Feature Analysis for Industrial Product Sounds
Using Discrete Meyer Wavelet

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Abstract—The process of wavelet decomposition into approximation and detail coefficients is used in many research fields, especially when signal de-noising is in focus. Extraction of features of different signal types is also an area where wavelets are often mentioned. Different wavelet families provide interesting results in feature analysis and further classification. In that regard, a wavelet that has attracted a significant interest is discrete Meyer. This paper presents the usage of discrete Meyer wavelet for feature analysis of industrial product sounds. More than 100 sounds of 6 different industrial products are tested. The most representative results are given here.

Index Terms—Audio feature analysis; Discrete Meyer wavelet; Approximation coefficients; Detail coefficients; Industrial product sound.

I. INTRODUCTION

Wavelets are often mentioned in literature as an algorithm that solved some problems in the Fourier transform [1-2]. If short time Fourier transform (STFT) and wavelet algorithms are compared, the window function are used in both cases for purpose of analysis and processing of signals. However, in the case of wavelets, window function is not of a fixed size. Width of the wavelet window can be changed as the transform is computed for every single spectral component, and that is the main difference in comparison with STFT [1-2].

Applicability of wavelets can be considered to be one of their advantages since a number of applications are made based on the wavelet algorithms. Some of them include echo cancelation, noise control, speech recognition, de-noising of audio and image signals, unknown system detection and others. Applications can be found in different fields, such as biomedical engineering, telecommunications, signal processing, computer engineering, etc [3-6].

The process of wavelet decomposition into approximation and detail coefficients (Fig. 1) is often used nowadays in feature extraction and analysis. Authors of this paper already presented the results of feature extraction method using wavelets for DC motor sounds in [7-9], where focus is on differences between non-faulty and faulty motors. In that regard, a lot of wavelet families are used, such as Haar, Daubechies, Coiflet, Symlet, biorthogonal, reverse biorthogonal and Meyer.

In the discrete wavelet processing, the decomposition process is done using the discrete wavelet transform (DWT), (see Fig. 1). This process is based on the use of low-pass (LP) and high-pass (HP) filters. Here, “A” stands for the approximation coefficients, “D” stands for the detailed coefficients and 2↓ is the down-sampling. Fig. 1 presents only decomposition up to level 2, but the whole process can be extended to a higher level of decomposition [4,5,10-11].

![Block diagram of wavelet decomposition](image)

This paper presents usage of the discrete Meyer wavelet for the purpose of feature extraction from different industrial product sounds. More than 100 sounds organized into 6 different classes (categories) are tested: fan, gearbox, slider, toycar, toystain and valve. Those sounds are taken from the DCASE 2021 challenge (task 2) [12]. The goal is to find certain similarities of the extracted features within the same class and certain differences among the classes in order to prepare the obtained features for future classification. The analysis is done in Matlab software package.

II. MEYER WAVELET

In 1985, Yves Meyer developed the first nontrivial orthogonal wavelet basis. This wavelet is in-group of orthogonal wavelets where are also Daubechies, Coiflet and Symlet wavelets. The main idea was to define wavelet and scaling function in the frequency domain using trigonometric functions. Both wavelet and scaling functions are presented in (1) and (2) [11,13-15]:

\[
\hat{\psi}(\omega) = \frac{1}{\sqrt{2\pi}} e^{-i\omega/2} \begin{cases}
\sin\left(\frac{\pi}{2} \sqrt{\frac{3}{2\pi}} |\omega| - 1\right) & \frac{2\pi}{3} \leq |\omega| \leq \frac{4\pi}{3} \\
\cos\left(\frac{\pi}{2} \sqrt{\frac{3}{4\pi}} |\omega| - 1\right) & \frac{4\pi}{3} \leq |\omega| \leq \frac{8\pi}{3} \\
0 & |\omega| \not\in \left[\frac{2\pi}{3}, \frac{8\pi}{3}\right]
\end{cases}
\]

\[
\hat{\varphi}(\omega) = \frac{1}{\sqrt{2\pi}} e^{-i\omega/2} \begin{cases}
\sin\left(\frac{\pi}{2} \sqrt{\frac{3}{2\pi}} |\omega| - 1\right) & \frac{2\pi}{3} \leq |\omega| \leq \frac{4\pi}{3} \\
\cos\left(\frac{\pi}{2} \sqrt{\frac{3}{4\pi}} |\omega| - 1\right) & \frac{4\pi}{3} \leq |\omega| \leq \frac{8\pi}{3} \\
0 & |\omega| \not\in \left[\frac{2\pi}{3}, \frac{8\pi}{3}\right]
\end{cases}
\]
The wavelet function, presented in (1), is symmetrical around point 1/2, while the scaling function (2) is symmetrical around the point 0 [12-14]. What is also important to mention is that the Meyer wavelet is an indefinitely differentiable with infinite support. In Fig. 2, the discrete Meyer wavelet function (which is often used in software packages) is presented.

