Probability of Detection and False Alarm Density Estimation in Target Tracking Systems

Ljubica Mijajilović and Željko Đurović

***Abstract* – In this paper an estimator of the probability of detection of targets whose movement is monitored, as well as the density of false alarms in their immediate environment is proposed. Knowledge of these parameters is necessary for the successful application of any object tracking algorithm. Probability of target detection and density od false alarms are usually unknown quantities, and they are time-varying. The estimator is based on the maximum-likelihood approach. Since the movement of more than one target is observed in this paper, the estimator estimates the probability of detection for each target separately.**

***Index Terms* – Multi Target Tracking System, Probability of Target detection, Density of False Alarms**

I. Introduction

The field of Multi target tracking began to develop in the sixties of the last century, and the expansion of efficient multi target tracking algorithms occurred after the invention of the Kalman filter [1]. The problem of object tracking boils down to the process of estimating the position, velocity, and acceleration of the object we are tracking in the presence of false alarms. A major challenge with such algorithms is the association of the obtained observations with detected targets. In that case, a certain matric is used to determine the association of each observation with the corresponding target, and then one of the single-target tracking algorithms is applied, usually the Kalman filter. Algorithms that have proven to be effective in solving the associated problems are the JPDAF (*Joint Probabilistic Data Association Filter*) [2] and MHT (*Multiple Hypothesis Tracking*) [3]. However, the effectiveness of all Multi-target tracking algorithms depends on knowing the probability od target detection and the density of false alarms. Since these parameters are often unknown, a new capture in research has opened, which is the estimation of the probability of target detection, as well as the estimation of the density of false alarms that occur in the observed space. The fact that the parameters of the probability od target detection and the density of false alarms are not stationary over time, significantly complicates their estimation.

This paper proposes a new approach for estimating the time-varying probability of detection of multiple targets, as well as the density of false alarms. False alarms are not uniformly distributed throughout the space, so the immediate surrounding of the objects is always observed for the estimation of both parameters.

In [4], an approach for estimating the probability of detection of a single target and the density of false alarms in its surroundings was proposed based on the maximum likelihood method, assuming that there is only one object of interest in the gate. This paper considers the case of tracking multiple targets whose trajectories are close enough that observations corresponding to different targets can appear in a single window function. This significantly complicates the process of estimating the desired parameters, as well as the data association process. It is important to note that the estimator of the probability of detection and the density of false alarms works completely independently of the object tracking algorithm, and therefore the estimator can be combined with a large number of algorithms.

II. System Description

The motion of two objects in environment where false alarms occur will be observed in this paper. Since the probability od target detection is always less than

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one, their observations will not appear in each scan. However, due to existence of false alarms, the process of associating observations and tracking targets becomes complicated. Since knowledge of target detection probability and false alarm density is essential for quality target tracking, the idea presented in the paper is to implement an estimator that would estimate these parameters over time. The idea for the estimator is to be based on maximizing the joint likelihood functions of all hypotheses that can be posed. Unlike classical MHT approaches, the paper uses a reduced number of hypotheses, resulting in a computationally less demanding estimator. As it cannot be assumed that the false alarm distribution is uniform throughout the space, only the immediate vicinity of the detected objects will be observed. Rectangular window functions were used for this purpose.

Figure 1 shows position predictions of the detected objects in the k-th scan. Window functions of corresponding dimensions are formd around the predictions, and since the target trajectories are very close, the gates around the target predictions overlap. The closer the targets are and the greater the overlap of their window functions, the higher the probability that their observations will be found in the overlap, which further worsens the performance of the tracking algorithm, and the performance of the estimator is also expected to deteriorate.

