Application of 5G Channel Coding Techniques in Smart Grid: LDPC vs. Polar coding for Command Messaging

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***Abstract*— A reliable, scalable, secure, and cost-effective telecommunication and information exchange system is of immense importance in a Smart Grid (SG) vision. 5G, as a novel technology, promises to bring numerous benefits in the energy sector, mostly in increasing overall energy efficiency, reliability, quality of service and accelerating the SG development. Channel coding, as a fundamental building block in any communications system, plays a substantial role in the realization of fast communication with minimum errors during data transfer in an imperfect channel environment. In this paper command messaging in SG has been considered and the performance of 5G channel coding techniques, Low-Density Parity-Check (LDPC), and Polar codes, in terms of the Bit Error Ratio (BER) for different code rates, have been investigated. The simulation results confirm the superiority of Polar coding in the case of transmitting a typical command message from the base station to the Phasor Measurement Unit (PMU) device.**

***Index Terms*—LDPC; Polar coding; 5G; Smart Grid; Command Message**

I. Introduction

The number of smart devices increases continuously, resulting in the creation of the Internet of Things (IoT) network in which smart devices are connected via the Internet. Existing mobile networks need to be enhanced in terms of capacity, data rate, latency, and other performances in order to successfully respond to the increased usage of mobile and smart devices and diversification of novel application requirements. Hence, the development of the fifth generation of mobile communications, commonly called the 5G, has been motivated by the increased usage of mobile and other smart devices and demands for low latency, highly reliable, and highly safe communication networks [1].

5G has made enormous progress during the last few years and it is anticipated that during the 2020s tremendous growth in the required connectivity, traffic volume, and scope of application scenarios will occur [1]. 5G will support a new radio access technology called 5G-NR (new radio) and an enhanced core network called NGC (Next Generation Core) [2]. It is expected that 5G will encompass a lot more than previous generations of mobile communications, becoming a user-centric concept instead of being an operator-centric (3G) or the service-centric concept (4G). 5G, compared with 3G and 4G systems, can support applications characterized by large connection density, very high traffic volume, and very high mobility.

The focus of 5G-NR can be split into three categories based on different user requirements [1-3]:

- Enhanced mobile broadband (eMBB) – in this scenario, providing higher data rates and enhancing the user experience are required (for high-capacity and ultra-fast mobile communication),
- Massive Machine-Type Communication (mMTC) – the focus is on supporting a huge number of devices with low costs, enhanced coverage and low energy use (for industrial and IoT applications),
- Ultra-Reliable and Low-Latency Communication (URLLC) – communication in this scenario needs to be extremely reliable with very low latencies (mission-critical applications). These applications require sub-millisecond over-the-air latency with packet rate failure of 10^{-5} (error rates lower than 1 packet loss in 10^5 packets) [4, 5].

In order to satisfy the requirements of these application scenarios, 5G wireless networks demand networks' structural improvements in terms of transmission's reliability, system's security, and the quality of offered services. Networks' densification, larger bandwidth, increased spectral efficiency, and new air interface are subjects of intensive research in order to achieve the data rate and capacity for 5G [6]. To achieve high capacity performance in a km^2, three important parameters should be improved [7]:

- A hundred times better data rates than previous generations,
- Less amount of latency (it should be 0.5 ms compared to 10 ms performed by 4G communication systems),
- A hundred times more connections (links).

The energy sector is one of the leading use cases where 5G technologies promise to bring numerous benefits. Increasing energy efficiency overall and accelerating the development of the Smart Grid (SG) are the predictions of how 5G will influence the energy sector [8]. However, the energy sector is quite challenging as it requires novel technologies to address a wide range of diverse requirements associated with different application scenarios. The expectations from 5G

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communication network utilization in SG are to support an increasingly diverse set of novel and emerging services as well as their faster development. Massive connectivity from production to consumption, high data rate, low latency, flexible and optimal deployment, increased scalability, reduced power consumption, improved security and privacy, and cost-effectiveness are additional benefits that 5G promises [9].