\[
\phi(\omega) = \frac{1}{\sqrt{2\pi}} \begin{cases} 
1 & \frac{2\pi}{3} \leq |\omega| \leq \frac{4\pi}{3} \\
\cos\left(\frac{\pi}{2} \nu\left(\frac{3}{2\pi}|\omega| - 1\right)\right) & |\omega| \leq \frac{2\pi}{3} \\
0 & |\omega| \geq \frac{2\pi}{3}
\end{cases}
\]

where the function \( \nu \) can be changed for the purpose of obtaining different wavelets, for example as done in (3) [12,14]:

\[
\nu(a) = a^4(35 - 84a + 70a^2 - 20a^3), a \in [0,1].
\]

The analysis is carried out in Matlab software package. When the wavelet decomposition is in focus, it is worthwhile to mention a few functions: `wavecalc` for wavelet decomposition, `appcoef` and `detcoef` for obtaining the approximation and detail coefficients, `abs` and `mean` for calculating absolute and mean values, respectively.

A specific measure (quantity) named feature difference is introduced in certain isolated cases. It is calculated as the mean value of a difference between wavelet-based features for different industrial products taken from all segments and normalized by the mean feature value [8-9]. This measure is used to indicate more closely the numerical differences among the obtained features in addition to what is shown graphically.

### III. METHODS OF ANALYSIS

Within the task 2 (Unsupervised anomalous sound detection for machine condition monitoring under domain shifted conditions) of DCASE 2021 challenge [12], a number of sounds of 7 different industrial machines (products) - fan, gearbox, pump, slide rail (slider), toycar, toytrain and valve are available. For each of the mentioned products, there are three datasets – development, additional training and evaluation datasets. The development dataset is split into three subsets, and in overall these development datasets contain more than 4000 sound samples of normal and anomalous operating product modes.

Among those sounds from the development datasets, about 100 sound samples from 6 classes – from all classes mentioned above except pump (approximately around 17 sounds per product) are chosen here for initial analysis. All sounds are recorded using the sampling frequency of 16 kHz. Length of the recorded signals is 10 s, and they include both the sounds of a machine and its associated equipment as well as environmental noise [12]. In this paper, the following abbreviations will be used: fan-f, gearbox-g, slider-s, toycar-tc, toytrain-tt, valve-v.

At the very beginning, all chosen sounds are analyzed in the time and frequency domain. Then, segmentation process is applied. Size of segments is 50 ms. In this paper, the overlap between neighbor segments of 50% (25 ms) is chosen like in some other previous papers [7-9,16]. Size of segments and overlap can vary from research to research. In some studies, authors propose no overlap between segments [17-18].

After segmentation, the signals are decomposed using the discrete Meyer wavelet. This wavelet is a unique wavelet in the sense that there are no more variations, as for instance in the Coiflet wavelet family, where there are 5 different types in Matlab or in the Symlet wavelet having 45 types, etc [19]. The Meyer wavelet is chosen here having in mind the previous authors’ researches where this wavelet gave interesting results. The detail coefficients obtained through the wavelet decomposition with different levels of decomposition (from 1 up to 8) are used for further analysis.

At each decomposition level and for every segment, the absolute values of the detail coefficients are calculated, and then the mean value of these absolute values is obtained. The set of the mean values of the details coefficients generated in the described way is used as a representative audio feature (wavelet-based feature). In some researches, such as [7-9,20-21], it is presented that the standard deviation can also be used, but authors of this paper showed previously, see [8-9], that the results based on the mean values and standard deviation are similar. This is why the standard deviation is omitted here.

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A specific measure (quantity) named feature difference is introduced in certain isolated cases. It is calculated as the mean value of a difference between wavelet-based features for different industrial products taken from all segments and normalized by the mean feature value [8-9]. This measure is used to indicate more closely the numerical differences among the obtained features in addition to what is shown graphically.

