An Approach of the Sequential Monte Carlo PHD Algorithm for Multi Target Tracking

Zvonko Radosavljević, Branko Kovacević and Dejan Ivković

***Abstract*—The paper provides algorithm testing for full automatic multi target tracking (initialization, maintenance and deletion) applied in a complex radar track-while-scan (TWS) system. The standard Sequential Monte Carlo (SMC) algorithm was applied with the Probability Hypothesis Density (PHD) filter. Heavy clutter together with moving targets is tested with two dynamic models constant velocity (CV) and coordinate turn (CT) model. The measured robustness performance of multi targets tracking is used to adjust parameters and increase tracking stability in a dense clutter environment. The clutter density, number of random occurred targets, targets load during the maneuver and the target detection probability were varied. The results of OSPA metric distance and confirmed true tracks (CTT) are given in this paper.**

***Index Terms*—Sequential Monte Carlo, Particle PHD, multi targets tracking.**

I. INTRODUCTION

The radar sensor measurements may either be a spurious (clutter) or a target measurement. The target existence and trajectory are not a priori known. Target measurements are only present in a scan with some probability of detection P_D<1 [1]. In a multi-target situation, the measurements may have also originated from one of various targets. Targets may enter and leave the surveillance region at any time, thus at any given moment the number of targets in the surveillance area is unknown. Automatic tracking in this environment initiates tracks using both target and clutter measurements. If a track follows a target, we call it a true track; otherwise we call it a false track [2]. To confirm likely tracks and terminate unlikely ones, a track quality measure is necessary. Some standard track quality measures include the probability of The tracks are initialized using measurements, thus both true tracks and false tracks simultaneously exist [3].

The multiple hypothesis tracker (MHT) [4] is one of the first widely used algorithm for target tracking in clutter. The measurement-oriented MHT, often known as the Reid algorithm [5], forms new tracks and measurement allocation hypotheses centered on global origin of measurements. The MHT uses statistical methods (track score) to discriminate between false and true tracks. The probability of target existence obtained by utilizing Markov chain propagation models and Bayes update is used as the track quality measure in Integrated Probabilistic Data Association (IPDA) of [6] and Integrated Track Splitting (ITS).

The state estimation problem of discrete-time stochastic systems with Markov switching parameters is the focus of interest in target tracking [7]. The Interacting Multiple Model (IMM) algorithm is a widely accepted algorithm for solving this problem, which is generally nonlinear. In the process of *IMM* algorithm, updating weights of models are derived from the mixture of probability density function (*pdf*) and probability masses. Many specific dynamic models of target motion have been developed for target tracking. The simplest model is constant-velocity (CV) models, or more precisely, “nearly-constant-velocity models”, which is a non-maneuver model [8].

The random finite set (RFS) approach to multi-target tracking is an alternative association-based methods and comparative discussion between the RFS approach and traditional multi target tracking methods has been given in [9], [10]. In the RFS formulation, the collection of individual targets is treated as a set-valued state, and the collection of individual observations is treated as a set-valued observation. Modeling set valued states and set-valued observations as RFSs allows the problem of dynamically estimating multiple targets in the presence of clutter and association uncertainty to be cast in a Bayesian filtering framework [11,12].

. The first systematic treatment of multi-sensor multi-target filtering, as part of a unified framework for data fusion using random set theory was finite set statistics (FISST). The alternative to optimal multi-target filtering is the PHD filter [13,14, 15]. It is a recursion propagating the 1st moment, called the intensity function or PHD, associated with the multi-target posterior.

This article proposes SMC implementations for both the Bayes multi-target filter and the PHD filter together with experimental results.

Measurements and false alarms are also represented as random sets in the observation model. Mahler [16] employed the random set framework to propose a PHD filter. This method perform the data association between observations and objects and some implementations of PHD filter are proposed by using SMC method [17, 18].