Command messaging plays an important role in the case of a new SG. There are many examples of typical applications such as remote control of Smart Meters (SM), Circuit Breakers (CB), PMUs, and other application-specific Intelligent Electronic Devices (IED). The benefits of the CB remote control system in SG, among many other systems, are illustrated in [10-12]. The focus of this paper is to determine the transmission reliability of standard-defined command messages [13] to PMUs in the case of standard-adopted 5G channel coding techniques [14].

Since data transmission occurs in an imperfect channel environment where noise, fading, and interference are present, channel coding plays a substantial role in achieving a higher data rate to realize a fast communication with minimum errors during data transfer. In order to be used efficiently in the communication systems, the selected 5G channel code should have the ability to support a wide range of block lengths and code rates and have an excellent Block Error Rate (BLER) and Bit Error Rate (BER) performances.

This paper represents an attempt to select and implement appropriate channel coding techniques for command messaging in SG. Hence, the paper is organized as follows. An overview of 5G channel coding techniques is presented in Section 2. Section 3 shows the implementation of channel coding techniques in a specific Smart Grid use case – for sending command messages, as well as simulation results. Summary of the performed research and directions for future research are given in the Conclusion.

II. Channel coding

Regarding block lengths, short data bits are typically used in IoT applications while broadband data applications use long data bits. When it comes to different code rates, low coding rates have been practiced in rural areas due to the sparse distribution of base stations, while in urban regions, due to the ultra-dense population, high coding rates have been used. When channel code supports a wide range of data block lengths and data code rates, it is possible to avoid using wasteful data bits and utilization of code rate that causes signal imperfections. Usage of wasteful data and the undesirable code rate will result in unwanted data bits transfer, hence wasting more spectrum, which has a bandwidth, time duration, and energy. This will badly influence on throughput, latency, and capability of error correction. Therefore, the flexibility of the chosen code scheme is an important factor [7]. In addition to better flexibility, low computation complexity, low latency, low cost, and high reliability are also desired for the coding scheme. Codes that show promising BER and BLER performances in a wide range of block lengths and coding rates are Low-Density Parity-Check (LDPC) codes and polar codes. Therefore, these codes are being considered for the 5G-NR physical layer. The channel coding, in this case, has been separated into channel coding of user information and channel coding of control information [1].

LDPC codes achieve a better result in case of data channels because they can efficiently support variable code rates, block lengths, and Hybrid Automatic Repeat Request (HARQ), with better decoding latency, throughput, and implementation complexity compared to other codes. On the other side, Polar codes are the most suitable for control channels because they offer the best error correction capability at the short messages used as control information while addressing a latency issue of successive cancellation decoding. Therefore, the 3rd Generation Partnership Project (3GPP) standardization has selected LDPC for uplink and downlink data channels and Polar codes for the uplink and downlink control channels, replacing the Turbo and Tail-Biting Convolutional Codes (TBCC) of LTE (Long Term Evolution), respectively [1, 15, 16]. More precisely, NR uses LDPC codes for user data which is transmitted on the Physical Downlink Shared Channel (PDSCH) and Physical Uplink Shared Channel (PUSCH). Polar codes are used for Uplink Control Information (UCI) transfer over the Physical Uplink Control Channel (PUCCH) or the PUSCH. In the downlink, Polar codes are used for encoding the Downlink Control Information (DCI) transmitted over the Physical Downlink Control Channel (PDCCH) and the payload in the Physical Broadcast Channel (PBCH) [1, 14, 17].

A. *LDPC Codes*

LDPC codes are efficient channel coding techniques that allow the correction of transmission errors. They were originally invented in presented by Gallager in 1963 [18] but have not been in use before 1996 when they were rediscovered by Mackay [19]. Mackay showed that the LDPC codes are linear block codes that can achieve empirical performance close to the Shannon limit for long words.

