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Analysis of the Impact of Passive Component Tolerances on Active Twin-T Notch Filter

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***Abstract*— This paper explores the impact of component tolerances on the behavior of an active twin-T notch filter, revealing complexities not encountered in simpler filters. Through simulations, it highlights the inadequacies of traditional error analysis and proposes a more precise Monte Carlo simulation approach. These insights emphasize the necessity of accounting for component tolerances, especially in applications like mains hum removal.**

Keywords— Notch filter, measurement uncertainty, transfer function, central frequency, mains hum, error analysis, maximum error, component tolerance, Monte Carlo.

I. Introduction

"Notch filter", also known as a band-stop filter, is a type of filter that attenuates only the frequencies within its stop band while leaving others unaltered. Often, a distinction is made between the terms "band-stop filter" and "notch filter", where the term "notch filter" implies a band-stop filter that attenuates a very narrow range of frequencies, i.e., it has a high Q factor.

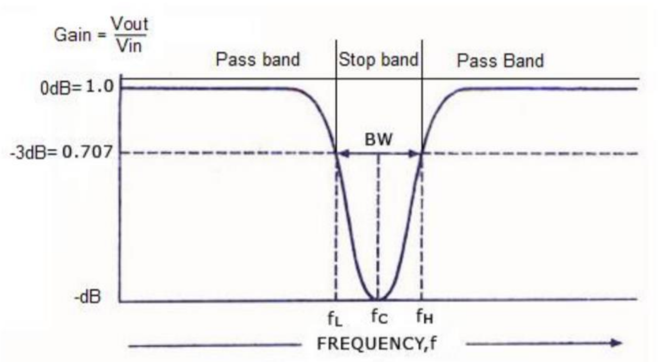

Fig. 1. General amplitude response of a band-stop filter [1]

The most common use of a notch filter is to eliminate interference arising from the mains power supply, also called "mains hum" or "power line interference." Power lines carry a voltage of 230 V and 50 Hz (or 120 V and 60 Hz depending on the region) which creates disturbances in electrical circuits through electromagnetic interference. These disturbances, for example, disrupt sound quality in the audio industry and affect the image of electrocardiograms if not eliminated. Therefore, it is crucial to remove these interferences without compromising the integrity of the useful signal. The frequency of the mains power supply has a maximum tolerance of 1% (0.5 Hz) [2], although generally, the error does not exceed 0.1 %, or 0.05 Hz [3]. This means that the frequency range to be suppressed is 49.9 Hz to 50.1 Hz, and for these purposes, it is ideal to use a notch filter which will, due to its very narrow stop band, eliminate interference without affecting the needed signal.

In the world of analog electronics, filters are constructed using a combination of various passive components (resistors, capacitors, and inductors) and active components (operational amplifiers). These components can be interconnected in various ways within an electric circuit, with different values of resistances, capacitances, and inductances, which define the characteristics of the filter. This means that, although a person wants to design a notch filter, they can achieve this in numerous ways. Therefore, for one type of filter, there are many topologies, each with its own advantages and disadvantages. For example, a notch filter can be implemented as a Multiple Feedback Notch, Bainter Notch, Twin-T Notch, or even a basic RLC circuit. The topic of this paper is the active Twin-T Notch filter and the analysis of changes in its characteristics due to tolerances of the passive components that comprise it. Only changes in the amplitude characteristic will be demonstrated in the paper, although it should be noted that tolerances also affect the phase characteristic.

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II. The Transfer Function

A. General notch filter transfer function

The transfer function is a mathematical equation that describes the relationship the between the output and the input of a system, in this case a filter circuit. The general form of the transfer function of a second-order notch filter is:[4]

$$H(s) = \frac{H_0(s^2 + \omega_0)}{s^2 + \frac{\omega_0}{Q}s + \omega_0}, \qquad (1)$$

where ω_0 is the central frequency of the filter (the frequency that will be most attenuated and around which the stop band is centered), Q is the parameter defining how narrow the stop band is (larger Q results in a narrower band), H_0 is gain, and s is the complex Laplace variable $s = \sigma + j\omega$. This transfer function simplifies the analysis of the twin-T notch filter under ideal conditions where components are matched, ensuring no deviation from the nominal value.