Alternative approach of the SMC is the Gaussian mixture, consisting of a weighted sum of Gaussian *pdf*, each with different means and covariance’s It is the natural form of the *pdf* of target state. Using such a structure, a mixture

Zvonko Radosavljević is with the Millitary Technical Institute of Belgrade, 1 Ratka Resnovica, 11030 Belgrade, Serbia (e-mail: zvonko.radosavljevic@gmail.com),

Branko Kovačević is with the School of Electrical Engineering, University of Belgrade, 73 Bulevar kralja Aleksandra, 11020 Belgrade, Serbia (e-mail: kovacevic_b@etf.rs),

Dejan Ivković is with the Millitary Technical Institute of Belgrade, Ratka Resnovića 1, 11030 Belgrade, Serbia (e-mail: divkovic555@gmail.com).

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component is created for every possible association, using every possible pairing of target and measurements with the mean and covariance calculated assuming that the particular hypothesis is true, and the weight calculated to represent the probability that the particular hypothesis is true.

Rest of the paper is organized as follows. The problem statement and models is presented in Section II. Section III derives the SMC probability data association filter (SMC PHD) followed by the results of simulations, presented from Section IV. Concluding remarks will be given in Section V.

II. Problem Statement

Consider dynamic state estimation problem, in which the state varies with time. Preamble involves determining the existence and the trajectory of possible targets in the surveillance space, by comparing random measurements received by the sensor with the applicable stochastic models. We use superscripts τ to denote tracks, and also targets followed by tracks.

A. Targets model

This model assumes that a target may exist and when it does it is always detectable with a given probability of detection P_D, or it may not exist [19]. During the targets maneuvering, the motion can be changed at random times. The trajectory of a target can be described at any time by one of predefined dynamic models. A linear model is considered. The targets trajectory state, for the linear system, at time k, evolves by:

$$x_k^\tau = F_k \, x_{k-1}^\tau + \nu_k^\tau \qquad (1)$$

(1)

where F_k is the propagation matrix, and the process ν_k^τ noise is a zero mean and white Gaussian sequence with covariance. At each scan k, the sensor returns a random number of the random target and clutter measurements. The measurement of the existing and detectable target is taken with a probability of detection,.

B. Measurements models

Measurements may originate from the targets as well as from other objects. The clutter measurements follow the Poisson distribution. We assume that the uniform intensity of the Poisson process at point y in the measurement space, termed here the clutter measurement density and denote by $\rho(y)$ is a priori known, or can be estimated using the sensor measurements. At time k, one sensor delivers a set of measurements denoted by $y_k = \{y_{k,j}\}_{j=1}^{M_k}$.

Denote by Y^k the sequence of selected measurement sets up to including time k, $Y^K = \{Y^{k-1}, y_{k,1}, ..., y_{k,j}, ..., y_{k,M_k}\}$.

C. Sensors model

At each scan the sensor returns a random number of the random target measurements and a random number of the random clutter measurements. The measurement of the existing and detectable target is taken with a probability of detection P_D is given by the following equation [20]:

$$y_k^\tau = Hx_k^\tau + w_k^\tau \qquad (2)$$

(2)

where H is measurements matrix and the measurements noise w_k^τ is zero mean and white Gaussian sequence with covariance matrix R [14].

III. GM PHD Filter Derivation

A. Probability Hypothesis Density Filter

The PHD of an RFS is the analog of the expectation of a random vector. The RFS Γ can be represented by a random counting measure M defined by [21] :

$$M_\Gamma(S) = NoE\{\Gamma \cup S\} \qquad (3)$$

where the notation $NoE\{Z\}$ denotes the number of elements in set of element Z. Assuming there are n targets in the multi target system, each having state **s**, the density p_Γ of M_Γ can be used to represent the RFS Γ [22]:

$$p_\Gamma(\mathbf{X}) = \sum_{i=1}^{n} \delta(\mathbf{s}_i - \mathbf{X}_i) \qquad (4)$$

where $\delta(\mathbf{s}_i - \mathbf{X}_i)$ denotes the Dirac delta function centered at **X**. The PHD is then the first moment of the above is:

$$D_\Gamma(\mathbf{X}) = E\{p_\Gamma(\mathbf{X})\} \qquad (5)$$

The first moment density PHD has the form:

$$D_{k|k}(\mathbf{X}|Y^k) = \int_{X_k \in \mathbf{X}} f(k|k)(\mathbf{X}|Y^k) d\mathbf{X} \qquad (6)$$