In general, LDPC codes are defined by a sparse parity check matrix that determines the transmission's performance. The theory of LDPC codes is related to graph theory and a parity check matrix is defined by a base graph along with a lifting size and cyclic shifts for the graph' edges. For NR, two base graphs are defined, along with eight sets of lifting sizes. In this way, a wide range of block lengths and code rates have been supported. The choice of the base graph depends on the size and code rate of the initial transmission [17, 20, 21].

Due to the sparsity of the parity check matrix, LDPC codes have relatively simple and practical decoding algorithms. Decoding is done by iteratively using the sum-product or using the belief propagation soft-decision decoder. Due to their excellent ability to attain performance near the Shannon limit, LDPC codes are currently being used in many communication systems such as DVB-S2 (satellite transmission, 10GBase-T Ethernet, 802.11n (Wi-Fi allowing

MIMO), 802.16e (Mobile WiMAX), etc. [6].

LDPC coding chain at the transmitter side for the 5G NR downlink shared transport channels contains following parts [17, 22-24]:

- CRC (Cyclic Redundancy Check) attachment to the transport block (payload for the physical layer) in order to provide error detection.
- Code block segmentation and code block CRC attachment - the transport block (including the 24 or 16 bit CRC) is segmented into multiple code blocks to reduce the complexity. Two LDPC base graphs are supported in the case of NR, one optimized for small transport blocks and one for larger transport blocks. After code block segmentation, each code block is appended with its own CRC and each code block is individually LDPC encoded.
- Channel coding using LDPC - In NR, new channel coding mechanisms are applied to enable error correction and error detection in the presence of noise, fading, and interference. Data channels use LDPC codes.
- Rate matching and code block concatenation – includes the stages of bit selection and interleaving defined for LDPC-encoded data and code block concatenation.

After these processing stages, the transport block is passed on the PDSCH for scrambling, modulation, layer mapping, and resource/antenna mapping. PDSCH supports QPSK, 16QAM, 64QAM, and 256QAM modulation schemes while PUSCH also supports pi/2-BPSK modulation in addition to those listed above for PDSCH.

At the receiver side, processing stages of the downlink shared channel correspond to those at the transmitter side.

- Rate recovery (rate dematching) - is performed on the receiver side to restore the encoded bit structure so that the LDPC decoding algorithm can decode the message. In this step, the inverse of the code block concatenation, bit interleaving, and bit selection stages have been performed.
- LDPC decoding – the LDPC decoder receives an LDPC encoded message that was transmitted over a channel. Due to noise presence, the received message values may differ from the message that was sent. LDPC code is designed to be able to correct errors and reconstruct the correct message from the received data. There are a variety of algorithms for decoding LDPC encoded messages: Belief propagation, Layered belief, Normalized min-sum, and Offset min-sum.
- Code block desegmentation and CRC decoding – in this step the input code block segments have been concatenated into one output data block. Any filler bits and type-24 bit CRC present in the input code block segments are removed. In other words, this process is the inverse of the LDPC code block segmentation and code block CRC attachment performed at the transmitter side.
- Transport block CRC decoding – as the final step, decoding, and removing CRC serves for message verification at both the code-block and transport block levels. If there are no CRC errors, the transport block is being recovered and decoded with no errors.

As PUSCH is used for the transmission of uplink shared channel and layer 1/2 control information, each transport block in the uplink goes through the following processing stages [18, 19]:

- CRC attachment to the transport block
- Code block segmentation
- Channel coding of data and control information
- Rate matching and code block concatenation
- Multiplexing of data and control information - ensures that control and data information are mapped to different modulation symbols,
- Channel interleaver - implements a time-first mapping of control modulation symbols and frequency-first mapping of data modulation symbols onto the transmit waveform.