B. Active Twin-T notch transfer function

First, the transfer function of the circuit must be derived. The analysis of the circuit begins by applying Kirchhoff's current law to nodes A and B. The impedance of the capacitor is described as $Zc = \frac{1}{j\omega C}$, where $j\omega = s$ in Laplace domain for sinusoidal waveforms. All voltages are complex numbers.

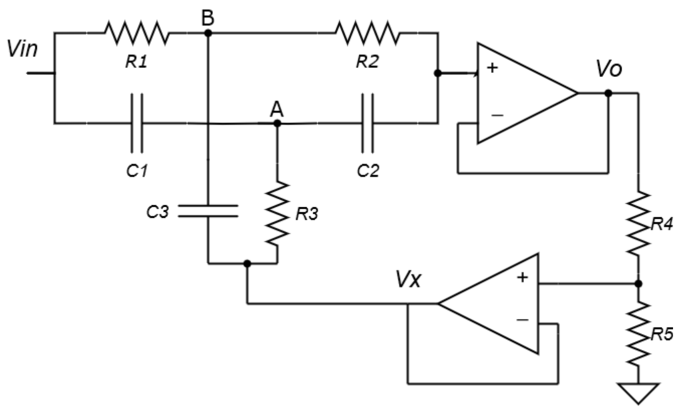

Fig 2. Active twin-T notch filter circuit

$$\frac{Vin - V_A}{\frac{1}{sC_1}} + \frac{Vo - V_A}{\frac{1}{sC_2}} + \frac{Vx - V_A}{R_3} = 0 \qquad (2)$$

$$\frac{Vin - V_B}{R_1} + \frac{Vo - V_B}{R_2} + \frac{Vx - V_B}{\frac{1}{sC_3}} = 0 \qquad (3)$$

Taking into account the characteristics of the ideal operational amplifier, the current at its inputs is zero, resulting in:

$$\frac{Vo - V_B}{\frac{1}{sC_2}} + \frac{Vx - V_B}{R_2} = 0 \qquad (4)$$

And finally, since both operational amplifiers are connected as buffers:

$$Vx = \frac{R_5}{R_4 + R_5} Vo \qquad (5)$$

Following similar steps of deriving the transfer function as in [5], without introducing matching components, we obtain the following expression for $H(s) = \frac{Vo}{Vin}$:

$$H(s) = \frac{s^3 C_1 C_2 C_3 R_1 R_2 R_3 + s^2 C_1 C_2 R_3 (R_1 + R_2) + s R_3 (C_1 + C_2) + 1}{s^3 C_1 C_2 C_3 R_1 R_2 R_3 + s^2\left(\left(1 - \frac{R_5}{R_5 + R_4}\right)(C_2 C_3 R_1 R_2 + C_1 C_3 R_1 R_3 + C_2 C_3 R_1 R_3) + C_1 C_2 R_3 (R_1 + R_2)\right) + s\left(\left(1 - \frac{R_5}{R_5 + R_4}\right)(C_2 R_1 + C_2 R_2 + C_3 R_1) + R_3 (C_1 + C_2)\right) + 1} \qquad (6)$$

This is the most general form of the transfer function of the active twin T notch filter. From this expression, it is not possible to directly derive equations describing the center frequency or Q factor of the filter; instead, this must be done indirectly through analysis of the frequency response of this function. However, this transfer function precisely describes the behavior of the circuit and holds true regardless of whether the components of the filter are matched or not, i.e., it is valid independently of the resistor and capacitor values. In the case where the values of passive components are carefully chosen in the following manner: $R_1 = R_2 = R$, $R_3 = R/2$, $C_1 = C_2 = C$, $C_3 = 2C$, the following expression is obtained:

$$H(s) = \frac{s^2 + \left(\frac{1}{RC}\right)^2}{s^2 + s\left(\frac{1}{RC}\right)\frac{4R_4}{R_4 + R_5} + \left(\frac{1}{RC}\right)^2}. \qquad (7)$$