The expected number of targets in region S is then:

$$N^{k|k} = E \cdot N_o E\{\Gamma(k) \cap S\} = \int_S D_{k|k}(\mathbf{X}|Y^k) d\mathbf{X} \qquad (7)$$

The PHD filter recursion is given in [23] and [24]. The predicted PHD is

$$D_{k|k}(\mathbf{Y}|Y^{k-1}) = b_{k|k-1}(\mathbf{Y}) + \int D_{k|k-1}(\mathbf{Y}|\mathbf{X}) D_{k-1|k-1}(\mathbf{X}|Y^{k-1}) d\mathbf{X} \qquad (8)$$

where

$$D_{k|k-1}(\mathbf{Y}|\mathbf{X}) = d_{k|k-1}(\mathbf{X}) f_{k|k-1}(\mathbf{Y}|\mathbf{X}) + b_{k|k-1}(\mathbf{Y}|\mathbf{X}) \qquad (9)$$

and $b_{k|k-1}(\mathbf{Y})$ is PHD of the spontaneous target birth, $d_{k|k-1}(\mathbf{X})$ is probability of target survival, $f_{k|k-1}(\mathbf{Y}|\mathbf{X})$ is transition probability density, $b_{k|k-1}(\mathbf{Y}|\mathbf{X})$: PHD of the targets

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spawned by existing targets. Given the new scan of data $Y^k = \{y_1,...,y_m\}$, the updated PHD is given by:

$$D_{k|k}(\mathbf{X}|Y^k) = \sum_{Y\in Y^k} \frac{P_D(\mathbf{X})D_k(\mathbf{Y})}{\lambda_k c_k(\mathbf{Y}) + P_D(\mathbf{X})D_k(\mathbf{Y})} \times D_k(\mathbf{X}|\mathbf{Y}) + (1 - P_D(\mathbf{X})D_{k|k-1}(\mathbf{X}|Y^{k-1}) \tag{10}$$

where

$$D_k(\mathbf{Y}) = \int f(k)[\mathbf{Y}|\mathbf{X}]D_{k|k-1}(\mathbf{X}|Y^{k-1})d\mathbf{X} \tag{11}$$

$$D_k(\mathbf{X}|\mathbf{Y}) = \frac{f(k)[\mathbf{Y}|\mathbf{X}]D_{k|k-1}(\mathbf{X}|Y^{k-1})}{D_k(\mathbf{Y})} \tag{12}$$

and $P_D(\mathbf{X})$ is probability of detection, λ_k is average number of false alarms per scan, assuming a Poisson distribution, $c_k(\mathbf{Y})$ is distribution of each of the false alarms, $f(k)[\mathbf{Y}|\mathbf{X}]$ is sensor likelihood function. At each time step, the PHD filter propagates not only the PHD, but also the expected number of targets. Consequently, estimation of the multitarget state is accomplished by searching for the $\min\{N^{k|k}\}$ largest peaks of $D_{k|k}(\mathbf{X}|Y^k)$.