B. Polar Codes

Polar codes are one of the newest channel coding schemes introduced by Arikan [25] in 2009 and they are the first provably codes that arrive near Shannon's limits of capacity with low encoding and decoding complexities. As 5G systems require significant improvements in channel capacity, Polar codes are the promising technique because of the advantages they provide, the capacity they produce, and the absence of error floor. Polar codes are approved as the channel coding algorithm for 5G control channels where the information blocks are small compared to data transmission and HARQ is not used. Polar codes replace the used Convolutional Codes in LTE. The idea of Polar codes is to transform several instances of the original radio channel into a set of virtual channels that tend to have either high reliability or low reliability. This means that the channels are either noiseless or completely noisy and the transmission of the data (information bits) is performed via noiseless channel while the pure-noise channel transmits fixed (known by the transmitter and the receiver) symbols. Decoding can be performed in several ways, typically using successive cancellation and list decoding. Successive cancellation decoding is not well suitable for short-to-medium block lengths, but list decoding substantially improves the performance of polar codes at those block lengths [3, 20, 26]. Polar codes have a recursive structure with low complexity and were proven to achieve the channel capacity for long block lengths. Therefore, during the Polar code's design, a maximum number of bits should be considered. In NR, the Polar code has been designed to support 512 coded bits (prior to rate matching) in the downlink. It can be handled with up to 140 information bits, which provides a sufficient margin for future extensions [6, 7, 27].

In the case of polar coding in the downlink, a transport block goes through the processing stages of [1, 17, 23, 24, 26]:

- CRC attachment,
- CRC interleaver - with the help of interleaver, a CRC

block is not attached at the end of DCI block but distributed among the DCI bits. This assists potentially early termination in the PDCCH decoding process.

- Radio Network Temporary Identifier (RNTI) coding - a 16-bit identifier that enables the user equipment to determine if the message is intended for it.
- Channel coding – coding channel data with Polar codes.
- Rate matching - for Polar code rate matching is defined per coded block and consists of sub-block interleaving, bit selection, and bit interleaving. Channel interleaving is only used for PUCCH where it improves the polar code's error correction capability when employing higher-order modulation schemes for transmission over fading channels, while it is not employed for PDCCH and PBCH. Rate matching is performed after polar coding and before the bits are transmitted over the channel. Rate matching is about matching the length of information blocks (the number of bits) to the desired transmission rate. How these information blocks can be manipulated depends on their size after encoding, which can either lead to shortening, puncturing, or repetition of the coded bits to achieve the desired number of transmission bits.
- After polar coding and rate-matching, bits are scrambled, modulated using QPSK, and mapped to the resource elements used for the PDCCH

At the receiver side, processing stages of PDCCH corresponds to those at the transmitter side.

In the uplink, UCI has been coded using different coding techniques depending on UCI length. After adequate coding, rate-matching and bit-level interleaving are performed. The PUCCH uses sequence selection, BPSK, or QPSK depending on the PUCCH format and the number of bits with $\pi/2$-BPSK available as a configurable option [17].

III. Channel Coding of command message in Smart Grid use case

Standard PMU device consists of three main parts: measurement, phasor computation (assisted by Global Positioning System or GPS receiver), and communication. The communication part is used for streaming data in the standard-defined format. The main information to be transmitted is in the form of data messages consisting of calculated and GPS-based synchronized phasors. Data messages are transmitted uplink to Phasor Data Concentrators (PDC). However, downlink transmission of command messages from PDC is required to remotely control PMU data streaming. The typical PMU-based data collection network is shown in Fig.1.

For simulation purposes, an example of a command message that affects the behavior of the Phasor Measurement Unit (PMU) has been considered (Table I) [13]. The example presented in Table I causes a PMU to begin transmission of data messages. The information (payload) message length is 16 bytes or 128 bits. This or very similar type of command message can be used for remote control of other IED in SG.