It is noticeable that in this special case, the transfer function simplifies to a second-order function. This form is much simpler and more reasonable, and moreover, expressions for ω_0 and Q can be directly derived from it. The issue arises in that this transfer function accurately describes the circuit only when the components are matched as described above, and any deviation from the nominal values of passive components renders this transfer function no longer 100% valid. However, although resistor and capacitor tolerances are inevitable, it turns out that this transfer function is a sufficiently good approximation in most cases if we choose components with a small tolerance limit. If we equate (7) with (1), we obtain the following expressions for the center frequency, Q and bandwidth:

$$\omega_0 = \frac{1}{RC}, \quad Q = \frac{R_4 + R_5}{4R_4}, \quad BW = \frac{\omega_0}{Q} \tag{8}$$

From these equations, it is easy to configure the filter to suppress desired frequencies by substituting *f0* into $\omega_0=2\pi f_0$, and selecting an appropriate combination of *R* and *C*. *Q* factor is adjusted in the similar fashion. As for the amount of attenuation at the desired frequency and the phase shift of the signal, we need to find the magnitude $|H(j\omega)|$ and argument arg($H(j\omega)$) of the transfer function. For this, it is again permissible to use the simplified equation (7) if the filter is designed with matched components.

III. The Analysis of Errors

Each component is defined by its nominal value and tolerance. For example, a resistor with a nominal value of 1000 Ω and a 1 % tolerance can actually have a value anywhere between 990 Ω and 1010 Ω. Exactly these deviations are the reason why the characteristics of the filter, such as the center frequency and maximum attenuation, differ from the desired values. For the purpose of calculating and demonstrating errors, an active twin T-notch filter is designed with the following nominal values: $R_1 = R_2 = 318.3$ kΩ, R_3=159.15 kΩ, $C_1 = C_2 =$ 10 nF, $C_3 = 20$ nF, $R_4 = 1$ kΩ and $R_5 = 99$ kΩ, which represents the case of matching component values. Assuming a tolerance of zero, the expressions from (8) are used, and the following results are obtained: $f_0 = 50.001$ Hz, $Q = 25$ and BW=2 Hz.

A. Theory and wrong way of calculating errors

If component tolerances are introduced into the previous example, the components become unmatched, and the equations from (8) are no longer valid. Therefore, the correct way to determine the errors is by using the transfer function from (6). There is a possibility that error analysis using formulas from (8) can be used as an approximation of the error boundaries or measurement uncertainty, so this method will also be addressed. The analysis begins by applying the formula for estimating the maximum error to (8). Maximum error [6] is defined as:

$$|\Delta y| \le \left|\frac{\partial y}{\partial x_1}\Delta x_1\right| + \left|\frac{\partial y}{\partial x_2}\Delta x_2\right| + \ldots + \left|\frac{\partial y}{\partial x_n}\Delta x_n\right|, \tag{9}$$

where y=f(x_1, x_2, … , x_n), Δy is the maximum error of indirectly calculated quantity, and Δx_i, i=1, 2, … , n are known errors of directly measured quantities x_1, x_2, … , x_n.. Once the expressions for the maximum error are obtained, it allows us to define an interval in which the indirectly measured quantities are certain to be within. For instance, $\omega_0 \pm \Delta\omega_0$ provides a range within which the true value of the central frequency is guaranteed to lie with 100% certainty. Consider the twin-T notch filter: each component deviates from its nominal value uniquely, with some deviations being positive and others negative. Consequently, it's improbable that every component will reach its maximum deviation, let alone in a manner where all errors accumulate. Hence, instead of relying solely on such a conservative maximum error formula, it's preferable to consider measurement uncertainty formula. Measurement uncertainty [6] takes into account not only the fact that each influential quantity contributes to the error of the indirectly determined quantity with its own error, but also the distribution function of those errors. The measurement uncertainty of an indirectly determined quantity is defined as:

