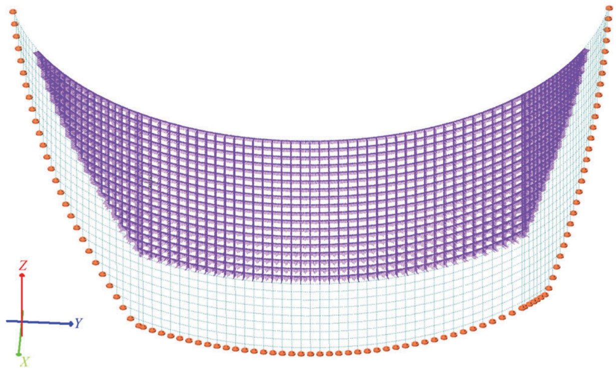


Fig. 5 Equivalent nodal forces

For the arches and cantilevers, nodal forces are obtained by integration over the domain of a fictitious shell element:

$$F_{a,j} = \int_{A_a} p_W(Z) \cdot N_{a,j}(s)\, dA_a \,,^2 \tag{3}$$

$$F_{c,i} = \int_{A_c} p_W(Z) \cdot N_{c,i}(y_{a,j}, z)\, dA_c \,.^2 \tag{4}$$

Here $p_w(Z)$ denotes hydrostatic pressure at depth Z. N_i and N_j are the FEM interpolation (shape) functions for radial displacement [12], [15], and dA_a, dA_c denote the differential surfaces of the fictitious shell associated with the arches and the cantilevers. Thus, the continuous hydrostatic action is consistently discretized into equivalent nodal forces, providing a reliable representation of the physical loading conditions within the structural model.

Effects of the self–weight (g) are carried by the vertical elements of the numerical model, in accordance with the construction technology of arch dams.

Since the contraction joints are injected only after the full construction height has been reached, and the arches in the dam become active due to temperature effects and hydrostatic pressure after injection, it is assumed that self–weight is not transferred laterally through the arches. Instead, the self–weight is carried exclusively in the vertical direction by the cantilevers, and this load is therefore modeled through the cantilever elements [3].

Because the dam structure together with the surrounding rock mass represents a massive three-dimensional body, the adopted cantilever elements do not cover the entire volume of the actual system. To obtain the correct self–weight in the numerical model, the volumetric weights assigned to the cantilever elements are reduced by applying reduction coefficients.

Reduction coefficients for the volumetric weights of the cantilevers are dimensionless quantities and depend solely on the geometry of the adjacent arch segments (l_l–left segment, l_r–right segment) and the cantilever thickness t_c. In the present case, the adopted cantilever thickness is $t_c = 1.00\,m$. The coefficients are defined as the ratio between the average length of the left and the right arch segments adjacent to the cantilever and the adopted cantilever thickness:

$$c_\gamma = \frac{l_l + l_r}{2 \cdot t_c} \,.^3 \tag{5}$$

Reduced volumetric weight is then obtained as:

$$\gamma^* = c_\gamma \cdot \gamma, \tag{6}$$

for the dam cantilevers:

$$\gamma_{c,i}{}^* = c_{\gamma,i} \cdot \gamma_c, \tag{7}$$

for the rock cantilevers:

$$\gamma_{r,i}{}^* = c_{\gamma,i} \cdot \gamma_r. \tag{8}$$

Here, i denotes the code of the corresponding cantilever (a vertical element with height h_i) in the numerical model, γ_c is the volumetric weight of the hydrotechnical concrete, and γ_r is the volumetric weight of the surrounding rock mass.

VII. Stress–Strain Analysis and Interpretation of the Results

Calculation was carried out according to first–order theory, assuming linear elastic behavior of both materials and structure. Results were considered for the load combination prescribed by *Eurocode* for the *serviceability limit state* (*SLS*), specifically the combination $1.0 \cdot g + 1.5 \cdot w$, where the self–weight (g) is treated as a favorable load and the hydrostatic pressure (w) as a variable load evaluated at the maximum water level [10]–[12].