B. Sequential Monte Carlo (SMC) Implementation of the Particle Probability Hypothesis Density Filter

SMC PHD algorithm is performed in next step: birth tagets Kalman prediction, spawned tagets Kalman prediction , update components and re sampling step. In this section, we describe the linear-Gaussian multiple target model and the recently developed Gaussian Mixture PHD filter. The multiple target models for the PHD recursion is described here. Each target follows a linear Gaussian dynamical model [25]:

$$p_{k|l-1,\xi} = N[x; F_{k-1,\xi}, Q_{k-1}] \tag{13}$$

$$g_k(y|x) = N[y; H_k(x), R_k] \tag{14}$$

where $N(.;m,P)$ denotes a Gaussian density with mean m and covariance P, F_{k-1} is the state transition matrix, Q_{k-1} is the process noise covariance, H_k is the observation matrix and $R(k)$ is the observation noise covariance. The survival and detection probabilities are state independent, $p_{S,k}(x) = p_{S_k}$ and $p_{D,k}(x) = p_{D,k}$. The intensities of the spontaneous birth and spawned targets are Gaussian mixtures [26]:

$$\gamma_k(x) = \sum_{i=1}^{J_\gamma(k)} w^i_{\gamma,k} N[x; m^i_{\gamma,k}, P^i_{\gamma,k}] \tag{15}$$

$$\beta_{k|k-1}(x|\varsigma) = \sum_{j=1}^{J_\beta(k)} w^j_{\beta,k} N[x; F^j_{\beta.k-1}\varsigma + d^j_{\beta,k-1} Q^j_{\beta,k-1}] \tag{16}$$

where $J_{\gamma,k}, w^i_{\gamma,k}, m^i_{\gamma,k}, P^i_{\gamma,k}$, $i = 1,..., J_{\gamma,k}$ are given model parameters that determine the shape of the birth intensity, similarly, $J_{\beta,k}, w^j_{\beta,k}, F^j_{\beta,k-1}, d^j_{\beta,k-1}$ and $Q^j_{\beta,k-1}$ $j = 1,..., J_{\beta,k}$ determine the shape of the spawning intensity of a target with previous state.

The PHD derivation involves multiple integrals that have no computationally closed form solutions for the case where individual targets follow a linear Gaussian dynamic model. Particle filtering techniques permit recursive propagation of the full posterior [6] and have been used for near-optimal Bayesian filtering. Sequential SMC implementation of the Particle PHD filter was given from [18].

For any $k \geq 0$ let $\{w_{i,k}, \xi_{i,k}\}_{i=1}^{L(k)}$ denote a particle approximation of the PHD.

The algorithm is designed such that the concentration of particles in a given region of the state space represents the expected number of targets in this region. We start with $L(k-1)$ particles, which are predicted forward to time *k*. At time *k*, we generate additional $J(k)$ new particles for exploring newborn targets. In the prediction step, we can get particle representation $\{w_{i,k|k-1}, \xi_{i,k}\}_{i=1}^{L(k-1)+J(k)}$. The update step maps the function with particle representation $\{w_{i,k|k-1}, \xi_{i,k}\}_{i=1}^{L(k-1)+J(k)}$ into one with particle representation $\{w_{i,k|k}, \xi_{i,k}\}_{i=1}^{L(k-1)+J(k)}$ by modifying the weights of these particles. Note that when implementing the resampling step, the weights are not normalized to 1 but sum to $\hat{N}^{k|k}$, the expected number of targets at time *k*.

Practical implementation of proposed algorithm, contain the procedure of the particle PHD filter is given as follows [27].

1) Step 1: Prediction Step

For all $i = 1,...,L(k-1)$ we compute sample:

$\tilde{\xi}_{i,k} \approx q_k[\cdot|\tilde{\xi}_{i,k-1}, Y^k]$

and compute the predicted weights:

$$\tilde{w}_{i,k|k-1} = \frac{\tau_k(\tilde{\xi}_{i,k}, \xi_{i,k-1})}{q_k(\tilde{\xi}_{i,k}|\xi_{i,k-1}, Y^k)} w_{i,k-1} \tag{17}$$

For $i = L(k-1)+1,..., L(k-1) + J_k$ sample $\tilde{\xi}_{i,k} \approx p_k[\cdot|Y^k]$ and compute the weights of newborn particles [28]:

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$$\tilde{w}_{i,k|k-1} = \frac{1}{J(k)} \frac{b_k[\tilde{\xi}_{i,k}]}{p_k[\tilde{\xi}_{i,k} | Y^k]} \tag{18}$$

where

$$\tau_k = d_{k|k-1} f_{k|k-1} + b_{k|k-1} \tag{19}$$

with $d_{k|k-1}$, $f_{k|k-1}$ and denoting $b_{k|k-1}$ the same meaning as in previous equations p_k and q_k are proposal densities and b_k denotes the PHD of the spontaneous target birth [29].