$$u(y)^2 = \left(\frac{\partial y}{\partial x_1}\mathrm{u}(x_1)\right)^2 + \left(\frac{\partial y}{\partial x_2}\mathrm{u}(x_2)\right)^2 + \ldots + \left(\frac{\partial y}{\partial x_n}\mathrm{u}(x_n)\right)^2 \tag{10}$$

where: u(y) is measurement uncertainty of indirectly measured quantity y, u(x_1), … , u(x_n) is measurement uncertainty of directly measured quantities x_1, … , x_n, and y is the function of (x_1, … , x_n). Applying (10) to (8) gives us the expressions for the measurement uncertainty of ω_0, Q, and BW. To calculate the measurement uncertainty, it is necessary to know the error distribution functions of the influential quantities. The final result of measurement uncertainty calculation is the interval $y \pm k \cdot \mathrm{u}(y)$ within which the true value of the indirectly determined quantity lies with a certain probability, rather than with 100% certainty. as in previous case. That probability depends on the distribution function of the measured quantity and the chosen interval for observation. For instance, if we assume that resulting error of ω_0 has normal distribution by central limit theorem [7], and we observe the confidence interval of k=2 standard deviations, the error would be expected in resulting interval of $y \pm 2 \cdot \mathrm{u}(y)$ with 95% probability. Finally, by applying all of these formulas to the example provided at the beginning of the chapter (50.001 Hz notch, Q=25, BW=2) and introducing various standard tolerances for resistors and capacitors, the following table is obtained:

Table 1: Maximum errors and uncertainties for various values of component tolerances (wrong way)

	Maximum error of ω_0, Q, BW (%)	Uncertainty uniform ω_0, Q, BW (%)	Uncertaitny normal ω_0, Q, BW (%)
$R_{\%}$=0.1, $C_{\%}$=1	1.10, 0.20, 1.20	0,58, 0.08, 0.59	0,33, 0.046, 0.34
$R_{\%}$=0.5, $C_{\%}$=2	2.5, 0.99, 3.01	1.18, 0.40, 1.25	0.69, 0.23, 0.73
$R_{\%}$=1, $C_{\%}$=5	6, 1.98, 7.02	2.94, 0.81, 3.05	1.70, 0.47, 1.76
$R_{\%}$=2, $C_{\%}$=10	12, 3.96, 14.04	5.89, 1.62, 6.11	3.40, 0.93, 3.53
$R_{\%}$=5, $C_{\%}$=20	25, 9.90, 30.10	11.90, 4.04, 12.60	6.87, 2.33, 7.27

The table shows values for maximum error, measurement uncertainty when uniform distribution is assumed for passive components, and finally for the case of assuming normal distribution (only to show that error distribution does affect the uncertainty value). This incorrect method of obtaining the maximum error and measurement uncertainty of the active twin

T notch filter serves as a good example to demonstrate how these expressions are mathematically derived and what they indicate about the measurement result. Additionally, it may serve as an initial approximation of the error, which will be further examined. In case of determining attenuation error and phase shift error, there is no generalized expression. Instead, this must be done by finding the magnitude and argument of the transfer function $H(s)$, and then applying equations (9) and (10) to those expressions. Since even using the wrong assumption that the simplified transfer function holds true and finding the total derivative of the magnitude and argument of that function is not straightforward and is also inaccurate, calculating attenuation error using (7) does not serve as approximation and will not be examined.

B. *The correct way of calculating errors*

The correct way to analyze changes in filter characteristics is by analyzing the transfer function (6), which is valid in the general case, even when the components are mismatched. From this function, equations directly describing ω_0, Q, and BW like (8) cannot be derived. Instead, it is necessary to find the magnitude and argument of (6) and then determine the amplitude and phase characteristics (Bode plots) via frequency response. The desired quantities can be determined from those plots. This analysis was performed using a program called LTspice. It provides insights into how each component individually influences the filter, as well as the conditions under which maximum error occurs and its magnitude. Once again, the analysis is done on the same 50 Hz notch example.