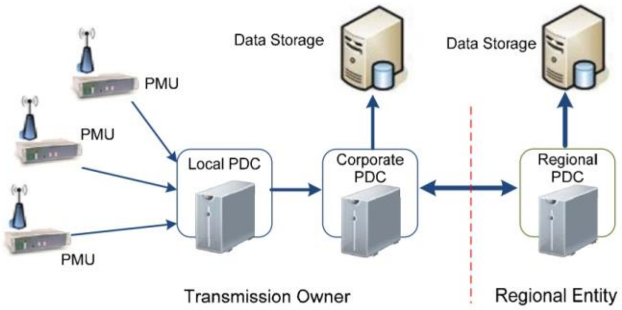

Fig. 1. Synchrophasor data collection network [13]

TABLE I
COMMAND MESSAGE EXAMPLE [13]

Field	Description	Example	Size (bytes)	Hexadecimal value
SYNC	Synchronization byte and version number	Command Message, version 1	2	AA 41
FRAMESIZE	Number of bytes in a frame	18	2	00 12
IDCODE	Data stream ID number, 16-bit integer, 1-65534	7734	2	1E 36
SOC	SOC time stamp	12:00 AM, 6.6.2006 = 1 149 591 600	4	44 85 60 30
FRACSEC	A fraction of second with Time quality.	No leap second pending or past, clock never locked, fractional time 0.77s	4	0F 0B BF D0
CMD	Defined commands are data on, data off, send header, send configuration, extended frame.	Turn on the data stream.	2	00 02
CHK	CRC-CCITT - 16-bit CRC calculated using the generating polynomial $X^{16} + X^{12} + X^5 + 1$.	-	2	CE 00

The command message under consideration is coded using LDPC and Polar coding techniques according to procedures recommended in the standard [14] and explained earlier in Section II. CRC part of the message is generated according to the same standard procedures. Code block segmentation is not required for short messages and thus it is not performed in simulations due to considered command message length. As the quality criterion of a channel code, BER of the coding schemes is plotted against the energy per bit to noise power spectral density ratio (E_b/N_0) for different code rates. MATLAB [23] is used to simulate the physical communication layer with LDPC and Polar codes and calculates the simulation results to BER vs. E_b/N_0 graphs (Fig. 2-4.). Command message transmission has been performed using QPSK over the Additive White Gaussian Noise (AWGN) channel. AWGN channel model variances are estimated from signal-to-noise ratio (SNR) values. The transmission parameters were set according to the 5G numerology. Each simulation was performed for 500 frames and continued until the BER of 10^{-4} is achieved.

Figure 2. shows the LDPC coding of command message and obtained BER performances for variable code rates.

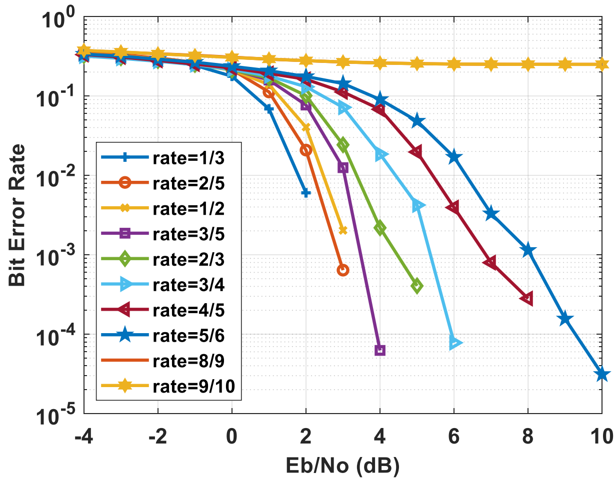

Fig. 2. LDPC - BER performance for variable code rates

BER performances for different code rates in case of Polar coding are given in Figure 3.