2) Step 2 Update Step

For each $y \in Y^k$ compute:

$$C_k(y) = \sum_{j=1}^{L(k-1)+J(k)} \psi_k[y, \tilde{\xi}_j(k)] \tilde{w}_{j,k|k-1} \tag{20}$$

For $i = 1,...,L(k-1) + J(k)$ update weights:

$$\tilde{w}_{i,k} = [1 - P_D + \sum_{y \in Y^k} \frac{\psi_k[y, \tilde{\xi}_{i,k}]}{\lambda_k c_k(y) + C_k(y)}] \tilde{w}_{i,k|k-1} \tag{21}$$

where $\psi_k(y,\xi) = P_D f_k(y|\xi)$.

3) Step 3 Resampling step

First, we compute the total mass by the [30]:

$$\hat{N}^{k|k} = \sum_{j=1}^{L(k-1)+J(k)} \tilde{w}j,_k \tag{22}$$

and than, resample $\{\frac{\tilde{w}_{i,k}}{\hat{N}^{k|k}}, \tilde{\xi}_i(k)\}_{i=1}^{L(k-1)+J(k)}$

to get $\{\frac{w_{i,k}}{\hat{N}^{k|k}}, \xi_{i,k}\}_{i=1}^{L(k-1)+J(k)}$.

In this filter, since the PHD is obtained for a frame at each scan, there is no state-to-state correlation between consecutive scans. All the measurements (including target-originated and clutter-originated measurements) are used equally weighted in the update step [31], [32].

This is not efficient for the use of the particles. Next, we introduce a track labeling technique combined with the PHD, so that we can use the information from the previous scan and choose better proposal densities and adjust the weights for the measurements accordingly in the update step.

Finally, mean and covariance are updated with the Kalman filter update equations,

$$m_{k|k}{}^j(y) = m_{k|k-1}{}^j(y) + K_k{}^j[y - H_k m_{k|k-1}{}^j(y)] \tag{23}$$

$$P^j{}_{k|k} = [I - K^j{}_k H_k] P^j{}_{k|k-1} \tag{24}$$

$$K_k{}^j = P^j{}_{k|k-1} H^T{}_k [H_k P^j{}_{k|k-1} H^T{}_k + R_k]^{-1} \tag{25}$$

IV. Results of simulations

The GM PHD experiments was performed and evaluated by 100 Monte Carlo (MC) runs over 2-dimensional test scenario. A target motion scenario (Fig.1) includes series of non-maneuvering and maneuvering flights modes.

Two targets (red circles) move in a straight line at a constant speed of 25 [m/s] and then in 14 scans they make a maneuver with an angular speed of 0.15 [rad/s], for a duration of 8 scans. Then they move again in a straight line 12 scans, so that the first target enters the maneuver with the same angular speed 17 scans into the second target 10 scans. At the end of the simulation, both wheels move in a straight line at a constant speed. Dimension of terrain surveillance is x=1000 [m] and y=1000 [m]. Clutter (blue dots) is have uniform distribution with density ρ =2e^{-5} [1/m^2].

The sampling period of radar sensor is *T=1s*. Duration of the scenario is 60 scans. Very important aspects to the implementation of SMC PHD algorithm is the selection of the model structures and their parameters.

The following two models, constant velocity (CV) model and constant acceleration (CT) model provide an adequate and self contained model set for tracking purposes. Hence, the model set for IMM filters has been selected as follows:M_1=*CV*, M_2=*CT*. Both models have the same state variables and this greatly simplifies the re-initialization operation of the IMM-GM-PHD filter.