Fig. 6 illustrates the exact positions of the analyzed arches and cantilevers in the dam structure. The corresponding stress and radial displacement diagrams for these elements are presented in the Tables II-V.

[2] The shape functions $N_{a,j}(s)$ and $N_{c,i}(y,z)$ interpolate the radial displacements of the arches and the cantilevers, respectively (displacements along the x-axis, in accordance with the local coordinate systems).

[3] For the physical consistency, the reduction coefficient must satisfy $c_\gamma \geq 1$, since adopting an excessively thick cantilever would otherwise lead to an artificial decrease of self–weight, which is not physically correct.

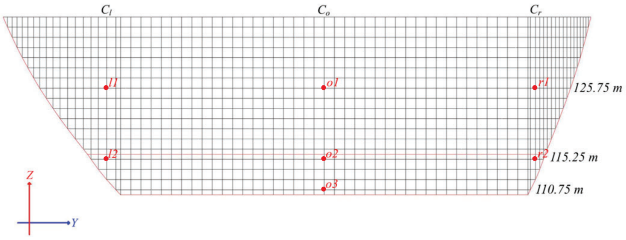
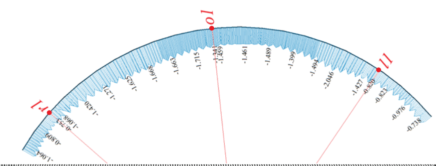
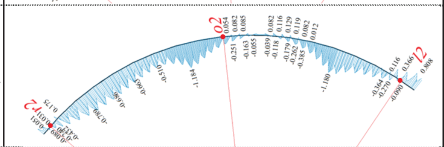
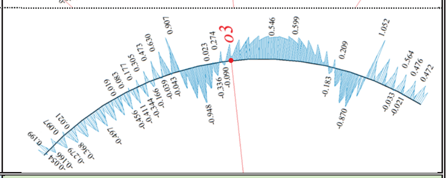
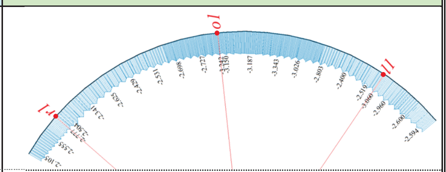
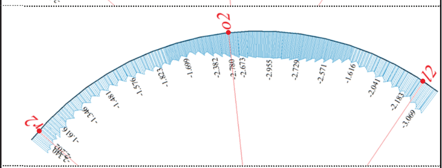
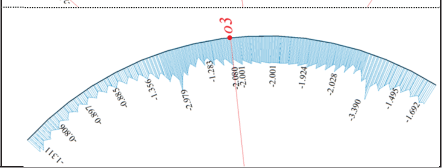


Fig. 6 The positions of the analyzed arches and cantilevers in the dam

TABLE II. NORMAL STRESSES OF THE ARCHES

Elevation	Upstream normal stresses σ_u [*MPa*]
Arch–125.75 m a.s.l.	
Arch–115.25 m a.s.l.	
Arch–110.75 m a.s.l.	
Elevation	**Downstream normal stresses σ_d [*MPa*]**
Arch–125.75 m a.s.l.	
Arch–115.25 m a.s.l.	
Arch–110.75 m a.s.l	

Normal compressive stresses are assigned a negative sign, while normal tensile stresses carry a positive sign. Radial displacements are considered negative when they shorten the arch radius and positive when they elongate it [3], [6]–[8].