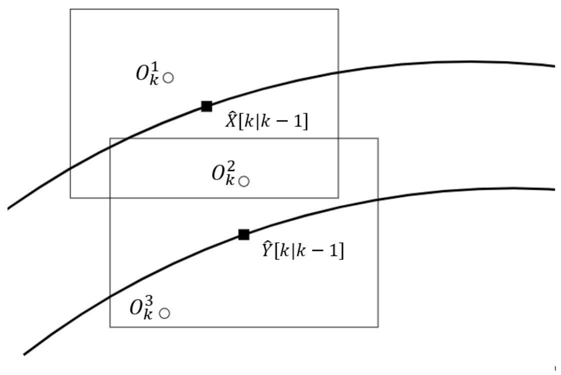

Figure 1 Illustration of window functions and observations in the k-th scan

In [4], one object was observed, so it was possible to construct only one or the most two hypotheses depending on weather an observation appear inside the gate or not. In the case of tracking multiple objects, the situation is much more complicated. In scan k shown in figure 1, three observations O_1, O_2 i O_3 satisfied the boundaries of the previously formed window functions, with observation O_1 located inside the gate of the first target; observation O_3 located in the gate of second target, and observation O_2 located in the intersection of those two gates. If all possible hypotheses were considered, there would be a total of eight in the case shown in figure 1.

- Targets are not detected.
- The first target is not detected, and observation O_2 is associated with the second target.
- The first target is not detected, and observation O_3 is associated with the second target.
- The second target is not detected, and observation O_1 is associated with the first target.
- The second target is not detected, and observation O_2 is associated with the first target.
- Both targets are detected. Observation O_1 is associated with the first target, and observation O_2 is associated with the second target.
- Both targets are detected. Observation O_1 is associated with the first target, and observation O_3 is associated with the second target.
- Both targets are detected. Observation O_2 is associated with the first target, and observation O_3 is associated with the second target.

As the likelihood of each hypothesis depends on the statistical distance between the target and the associated observation, it is obvious that hypotheses in which the closest observations are not assigned to the targets will have significantly lower likelihood. Therefore, such hypotheses will be eliminated at the outset. Instead of the initially proposed eight hypotheses in the case shown in figure 1, only four hypotheses are considered now:

- Targets are not detected.
- The first target is not detected. The observation that is statistically the closest is associated with the second target.
- The second target is not detected. The observation that is statistically the closest is associated with the first target.
- Both targets are detected. The observations that are statistically the closest are associated with them.

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In fact, the number of proposed hypotheses can range from one to four depending on the number and arrangement of incoming observations.

In order to propose hypotheses and define joint likelihood functions, the following notations will be adopted:

- M_1 – represents the total number of observations within the gate of the first object
- M_2 – represents the total number of observations within the gate of the second object
- M_{12} – represents the total number of observations in the intersection of two gates

The following are listed all the cases that may occur.

Case 1) $M_1 = 0, M_2 = 0, M_{12} = 0$

In this case, there is only one possible hypothesis which assumes that neither target has been detected. The likelihood function corresponding to this case is:

$$L(H_1^j) = (1 - P_{d1})(1 - P_{d2}) \frac{(\lambda V_g)^{M_j} e^{-\lambda V_g}}{M_j!} \frac{1}{V_g^{M_j}} \tag{1}$$

Case 2) $M_1 > 0, M_2 = 0, M_{12} = 0$

In the context of the second case, two hypotheses must be considered.

a. The first target is detected.

$$L(H_2^j) = P_{d1}(1 - P_{d2}) \frac{(\lambda V_g)^{M_j - 1} e^{-\lambda V_g}}{M_j - 1!} \cdot \frac{1}{V_g^{M_j - 1}} \frac{e^{-\frac{d_X(j)^2}{2}}}{(2\pi)^{0.5n} |S_x[j|j-1]|^{0.5}} \tag{2}$$

b. Targets are not detected.

$$L(H_2^j) = (1 - P_{d1})(1 - P_{d2}) \frac{(\lambda V_g)^{M_j} e^{-\lambda V_g}}{M_j!} \frac{1}{V_g^{M_j}} \tag{3}$$

Case 3) $M_1 > 0, M_2 > 0, M_{12} = 0$

In the context of the third case, four hypotheses must be considered.

a. Both targets are detected.