If The system input is modeled as follows: $X = [x \quad \dot{x} \quad y \quad \dot{y}]$ is vector state, x, y are the Cartesian coordinates of the target position, $\dot{x}, \dot{y}$ are the appropriate velocities. Transition matrix (F_{CV} - constant velocity model and F_{CT}- coordinate turn model) and process noise matrix are given by:

$$\mathbf{F}_{CV} = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{26}$$

$$\mathbf{F}_{CT} = \begin{bmatrix} 1 & \sin wT / w & 0 & \frac{\cos wT - 1}{w} \\ 0 & \cos wT & 0 & -\sin wT \\ 0 & \frac{1 - \cos wT}{w} & 1 & \frac{\sin wT}{w} \\ 0 & \sin wT & 0 & \cos wT \end{bmatrix} \tag{27}$$

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$$\mathbf{Q}(k) = q\begin{bmatrix} T^3/3 & T^2/2 & 0 & 0 \\ T^2/2 & T & 0 & 0 \\ 0 & 0 & T^3/3 & T^2/2 \\ 0 & 0 & T^2/2 & T \end{bmatrix} \qquad (28)$$

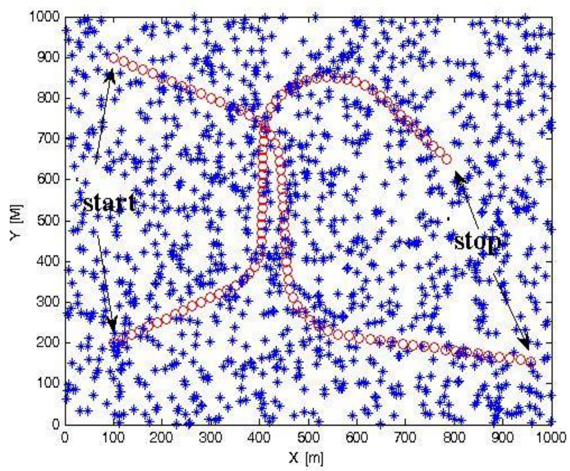

Fig. 1. Two targets simulation scenario.

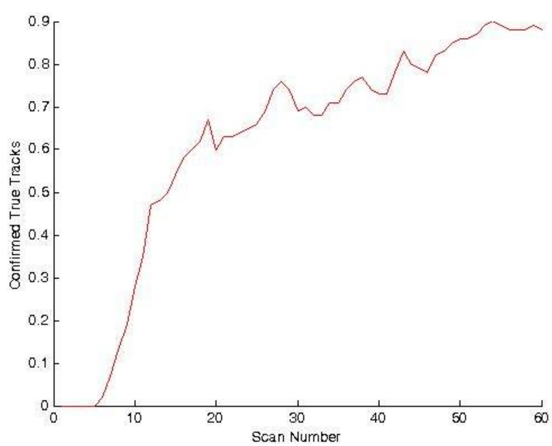

Fig. 2. Confirmed true tracks diagram.

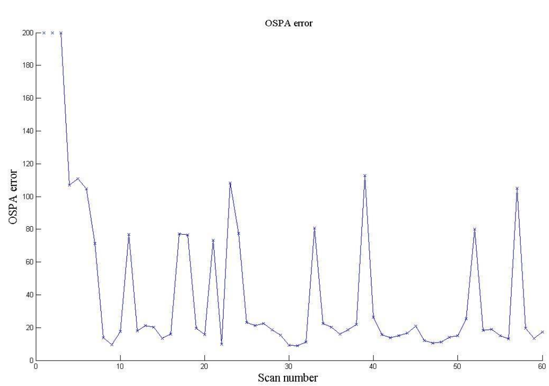

Fig. 3. OSPA diagram for experimental scenario .

Fig. 2. shows a diagram of the confirmed true tracks. It can be seen that effective tracking of maneuvering targets is possible under conditions of heavy clutter The diagram in Fig. 2. is a smallpox diagram.