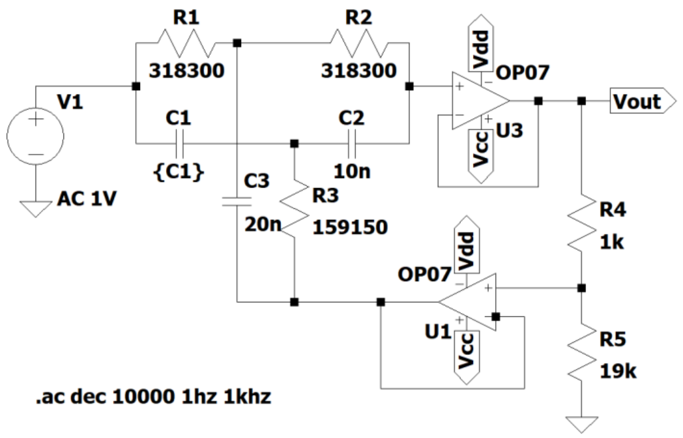

Fig. 3. 50 Hz notch filter in LTspice

Each passive component affects the transfer function of the filter with its tolerance by changing the expected ω_0, BW and attenuation. For example, with parameter analysis we get how tolerance in C_1 changes the observed quantities (Red = -2 % tolerance, Blue = 0 % and Green = +2 % tolerance), this is shown in fig. 4.

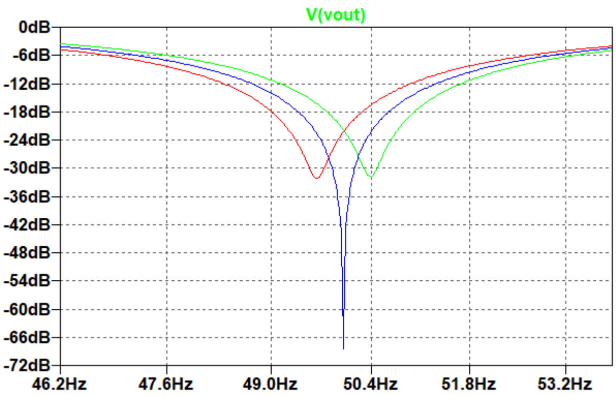

Fig. 4. Changes in twin T notch transfer function in respect to deviation of C1

By studying the obtained frequency responses from Fig. 4, it was discovered that ω_0 increases with positive deviation from the nominal value of C_1 and decreases with negative deviation. The maximum attenuation, as well as attenuation at 50 Hz, decreases regardless of the sign of deviation, and the bandwidth behaves similarly to ω_0. Analyzing each component in this way yields the results shown in table 2.

TABLE 2: TRANSFER FUNCTION CHANGES DEPENDING ON COMPONENT AND SIGN OF TOLERANCE

Component and sign of tolerance			ω_0	BW	Attenuation
	R_1	+	Increases	Increases	Decreases
		-	Decreases	Decreases	Decreases
	R_2	+	Increases	Increases	Decreases
		-	Decreases	Decreases	Decreases
	R_3	+	Increases	Increases	Decreases
		-	Decreases	Decreases	Decreases
	C_1	+	Increases	Increases	Decreases
		-	Decreases	Decreases	Decreases
	C_2	+	Increases	Increases	Decreases
		-	Decreases	Decreases	Decreases
	C_3	+	Increases	Increases	Decreases
		-	Decreases	Decreases	Decreases
	R_4	+	Unchanged	Increases	Increases
		-	Unchanged	Decreases	Decreases
	R_5	+	Unchanged	Decreases	Decreases
		-	Unchanged	Increases	Increases