$$L(H_3^j) = P_{d1} P_{d2} \frac{(\lambda V_g)^{M_j - 2} e^{-\lambda V_g}}{M_j - 2!} \frac{1}{V_g^{M_j - 2}} \frac{e^{-\frac{d_X(j)^2}{2}}}{(2\pi)^{0.5n} |S_x[j|j-1]|^{0.5}} \frac{e^{-\frac{d_Y(j)^2}{2}}}{(2\pi)^{0.5n} |S_Y[j|j-1]|^{0.5}} \tag{4}$$

b. The first target is detected.

$$L(H_3^j) = P_{d1}(1 - P_{d2}) \frac{(\lambda V_g)^{M_j - 1} e^{-\lambda V_g}}{M_j - 1!} \frac{1}{V_g^{M_j - 1}} \frac{e^{-\frac{d_X(j)^2}{2}}}{(2\pi)^{0.5n} |S_x[j|j-1]|^{0.5}} \tag{5}$$

c. The second target is detected.

$$L(H_3^j) = (1 - P_{d1}) P_{d2} \frac{(\lambda V_g)^{M_j - 1} e^{-\lambda V_g}}{M_j - 1!} \frac{1}{V_g^{M_j - 1}} \frac{e^{-\frac{d_Y(j)^2}{2}}}{(2\pi)^{0.5n} |S_Y[j|j-1]|^{0.5}} \tag{6}$$

d. Targets are not detected.

$$L(H_3^j) = (1 - P_{d1})(1 - P_{d2}) \frac{(\lambda V_g)^{M_j} e^{-\lambda V_g}}{M_j!} \frac{1}{V_g^{M_j}} \tag{7}$$

Case 4) $M_1 = 0, M_2 > 0, M_{12} = 0$

In the context of fourth case, two hypotheses are being considered.

a. The second target is detected.

$$L(H_4^j) = (1 - P_{d1}) P_{d2} \frac{(\lambda V_g)^{M_j - 1} e^{-\lambda V_g}}{M_j - 1!} \frac{1}{V_g^{M_j - 1}} \frac{e^{-\frac{d_Y(j)^2}{2}}}{(2\pi)^{0.5n} |S_Y[j|j-1]|^{0.5}} \tag{8}$$

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b. Targets are not detected.

$$L\left(H_3^j\right) = (1 - P_{d1})(1 - P_{d2})\frac{\left(\lambda V_g\right)^{M_j} e^{-\lambda V_g}}{M_j!}\frac{1}{V_g^{M_j}} \quad (9)$$

Case 5) $M_1 = 1, M_2 = 1, M_{12} = 1$

As there is only one observation in the gate intersection in this case, its association with only the nearest target will be considered.

a. We adopt the assumption that the target closest to the observation has been detected. There are two options for setting the likelihood function:

$$L\left(H_5^j\right) = P_{d1}(1 - P_{d2})\frac{\left(\lambda V_g\right)^{M_j-1} e^{-\lambda V_g}}{M_j - 1!} \frac{1}{V_g^{M_j-1}}\frac{e^{-\frac{d_X(j)^2}{2}}}{(2\pi)^{0.5n}|S_x[j|j-1]|^{0.5}} \quad (10)$$

Or

$$L\left(H_5^j\right) = (1 - P_{d1})P_{d2}\frac{\left(\lambda V_g\right)^{M_j-1} e^{-\lambda V_g}}{M_j - 1!} \frac{1}{V_g^{M_j-1}}\frac{e^{-\frac{d_Y(j)^2}{2}}}{(2\pi)^{0.5n}|S_Y[j|j-1]|^{0.5}} \quad (11)$$

b. Targets are not detected.

$$L\left(H_5^j\right) = (1 - P_{d1})(1 - P_{d2})\frac{\left(\lambda V_g\right)^{M_j} e^{-\lambda V_g}}{M_j!}\frac{1}{V_g^{M_j}} \quad (12)$$

Case 6) $M_1 = 1, M_2 > 1, M_{12} = 1$

In the context of sixth case, four hypotheses are being considered.

a. Both targets are detected. The observation from the gate intersection assigned to the first target, while the nearest observation excluding the one from the intersection is assigned to the second target.