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V. Conclusion

In this paper, we present the concept of SMC PHD full automatic multiple targets tracking for radar system approach. The proposed algorithm is numerical tested and could be applied for modernization of a TWS radar in meter frequency range. The full automatic SMC PHD multi target tracking algorithm is tested and illustrated. A standard SMC PHD derivation is defined in a real radar resolution cell. Simulation results with two-dimensional scenario showed that the proposed algorithm ends up with better performance and less computational load than the standard PHD filter and with better performance than the traditional MHT/assignment algorithms. Also, SMC PHD derivation of all track status parameters is given.

Whole target tracking procedure is tested by the extensive simulation. Future research should better determine the association of radar received measurements with existing targets as well as automatic initialization of targets.

In the future, the theoretical constraints of the proposed tracking algorithm have been discussed in the case of crossing targets. It is anticipated that the problem of retaining the correct target identity in this scenario can be resolved by considering the previous trajectories of targets. Results of simulation, given in the paper, show that the proposed algorithms has a good target tracking performance.

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References

[1] D. Reid, An algorithm for tracking multiple targets” IEEE Trans. AC, vol. AC-24, no. 6, pp. 843–854, 1979.

[2] Y. Bar-Shalom and T. E. Fortmann, Tracking and Data Association. Academic Press, San Diego, 1988.

[3] Ristic, B., Arulampamm, S., Gordon, N. Beyond the Kalman Filter.Dedham, MA: Artech House, 2004.

[4] Musicki, D., Evans, R. J.and Stankovic, S. Integrated probabilistic data association. IEEE Transactions on Automatic Control, 39, 6 (June 1994), 1237—1241.

[5] T. L. Song, D. Mušicki, D.S. Kim and Z.Radosavljević, Gaussian mixtures in multi-target tracking: a look at Gaussian mixture probability hypothesis density and integrated track splitting, IET proceedings on Radar Sonar and Navigation, Vol 6, no 5, pp. 359-364, June 2012.

[6] Z. Radosavljević,,IPDA Filters in the Sense of Gaussian Mixture PHD Algorithm, Scientific Technical Review, 2016,Vol.66,No.3, pp.34-40, Belgrade 2016, UDK : 681:5.017:623:746.3, COSATI:17-09, 12-01.

[7] Z. Radosavljević, D. Ivković and B. Kovačevic: ‘Integrated Particle Filter for Milti Target Tracking’, Proceedings of 8th International Conference of Electrical, Electronics, and Computing Engineering, IcETRAN 2021, Etno Village Stanisici, Republic of Srpska. Bosnia and Herzegovina, September 8-10, 2021.

[8] B. Kovačević, Z. Radosavljević, D.Ivković, MHT and ITS: A Look at Single Target Tracking in Clutter, EUSAR 2018: 12th European Conference on Synthetic Aperture Radar Electronic Proceedings, 04 - 07 June, 2018, Aachen, Germany, pp 971-977. ISBN-10: 3800746360.

[9] Z. Radosavljević, B. Kovačević, The Method of Integrated Particle Filter and Applications Capability for Targets Tracking, Invited paper, Proceedings of 17th International Symposium INFOTEH-JAHORINA,,21-23 March 2018.

[10] Z. Radosavljević, B. Kovačević, A Survey on Sequential Monte Carlo Particle PHD Algorithm, Proc. of 4. International Conferences , and 61. Conferences ETRAN, Kladovo, Serbia, June 2017.

[11] Z. Radosavljević, B. Kovačević, Analysis of Marginal Probability of Detection in Integrated Track Splitting Algorithm, Proc. of XVI Conference INFOTEH 2017, East Sarajevo, March 2017.

[12] S. Blackman, Multiple hypothesis tracking for multiple target tracking, IEEE A & E Systems Magazine, vol. 19, no. 1, part 2, pp. 5–18, 2004.

[13] I. Goodman, R. Mahler and H. Nguyen, Mathematics of Data Fusion. Kluwer Academic Publishers, 1997.