Table 2 shows how each component affects the transfer function but doesn't show how much. To demonstrate the extent to which each component affects these characteristics, a parametric analysis was conducted for each component separately. The tolerance limits ranged from -5 % to +5 % with an increment of 0.1 %. LTspice performed frequency analysis for each deviation step from the nominal value of all components. By extracting data (ω_0, BW...) for each step using Python, graphs like Fig. 5 are obtained. From these graphs, the following results are obtained: C_1, C_2, R_1, and R_2 affect the center frequency and bandwidth in approximately the same manner, with similar attenuation characteristics (differences between C_1, C_2, and R_1, R_2 for large deviations from the nominal value). C_3 and R_3 influence the center frequency and bandwidth to a similar extent (and to a slightly lesser degree than C_1, C_2, R_1, and R_2), but when

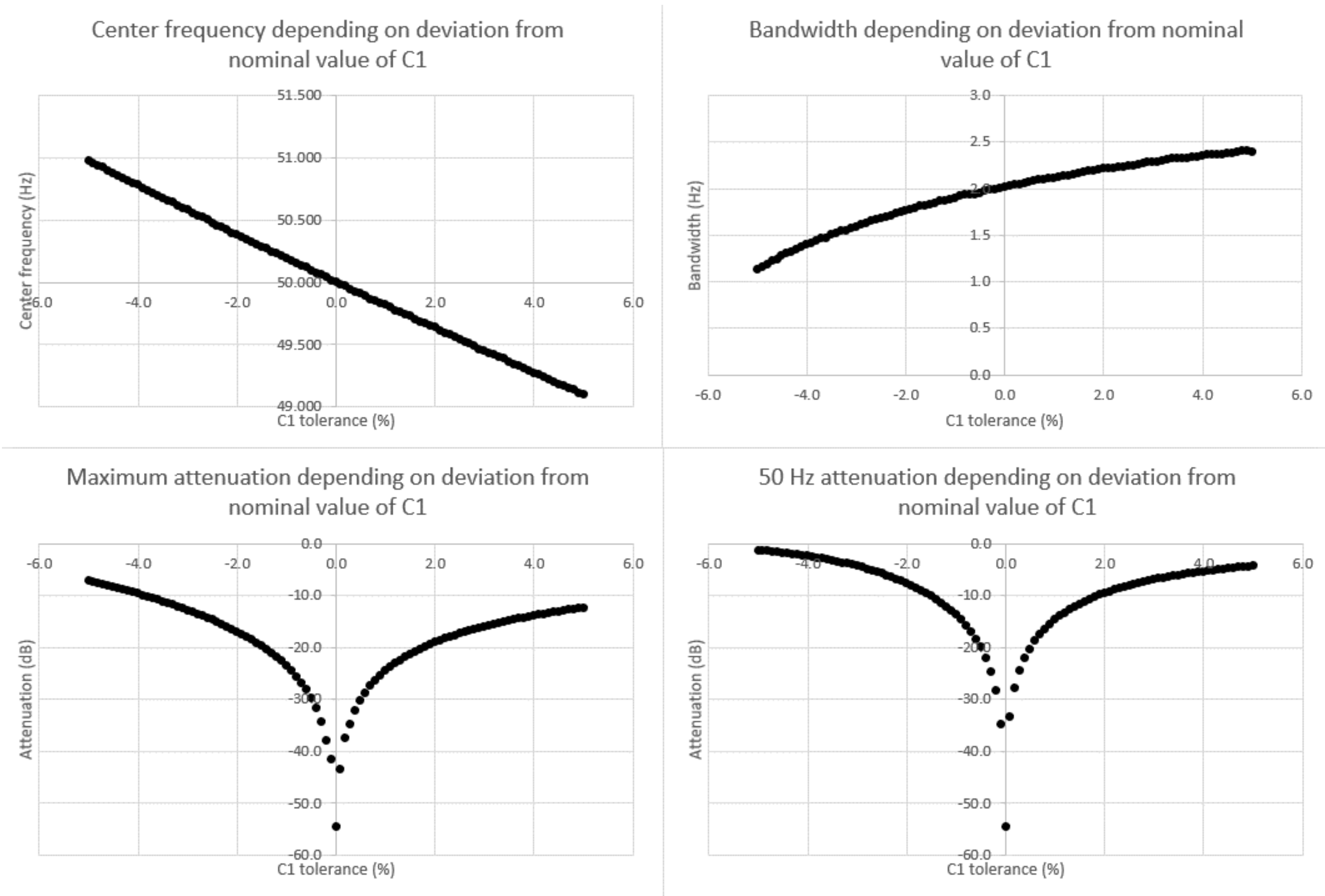

Fig. 5. Graphs showing impact of C_1 tolerance on transfer function

it comes to attenuation, they affect it in different amounts. However, tolerances in both components significantly impact the amount of attenuation in the system and the shape of the transfer function (more than the other components). Deviation in R_4 and R_5 only leads to small changes in bandwidth and attenuation. These resistors serve to define the bandwidth, so their tolerances do not create significant issues and are negligible. They have no impact on ω_0 and do not compromise the appearance and characteristics of the transfer function. The influence of component tolerances on the characteristics of the transfer function increases with a higher Q factor, meaning that as the feedback with resistors R_4 and R_5 increases, deviations from nominal values of other components will have a stronger impact on essential characteristics. They will more significantly reduce the filter attenuation, and alter the shape of the transfer function, but ω_0 error will remain the same regardless of Q. This is shown in table 3, which shows impact of C_1 on transfer function in cases of Q=25 and Q=5 for deviation of -5 %. It is important to mention