$$\left(H_6^j\right) = P_{d1}P_{d2}\frac{\left(\lambda V_g\right)^{M_j-2} e^{-\lambda V_g}}{M_j - 2!}\frac{1}{V_g^{M_j-2}} \frac{e^{-\frac{d_X(j)^2}{2}}}{(2\pi)^{0.5n}|S_x[j|j-1]|^{0.5}}\frac{e^{-\frac{d_Y(j)^2}{2}}}{(2\pi)^{0.5n}|S_Y[j|j-1]|^{0.5}} \quad (13)$$

b. The first target is detected, while the second is not detected.

$$L\left(H_6^j\right) = P_{d1}(1 - P_{d2})\frac{\left(\lambda V_g\right)^{M_j-1} e^{-\lambda V_g}}{M_j - 1!} \frac{1}{V_g^{M_j-1}}\frac{e^{-\frac{d_X(j)^2}{2}}}{(2\pi)^{0.5n}|S_x[j|j-1]|^{0.5}} \quad (14)$$

c. The second target is detected, while the first is not.

$$L\left(H_6^j\right) = (1 - P_{d1})P_{d2}\frac{\left(\lambda V_g\right)^{M_j-1} e^{-\lambda V_g}}{M_j - 1!} \frac{1}{V_g^{M_j-1}}\frac{e^{-\frac{d_Y(j)^2}{2}}}{(2\pi)^{0.5n}|S_Y[j|j-1]|^{0.5}} \quad (15)$$

d. Targets are not detected.

$$L\left(H_6^j\right) = (1 - P_{d1})(1 - P_{d2})\frac{\left(\lambda V_g\right)^{M_j} e^{-\lambda V_g}}{M_j!}\frac{1}{V_g^{M_j}} \quad (16)$$

Case 7) $M_1 > 1, M_2 = 1, M_{12} = 1$

In the context of seventh case, four hypotheses are being considered.

a. Both targets are detected. The observation from the gate intersection assigned to the second target, while the nearest observation excluding the one from the intersection is assigned to the first target.

$$\left(H_7^j\right) = P_{d1}P_{d2}\frac{\left(\lambda V_g\right)^{M_j-2} e^{-\lambda V_g}}{M_j - 2!}\frac{1}{V_g^{M_j-2}} \frac{e^{-\frac{d_X(j)^2}{2}}}{(2\pi)^{0.5n}|S_x[j|j-1]|^{0.5}}\frac{e^{-\frac{d_Y(j)^2}{2}}}{(2\pi)^{0.5n}|S_Y[j|j-1]|^{0.5}} \quad (17)$$

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b. The first target is detected, while the second is not detected.

$$L(H_7^j) = P_{d1}(1 - P_{d2})\frac{(\lambda V_g)^{M_j-1}e^{-\lambda V_g}}{M_j - 1!} \frac{1}{V_g^{M_j-1}} \frac{e^{-\frac{d_X(j)^2}{2}}}{(2\pi)^{0.5n}|S_x[j|j-1]|^{0.5}} \quad (18)$$

c. The second target is detected, while the first is not.

$$L(H_7^j) = (1 - P_{d1})P_{d2}\frac{(\lambda V_g)^{M_j-1}e^{-\lambda V_g}}{M_j - 1!} \frac{1}{V_g^{M_j-1}} \frac{e^{-\frac{d_Y(j)^2}{2}}}{(2\pi)^{0.5n}|S_Y[j|j-1]|^{0.5}} \quad (19)$$

d. Targets are not detected.

$$L(H_7^j) = (1 - P_{d1})(1 - P_{d2})\frac{(\lambda V_g)^{M_j}e^{-\lambda V_g}}{M_j!}\frac{1}{V_g^{M_j}} \quad (20)$$

Case 8) $M_1 > 1, M_2 > 1, M_{12} \geq 1$

In the context of the eighth case, four hypotheses are being considered.

a. Both targets are detected.

$$(H_8^j) = P_{d1}P_{d2}\frac{(\lambda V_g)^{M_j-2}e^{-\lambda V_g}}{M_j - 2!} \frac{1}{V_g^{M_j-2}} \frac{e^{-\frac{d_X(j)^2}{2}}}{(2\pi)^{0.5n}|S_x[j|j-1]|^{0.5}} \frac{e^{-\frac{d_Y(j)^2}{2}}}{(2\pi)^{0.5n}|S_Y[j|j-1]|^{0.5}} \quad (21)$$

b. The first target is detected, while the second is not.