TABLE III. RADIAL DISPLACEMENTS OF THE ARCHES

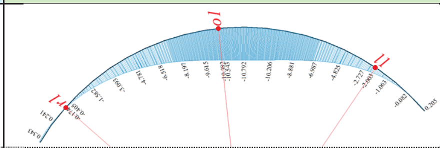
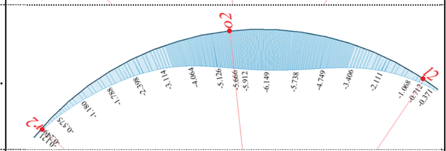
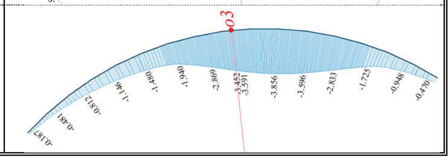
Elevation	Radial displacement ΔR [*mm*]
Arch–125.75 m a.s.l.	
Arch–115.25 m a.s.l.	
Arch–110.75 m a.s.l	

TABLE IV. NORMAL STRESSES OF THE CANTILEVERS

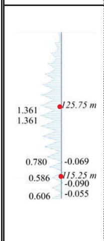
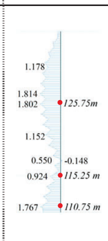
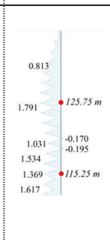
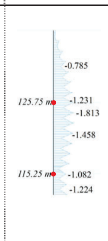
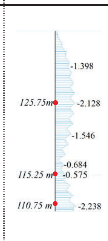
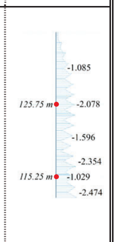
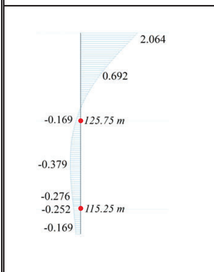
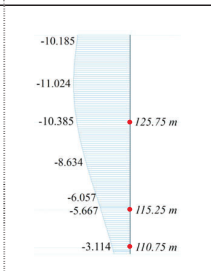
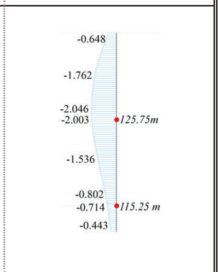
Upstream normal stresses σ_u [*MPa*]			Downstream normal sresses σ_d [*MPa*]		
C_r	C_o	C_l	C_r	C_o	C_l
					

TABLE V. RADIAL DISPLACEMENTS OF THE CANTILEVERS

Radial displacement ΔR [*mm*]		
C_r	C_o	C_l
		

Based on the obtained results it can be consluded:

- Dam elements are predominantly in compression.
- Local tensile stresses may occur upstream at the contact zone with the alluvial deposit on the arches, but these are not significant.
- Already at the first adjacent arch, which is not in direct contact with the alluvial deposit, exclusively compressive stresses are observed.
- The downstream contour shows compressive stresses in all elements.

Calculated deformations are small relative to the dam height, so the change in geometry after deformation does not significantly affect the distribution of internal forces. Therefore, the assumption of first–order theory is justified, and the principle of superposition of effects for various load cases is valid.

The conducted analysis confirms that the structure is globally in compression, with negligible local tensile zones, and that the small deformations validate the initial assumption of first-order theory.

Since the tensile stresses in the dam elements are lower than the tensile strength of concrete, cracking will not occur, which is a fundamental criterion in the design of arch dams.

On this basis, the conclusion can clearly emphasize the methodological contribution and the practical applicability of the proposed procedure.

VIII. Conclusion

The methodology applied in this research is original and independent of geometry, meaning that it can be applied to any adopted geometric configuration of the numerical model. The obtained results confirm a realistic stress distribution and radial displacements, while the simplified geometric model preserves the essential stiffness characteristics of both the dam and the surrounding rock mass. The originality of this approach lies in the specific manner of load distribution onto the dam elements, which ensures a reliable interaction between the structure and the foundation. In addition to the originality, methodology provides a practical framework for the routine engineering analyses by reducing modeling complexity while ensuring reliable results.

Methodology aligns with *Eurocode* [11], [12] and *USBR* [6] guidelines, ensuring that the obtained results are both scientifically rigorous and practically applicable.

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