that a filter with a high Q factor should not be tested under conditions where components have large tolerances because they lead to significant distortion of the transfer function. The active twin T notch filter no longer behaves like a band reject filter but instead completely loses its characteristics. For the previously mentioned 50 Hz filter example, this occurs when the components deviate by approximately -8 % for C_1 and C_2, +8 % for R_1 and R_2, -4 % for R_3, and +4 % for C_3.

TABLE 3. COMPARISON OF THE INFLUENCE OF C1 WITH A DEVIATION OF -5% FOR FILTERS WITH Q=25 AND Q=5

	Q=25	Q=5
ω_0 error (%)	1.9	1.9
BW error (%)	44	2.8
Attenuation 50 Hz error (%)	97.52	80.03
Attenuation max error (%)	86.9	65.45

Now that it's clear how each component individually affects the filter's characteristics, we can examine the errors resulting from deviations of all components simultaneously, as this is the case in reality. Once again, we will determine the maximum error and measurement uncertainty due to component tolerances as in Chapter 3, but in the correct manner. These errors will not be calculated using formulas 1 and 2 because they should be applied to the argument and modulus of transfer function (5), which is overly complicated and unnecessary. Instead, a Monte Carlo simulation was performed using MATLAB. To calculate measurement uncertainty, 100000 filters were simulated with passive components than had randomly generated deviations within their tolerance limits. Deviations were created using the function “rand”, and they were uniformly distributed. For each filter, key values (ω_0, BW...) were extracted. Maximum error was determined in a similar manner, using simulation.

TABLE 4: MAXIMUM ERRORS AND UNCERTAINTIES FOR VARIOUS VALUES OF COMPONENT TOLERANCES (MONTE CARLO)

	Maximum error of ω_0, Attenuation, *BW* (%)	Uncertainty uniform ω_0, Attenuation, *BW* (%)
$R_{\%}$=0.1, $C_{\%}$=1	1.0, 84.5, 34.5	0.34, 15.1, 8.9
$R_{\%}$=0.5, $C_{\%}$=2	2.1, 20.2, 99.1	0.68, 15.5, 20.2
$R_{\%}$=1, $C_{\%}$=5	Too large tolerances	Too large tolerances
$R_{\%}$=2, $C_{\%}$=10	Too large tolerances	Too large tolerances
$R_{\%}$=5, $C_{\%}$=20	Too large tolerances	Too large tolerances