$$L(H_8^j) = P_{d1}(1 - P_{d2})\frac{(\lambda V_g)^{M_j-1}e^{-\lambda V_g}}{M_j - 1!} \frac{1}{V_g^{M_j-1}} \frac{e^{-\frac{d_X(j)^2}{2}}}{(2\pi)^{0.5n}|S_x[j|j-1]|^{0.5}} \quad (22)$$

c. The second target is detected, while the first is not.

$$L(H_8^j) = (1 - P_{d1})P_{d2}\frac{(\lambda V_g)^{M_j-1}e^{-\lambda V_g}}{M_j - 1!} \frac{1}{V_g^{M_j-1}} \frac{e^{-\frac{d_Y(j)^2}{2}}}{(2\pi)^{0.5n}|S_Y[j|j-1]|^{0.5}} \quad (23)$$

d. Targets are not detected.

$$L(H_8^j) = (1 - P_{d1})(1 - P_{d2})\frac{(\lambda V_g)^{M_j}e^{-\lambda V_g}}{M_j!}\frac{1}{V_g^{M_j}} \quad (24)$$

III. Proposal of algorithm

Step 1: Define an integer variable N which corresponds to the number of scans that will be considered in the experiment. After each scan, parameter estimates of P_{d1}, P_{d2} and λ will be performed.

Step 2: After each scan, observations that satisfy the boundaries of pre-defined rectangular gate are selected. Based on positions of the observations, hypotheses are generated, of which there can be a maximum of 4. At the same time, the hypothesis matrix is updated.

Step 3: Each column of the hypothesis matrix corresponds to one integral hypothesis. N_h is the total number of integral hypotheses. The complex likelihood function $L(H_i), i = 1, \dots, N_h$ of each integral hypothesis is calculated. The complex likelihood function in the k-th scan will be of the form:

$$L(H_i) = (1 - P_{d1})^{N-\sum p_j}(1 - P_{d2})^{N-\sum q_j}P_{d1}^{\sum p_j}P_{d2}^{\sum q_j} \frac{(\lambda_{fa}V_g)^{\sum(M_j-p_j)}e^{-N\lambda_{fa}V_g}}{\prod(M_j-p_j)!}\frac{1}{V_g^{\sum(M_j-p_j)}} \cdot \frac{(\lambda_{fa}V_g)^{\sum(M_j-q_j)}e^{-N\lambda_{fa}V_g}}{\prod(M_j-q_j)!}\frac{1}{V_g^{\sum(M_j-q_j)}} \times \prod f_{X,j}(Z_j)^{p_j} \times \prod f_{Y,j}(Z_j)^{q_j} \quad (25)$$

Where $f_{X,j}(\cdot)$ and $f_{Y,j}(\cdot)$ are the corresponding probability density functions, and Z_j is the observation that is statistically closest to the assigned target. M_j represents the total number of observations within the boundaries of the first and second gates.

In order to maximize this complex likelihood function with respect to the unknown parameters, the following equations need to be solved:

$$\frac{\partial L(H_i)}{\partial P_{d1}} = 0, \frac{\partial L(H_i)}{\partial P_{d2}} = 0 \ i \ \frac{\partial L(H_i)}{\partial \lambda_{fa}} = 0 \qquad (26)$$

After solving the previous equations, the following expressions are obtained:

$$\hat{P}_{d1} = \frac{\sum p_j}{N}, \hat{P}_{d2} = \frac{\sum q_j}{N}, \hat{\lambda}_{fa} = \frac{\sum(M_j - q_j)}{NV_g} \qquad (27)$$

The previously mentioned expressions optimize the likelihood of each integral hypothesis. The values obtained using the aforementioned expressions are used to calculate the likelihood of each integral hypothesis.

Step 4: Out of all optimal likelihoods, the highest one is selected, and based on it, the optimal parameters of the estimator P_{d1}, P_{d2} i λ_{fa} are determined.