### IV. RESULTS

The initial analysis starts with observation of signals in the time and frequency domain. In Fig. 3, six random audio signals (one from each type of the industrial products) are presented in the time domain. If the graphs are closely observed, it can be pointed out that differences between these signals are small. Thus, the time domain signals of fan and gearbox are very similar. The same is valid for the signals of slider, toycar and valve, although some more transients are present here. An exception is the signal of toytrain, which has a rather different shape and amplitude comparing to others. In the next four figures, lines associated to these products are plotted by the same color scheme as done in Fig. 3: red color for fan, blue color for gearbox, black color for slider, green color for toycar, magenta color for toytrain, cyan color for...
Fig 3. Audio signals (sounds) of industrial products in time domain: (a) fan, (b) gearbox, (c) slider, (d) toycar, (e) toytrain and (f) valve.

Spectra of the signals from Fig. 3 are presented in Fig. 4. In the frequency domain, the situation is rather different and differences between spectra are more prominent than in the time domain. Again, like in the time domain, the sound of toytrain product sound stands out from the others (see Fig. 4). The trend in spectra for all other products is similar in the frequency range above 125 Hz, where the maximum deviation of any spectra of these products is not greater than approximately 10 dB. On the other hand, the spectrum of toycar shows a significant rolloff below 125 Hz, which is not the case with spectra of other products.

The results obtained in the frequency domain indicate a very favorable trend of what could happen when the signal decomposition by the discrete Meyer wavelet is done. In Fig. 4, the areas from D1 to D8 (from right to left) are also indicated. Those areas are related to the frequency ranges of the detail wavelet coefficients after decomposition at a particular level. Thus, the frequency range from 4 kHz to 8 kHz is correlated with the detail coefficients at the decomposition level 1 (D1). At the next decomposition level, the lower frequency range (up to 4 kHz) is split into two halves, so that the detail coefficients at the decomposition level 2 (D2) are extracted from the frequency range from 2 kHz to 4 kHz. This trend continues through the next decomposition levels [9] up to the last one, which is here the level 8. For all 8 decomposition levels, the related frequency ranges are given in Table I.

The illustration of the product sound characteristics in both time and frequency domain is given through the spectrograms presented in Fig. 5. The trend seen in the spectra given in Fig. 4 of the level reduction with frequency is also visible in the spectrograms. However, here some unique time-stamps of the signals are stressed, such as fluctuation in time of the slider or non-stationary amplitude of the toytrain.

<table>
<thead>
<tr>
<th>Decomposition level</th>
<th>Frequency range</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4 kHz – 8 kHz</td>
</tr>
<tr>
<td>2</td>
<td>2 kHz – 4 kHz</td>
</tr>
<tr>
<td>3</td>
<td>1 kHz – 2 kHz</td>
</tr>
<tr>
<td>4</td>
<td>500 Hz – 1 kHz</td>
</tr>
<tr>
<td>5</td>
<td>250 Hz – 500 Hz</td>
</tr>
<tr>
<td>6</td>
<td>125 Hz – 250 Hz</td>
</tr>
<tr>
<td>7</td>
<td>62.5 Hz – 125 Hz</td>
</tr>
<tr>
<td>8</td>
<td>31.25 Hz – 62.5 Hz</td>
</tr>
</tbody>
</table>

After analysis in the time and frequency domain, the chosen signals are decomposed using the discrete Meyer wavelet. Fig. 6 presents the values of the extracted wavelet-based feature for all six products, that is, signals whose time and frequency representations are given in Figs. 3, 4 and 5. All signals are segmented, wavelet decomposed and then the mean value of the absolute values of the obtained detail coefficients is calculated for each particular segment. At each decomposition level, some differences between the wavelet-based features (detail coefficients) of different products can be observed.
These differences are smaller for some decomposition levels and comparing certain products, while they are larger at some other levels and for some other products. Also, the stationarity of the features of different products is not the same, and it might be used as one more parameter for differentiating the products. The previously mentioned emphasizes the need to make a more detailed analysis between the signals and the products. The previously mentioned emphasizes the need to make a more detailed analysis between the signals and the products. The previously mentioned emphasizes the need to make a more detailed analysis between the signals and the products.