By comparing tables 1 and 4 we can see that the wrong way of calculating measurement uncertainty and maximum error yields incorrect results in both cases and cannot be used as an approximation. In Table 4, the phrase "Too large tolerances" indicates that the tolerances of passive components cause deviations from nominal values to be so significant that, during simulation, the Twin-T notch filter entirely loses its characteristics and ceases to function as a band reject filter, rendering the errors effectively infinite or at least undeterminable. As for the error distribution function of ω_0, *BW*, and attenuation, it was assumed from the previous explanations that it will follow a normal distribution due to the central limit theorem. This is the case because the filter involves a large number of influential variables, each contributing to the error in a similar manner (except for C_3 and R_3, which have a slightly higher impact on attenuation). This assumption has now been confirmed by the Monte Carlo simulation, as shown in the histogram in Fig. 6. The distribution function is normal for errors of all observed quantities, but on Fig. 6 ω_0 error distribution is shown, in case of *R*=0.1 % and *C*= 1% tolerances.

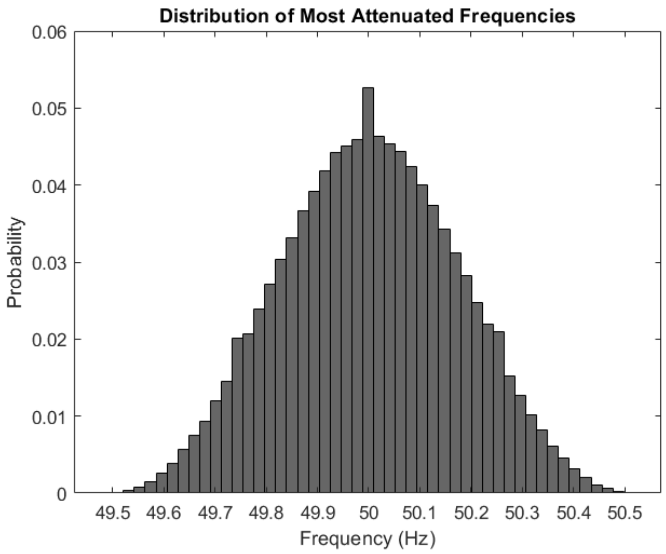

Fig. 6. Histogram for ω_0, 10^6 simulations, 50 bars

IV. CONCLUSION

There are a few problems with an active twin-t notch filter that make the analysis of its characteristics unique and difficult. Taking into consideration an integrator or differentiator, tolerances of passive components also alter the filter characteristics such as cutoff frequency, attenuation, phase shift, and so forth. However, in the case of these filters, tolerances do not change the order of the transfer function and do not complicate the analysis of the entire circuit. Instead, only the coefficients associated with poles and zeros change, so there always exists a generalized expression for determining ω. This differs with the twin T notch filter. It is a simple second-order transfer function commonly found in most filter textbooks, valid if and only if the filter components are matched. In this case, pole-zero cancellation occurs, and expressions for *Q*, ω_0, etc… are applicable. Any deviation from this case renders this transfer function inaccurate because it becomes third-order, and there are no direct expressions describing ω_0 and *Q*, significantly complicating the understanding of this filter's behavior. The most general form of the transfer function of this filter is provided in the paper for understanding the circuit's behavior, and an incorrect method of thinking and calculating errors arising from component tolerances is presented, which was found to have little practical utility in estimating these errors. Another difference of the twin-T notch filter from the previously mentioned ones is that in, for example, an integrator, a small change in cutoff frequency is not as crucial because the filter already cuts off a wide range of frequencies. In contrast, the notch filter is intentionally designed with a narrow stopband, so any change in ω_0 can render this filter completely useless. For instance, if we create a 50 Hz notch filter to remove mains hum, and the center frequency shifts by + or -1 Hz, the filter becomes entirely ineffective because the mains frequency typically varies in a very narrow range around 50 Hz, often by 0.1 Hz. This is why measurement uncertainty table can be used to determine if component tolerances are low enough to even begin making the filter, and if they are, which deviations from wanted results can be expected. Table gives error boundaries and uncertainties for the 50 Hz example, and it can be reproduced for any wanted Twin-T notch filter. Histograms confirm that the change of characteristics due to component tolerances indeed does follow a normal distribution, and this information can be used while designing this filter.

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