Step 5: As the number of integral hypotheses increases from scan to scan, it is necessary to reduce the number of hypotheses. For this purpose, the Branch and Bound algorithm [5] is used.

It is assumed that the same assumptions as in the paper [4] apply.

IV. Simulation results

In order to demonstrate the performance of the proposed estimator, 1000 Monte Carlo simulations were conducted. One simulation consists of $N = 30$ consecutive scans. The motion of two objects with actual detection probabilities $P_{d1} = 0.95$ and $P_{d2} = 0.75$ was observed. The actual false alarm density was $\lambda_{fa} = 10^{-5}$. Figure 2 shows the estimated detection probabilities of both targets.

The variance of the obtained estimates is shown in Figure 3. It is expected that the variance increases as the actual target detection probability decreases. However, the variance values are high, especially for the second target with an actual detection probability of $P_{d2} =$ 0.75. Further research would involve correcting the proposed estimator to reduce the variance of the detection probability estimates. The estimate of the false alarm density is shown in Figure 4.

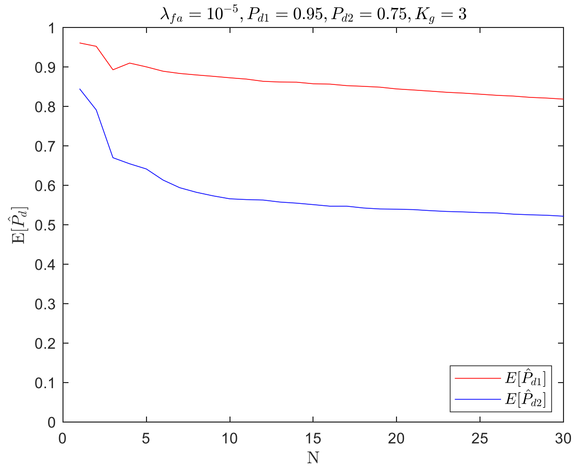

Figure 2 Mean estimate of target detection probability

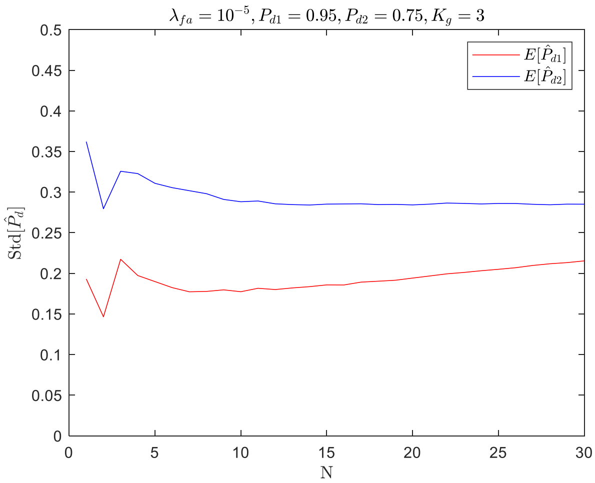

Figure 3 Standard deviation of the obtained estimates from figure 2

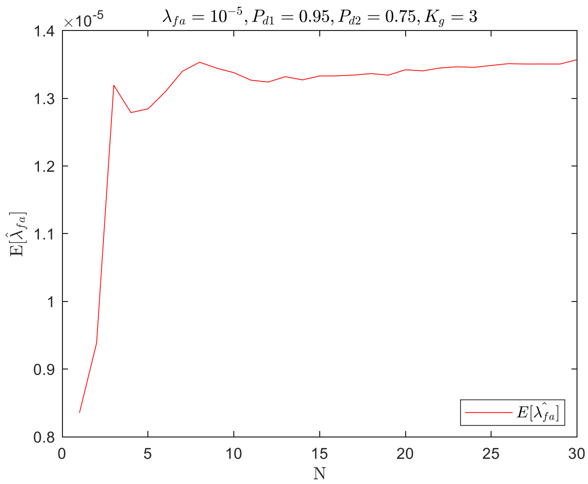

Figure 4 Mean estimate of false alarm density

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