In Tables II and III, the numerical values of the feature difference measure are summarized, where indexes in the numerical values represent the level of decomposition from which that specific case is obtained. These differences can be used to consider more closely the relations between particular products. Table II presents the feature differences between the products belonging either to the same class or different classes, where these differences are prominent. On the other hand, Table III presents the feature differences between the products where the differences are rather small when the products belong to the same class, but also when the products belong to different classes.

**TABLE II**

<table>
<thead>
<tr>
<th>Diff.</th>
<th>f</th>
<th>g</th>
<th>s</th>
<th>tc</th>
<th>tt</th>
<th>v</th>
</tr>
</thead>
<tbody>
<tr>
<td>f</td>
<td>0.79</td>
<td>1.12</td>
<td>1.15</td>
<td>1.35</td>
<td>1.76</td>
<td>1.01</td>
</tr>
<tr>
<td>g</td>
<td>1.12</td>
<td>0.86</td>
<td>1.13</td>
<td>1.32</td>
<td>1.84</td>
<td>1.24</td>
</tr>
<tr>
<td>s</td>
<td>1.15</td>
<td>1.13</td>
<td>0.76</td>
<td>1.31</td>
<td>1.91</td>
<td>0.85</td>
</tr>
<tr>
<td>tc</td>
<td>1.35</td>
<td>1.32</td>
<td>1.31</td>
<td>0.84</td>
<td>1.91</td>
<td>1.36</td>
</tr>
<tr>
<td>tt</td>
<td>1.76</td>
<td>1.84</td>
<td>1.91</td>
<td>1.06</td>
<td>1.73</td>
<td>1.03</td>
</tr>
<tr>
<td>v</td>
<td>1.01</td>
<td>1.24</td>
<td>0.85</td>
<td>1.36</td>
<td>1.73</td>
<td>1.03</td>
</tr>
</tbody>
</table>

Analyzing Table II, it can be concluded that the smallest feature difference is obtained within the same class, that is going along the diagonal except for the product valve denoted as “v”, which has slightly smaller values for fan and slider meaning that this feature of the valve is more similar to the feature of those two products - fan and slider. But, in other products, it can be clearly seen that the features are much more similar within the class than between the classes.

The trend seen in Table II can not be seen in Table III where the features differences are rather small independently whether the compared products belong to the same class or different classes. The only exception is the product toytrain that has considerably larger feature differences when the product is compared to other product belonging to other classes. All these trends seen in the feature difference measure are in line with the noticed specific properties of spectra of these products.

Table III presents the feature differences from Tables II and III in the plot format. Differences having moderate values (called prominent differences) are given by blue curves, while small differences are given by red curves. Here, Fig. 7(b) presents only 6 cases randomly extracted from Tables II and III used to illustrate prominent and small differences between the classes. The values for prominent differences are smaller within the same class, Fig. 7(a) and diagonal elements of Table II, in comparison with the values between the classes, Fig. 7 (b). On the other hand, the values for small differences are rather small independently whether are calculated within the same class or between the classes except for the toytrain as discussed above. The latter shows that the differences of the wavelet-based feature between the classes can be small in some extreme cases representing minority of all cases.

**TABLE III**

<table>
<thead>
<tr>
<th>Diff.</th>
<th>f</th>
<th>g</th>
<th>s</th>
<th>tc</th>
<th>tt</th>
<th>v</th>
</tr>
</thead>
<tbody>
<tr>
<td>f</td>
<td>0.007</td>
<td>0.005</td>
<td>0.009</td>
<td>0.008</td>
<td>0.24</td>
<td>0.004</td>
</tr>
<tr>
<td>g</td>
<td>0.005</td>
<td>0.002</td>
<td>0.001</td>
<td>0.005</td>
<td>0.84</td>
<td>0.002</td>
</tr>
<tr>
<td>s</td>
<td>0.009</td>
<td>0.001</td>
<td>0.001</td>
<td>0.03</td>
<td>0.71</td>
<td>0.001</td>
</tr>
<tr>
<td>tc</td>
<td>0.008</td>
<td>0.005</td>
<td>0.03</td>
<td>0.005</td>
<td>0.53</td>
<td>0.001</td>
</tr>
<tr>
<td>tt</td>
<td>0.24</td>
<td>0.84</td>
<td>0.71</td>
<td>0.53</td>
<td>0.001</td>
<td>0.5</td>
</tr>
<tr>
<td>v</td>
<td>0.004</td>
<td>0.002</td>
<td>0.001</td>
<td>0.001</td>
<td>0.5</td>
<td>0.007</td>
</tr>
</tbody>
</table>