[14] R. Mahler, Multi-target Bayes filtering via first-order multi-target moments, IEEE Trans. AES, vol. 39, no. 4, pp. 1152–1178, 2003.

[15] Doucet, A., Godsill, S. J., and Andrieu, C. On sequential Monte Carlo sampling methods for Bayesian filtering. Statistics and Computing, 10 (2000), pp.197-208.

[16] Vo.B.Singh, S. and Doucet, A. Sequential Monte Carlo implementation of the PHD filter for multi-target tracking. In Proceedings of the International Conference on Information Fusion, Cairns, Australia, July.

[17] R. Mahler, “Multitarget Bayes filtering via first-order multitarget moments,” IEEE Trans. Aerosp. Electron. Syst., vol. 39, no. 4, pp. 1152–1178, Oct. 2003.

[18] B. Vo, S. Singh, and A. Doucet, “Sequential Monte Carlo methods for multitarget filtering with random finite sets,” IEEE Trans. Aerosp. Electron.Syst., vol. 41, no. 4, pp. 1224–1245, Oct. 2005.

[19] D. Clark and J. Bell, “Convergence results for the particle PHD filter,” IEEE Trans. Signal Process., vol. 54, no. 7, pp. 2652–2661, Jul. 2006.

[20] A. Johansen, S. Singh, A. Doucet, and B.Vo, “Convergence of theSMC implementation of the PHD filter,” Methodol. Comput. Appl. Probab., vol. 8, no. 2, pp. 265–291, 2006.

[21] R. Mahler, “An introduction to multisource-multitarget statistics and applications,” Lockheed Martin Technical Monograph, 2000.

[22] Bar-Shalom, Y.; Daum, F.; Huang, J. The Probabilistic Data Association Filter-Estimation In the Presence of Measurement Origin Uncertainty. IEEE Contr. Syst. Mag. 2009, 29, 82–100.

[23] S. Challa, R. Evans, M. Morelande, and D. Musicki, Fundamentals of Object Tracking. Cambridge University Press, 2011.

[24] Welch, G.; Bishop, G. An Introduction to Kalman Filter; Department of Computer Science University of North Carolina: Chapel Hill, NC, USA, 2006.

[25] Jan, S.S., Kao Y.C., Radar Tracking with an Interacting Multiple Model and Probabilistic Data Association Filter for Civil Aviation Applications, Sensors 2013, 13, 6636-6650; doi:10.3390/s130506636.

[26] K. Panta, B. Vo, and S. Singh. Improved probability hypothesis density filter (PHD) for multitarget tracking. Proceedings ICISIP, Bangalore 12th-15th December 2005.

[27] B. N. Vo, S. Singh, and A. Doucet, “Sequential Monte Carlo methods for Bayesian multi-target ┐ ltering with random ┐ nite sets,” IEEE Trans.Aerospace and Electronic Systems, vol. 41, no. 4, pp. 1224–1245, Oct 2005.

[28] W. K. Ma, B. Vo, S. Singh, and A. Baddeley, “Tracking an unknown time-varying number of speakers using TDOA measurements: a random finite set approach,” IEEE Trans Signal Processing, vol. 54, no. 9, 2006.

[29] B. N. Vo, S. Singh, and W. K. Ma, “Tracking multiple speakers with random sets,” in ICASSP, Montreal, Canada, 2004.

[30] O. Erdinc, P. Willet, and Y. Bar-Shalom. Probability Hypothesis Density Filter for Multitarget Multisensor Tracking. Proc. FUSION 2005.

[31] T. Zajic and R. Mahler, “A particle system implement of the PHD multitarget tracking ┐ lter,” in Signal Processing, Sensor Fusion and target Recognition XII, SPIE Proc, 2003, pp. 291–299.

[32] B. N. Vo and W. K. Ma, “The Gaussian mixture probability hypothesis density ┐ lter,” IEEE Transaction Signal Processing, vol. 54, no. 11, 2006.