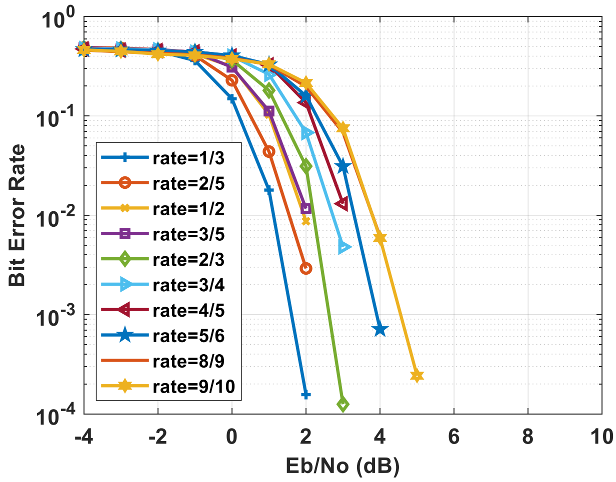

Fig. 3. Polar codes - BER performance for variable code rates

Figure 4 presents the comparative analysis of LDPC and Polar coding techniques in terms of the BER for several code rates. Rate matching function is set to enable BER analysis in the case of equal code rates for LDPC and Polar coded message.

All three figures (Fig. 2-4.) illustrate a typical curve - BER decreases with the increase of E_b/N_0. The waterfall regions (regions where BER falls clearly after a certain E_b/N_0) for both coding schemes indicate that in the case of a Polar code application, the error correction requirements of BER of 10^{-4} can be achieved at lower E_b/N_0, compared with LDPC code use. Another advantage of Polar codes over LDPC codes is a good error floor performance. The error floor region covers the region where performance flattens - BER does not fall as quickly as before. LDPC codes with good waterfall characteristics suffer most from the error floor problem. Fig. 4. clearly presents that Polar code outperforms the LDPC code for all investigated code rates of {1⁄3, 1⁄2, 2⁄3, 5⁄6}, in case of transmitting the typical command message from base station to PMU device.

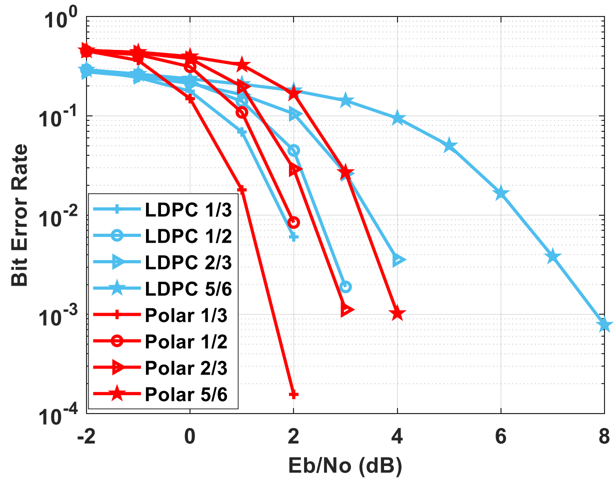

Fig. 4. BER performance for variable code rates: LDPC vs Polar coding

Simulation results confirm that Polar codes offer better error correction capabilities in comparison with LDPC codes in low BER area which is significant for transmitting short control messages. The use of Polar coding for command messaging in 5G-empowered SG applications has been justified according to error correction capability criterion.

IV. Conclusion

There is no doubt that 5G will completely revolutionize the entire energy sector. An immense number of smart devices deployed in the SG vision will require fast and secure data exchange. 5G channel coding techniques will play a major role in achieving fast communication with minimum errors in a variety of 5G-empowered applications. In order to demonstrate the selection of the appropriate channel coding scheme, the transfer of a typical command message from the base station to the PMU device has been investigated. The BER results obtained using QPSK for communication over AWGN, as a function of the E_b/N_0 shows that the Polar code achieves superior BER performance compared to the LDPC code in case of the command message transmission. Based on the results, Polar codes have been recommended for the use in control channels in 5G-based SG applications, over which short messages, such as command message, could be transmitted with lower BER.

Future work will be focused on selecting an appropriate channel coding scheme for the other types of messages used for the exchange of measurement data between power system equipment. The evaluation of the performances and complexity of the LDPC and Polar codes is a quite challenging task due to their dependency on the underlying hardware and the decoding algorithm. These issues will be also the focus of further research.

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