It is beneficial to look at the differences in the obtained detail coefficients (wavelet-based features) from different
angles. As in all other cases of the wavelet-based features presented here, these features are obtained by applying the discrete Meyer wavelet to each signal segment. Fig. 8 presents prominent differences in the detail coefficients at some decomposition levels between the signals from same class (the same product type). The level of decomposition and class are indicated in the title of each graph, as it is also done for next figures of the same type. The blue curve is related to the first signal (product), while the red curve is related to the second signal (product). The presented differences are obvious, like the ones given in Table II.

A rather specific situation is presented for the toycar in Fig. 8, where the detail coefficients of these two toycar signals differ in a part of the segments (part of the signal), while this is not the case in the rest of the segments (rest of the signal) where the differences are negligible.

The opposite case is given in Fig. 9 presenting the negligible differences between the detail coefficients obtained from the signals belonging to the same class (the same type of industrial product). This figure can be used as a graphical confirmation of the results listed in Table III.

Fig. 10 presents some illustrative examples of the wavelet-based features - detail coefficients at particular levels of decomposition where the differences between the signals from different classes (types of industrial products) are obvious. Those difference are rather prominent, and they are greater than differences seen between the products belonging to the same class. Smaller differences between the detail coefficients (wavelet-based features) for signals from different classes (types of industrial products groups) are shown in Fig. 11. The found differences could not be considered to be negligible. They are smaller than those given in Fig. 10, but greater than those given in Fig. 9. What is also worth mentioning is that these wavelet-based features have some specific behavior along the time axis. For some of the classes (product types), such as the toytrain and toycar, it is not an easy task to find small differences between the wavelet-based features. These signals are not as stationary in time as other signals, and this is why their wavelet-based features show significant fluctuations of values along the time axis, that is, going through the segments.

Fig 8. Wavelet-based features generated from the detail coefficients after applying discrete Meyer wavelet to segmented signals obtained for signals from the same class (type of industrial product); the case where the differences are obvious.

Fig 9. Wavelet-based features generated from the detail coefficients after applying discrete Meyer wavelet to segmented signals obtained for signals from the same class (type of industrial product); the case where the differences are negligible.

Fig 10. Wavelet-based features generated from the detail coefficients after applying discrete Meyer wavelet to segmented signals obtained for signals from different classes (types of industrial products) (extracted illustrative examples); the case where the differences are obvious.

Fig 11. Wavelet-based features generated from the detail coefficients after applying discrete Meyer wavelet to segmented signals obtained for signals from different classes (types of industrial products) (extracted illustrative examples); the case where the differences are small.
Analyzing Figs. from 8 to 11, it can be concluded that the presented prominent differences between the wavelet-based features are related to the cases when the feature curves differ from each other, while the negligible differences are related to the cases where the feature curves are overlapped (or close to each other) in the majority of time segments.

V. CONCLUSION

Wavelets represents a technique that is more and more popular nowadays in diverse application including audio feature extraction. Since there are a number of different wavelet families, it is a rather difficult task to select the most appropriate one for a particular purpose. This paper presents usage of the discrete Meyer wavelet in signal decomposition to approximation and detail coefficients, where the latter are used as the wavelet-based features. The goal is to get relevant audio features able to provide a large enough distinction among industrial products based on sound they generate.

More than 100 sound samples of industrial products (machines) belonging to six different classes are used in this study. The decomposition of every sound using the discrete Meyer wavelet goes from level 1 to level 8. All signals are first segmented, where the overlap between two segments is 50% of the segment length.

In this phase of the research, only manual numerical and graphical analysis is performed. The results show that the detail coefficients used as a wavelet-based feature provide a significant difference between the classes (types of industrial products) in majority of cases. Certain differences exist even between the sound samples from the same class, but typically they are smaller than the ones between the classes. Some illustrative examples showing small differences both within the same class and between different classes are identified and presented. This confirms that even when different industrial products are analyzed, it is possible to get very similar characteristics in the time and frequency domain, but also in the domain of audio features (in this case wavelet-based feature). This is why a more detailed analysis of statistics of the extracted wavelet-based features will be done in the next phase of the research.

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REFERENCES


