ON PIEZOMAGNETISM AT VISCOPLASTICITY OF FERROMAGNETICS

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Abstract - The paper deals with viscoelasticity of ferromagnetic materials. Tensor representation is applied to a set of evolution equations comprising the plastic stretching and residual magnetization tensors. Small magnetoelastic strains of isotropic translators are considered in detail in two special cases of finite as well as small plastic strain. A special emphasis is given to piezomagnetism effects in the case of uniaxial cycling strain.

INTRODUCTION

The principal objective of this work is to towards a simplified approach of inelasticity of ferromagnetics serving primarily to subsequent nondestructive electromagnetic examination of inelastic behavior of nuclear reactor steels (cf. [1, 2]).

In this paper like in [3, 4, 5] associativity of flow rules (the normality of the plastic strain rate tensor onto a yield surface has not been taken as granted even if such an approach is accepted in the majority of the papers dealing with the subject. Evolution equations (exposed in the second section of this paper) are based on the appropriate geometry of deformation and the extended irreversible thermodynamics. This geometry is founded on the continuum theory of dislocations (compare with [6, 7]) and is shortly reviewed in the sequel.

As a prerequisite, a correct geometric description of an inelastic deformation process analyzed is necessary. Consider a crystalline body in a real configuration \((k)\) with dislocations and an inhomogeneous temperature field \(T(X, t)\) (where \(t\) stands for time and \(X\) for the considered particle of the body) subject to surface tractions. Corresponding to \((k)\) there exists, usually, an initial reference configuration \((K)\) with (differently distributed) dislocations at a homogeneous temperature \(T\) without surface tractions. Due to these effects such a configuration is not stressfree but contains an equilibrium residual stress (often named as "back-stress").

It is generally accepted that linear mapping function \(F(\cdot, t) : (K) \rightarrow (k)\) is compatible second rank total deformation gradient tensor. Herein time \(t\) as scalar parameter allows for family of deformed configurations \((k)\). In the papers dealing with continuum representations of dislocation distributions configuration \((k)\) is imagined to be cut into small elements denoted by \((n)\), these being subsequently brought to the temperature of \((K)\) free of neighbors. The deformation tensor \(F_e(\cdot, t) : (n) \rightarrow (k)\) obtained in such a way is incompatible and should be called the thermoplastic distortion tensor whereas \((n)\)-elements are commonly named as natural state last reference configurations (cf. for instance [8]). Of course, the corresponding plastic distortion tensor

\[
F_p(\cdot, t) := F_e(\cdot, t)^{-1} \cdot F(\cdot, t),
\]

is also incompatible. Herein \(F\) is found by comparison of material fibres in \((K)\) and \((k)\) while \(F_e\) is determined by crystallographic vectors in \((n)\) and \((k)\). Multiplying above formula from the left hand side by \(F(\cdot, t)\) we reach at Kröner's decomposition rule [8] which is often wrongly named as Lee's decomposition formula. It is worthy of note that \(\text{curl} F_p(\cdot, t)^{-1} \neq 0\) and this incompatibility is commonly connected to an asymmetric second order tensor of dislocation density.

In the paper [7] the authors connected to the natural state elements magnetization vectors in such a way that they are incoincid in \((n)\) and inhomogeneous in \((k)\) the inhomogeneity being responsible for magnetostrictive strains. Such an assumption is very much in accord with the above geometrical argument and is accepted in the sequel.

ISOTROPIC VISCOPLASTICALLY DEFORMED INSULATORS

We consider an isotropic body under the following very simplifying assumptions:

(A1) elastic strain, reversible and irreversible magnetization are small of the same order but plastic strain itself is finite (cf. also [3]);

(A2) thermal and electric effects are neglected.

Such assumptions correspond to the so called piezomagnetism processes when magnetization is generated by straining processes (cf. e.g. [10] and [11]).

Let us take into account that by its very nature the mechanical stress disappears when pure elastic strain vanishes and, similarly, the local magnetic field equals to zero if the reversible magnetization vanishes. Then, according to [7] it is reasonable to introduce magnetostrictive strain by means of

\[
E_0 := E - E_\Phi = F_p^t \cdot (F_p F_e - I) \cdot F_p,\]

(2)
\[ E_0 = E_0 + \mathbf{L} : (\mathbf{M}_0 \circ \mathbf{M}_0) = E_0 + \mathbf{E}_{\text{mag}}. \]  

Here \( \mathbf{L} \) is the fourth rank tensor of magnetostriiction constants symmetric only in indices of the first as well as the second pair whereas the notation \( \mathbf{M}_0 \) stands for the unit vector of the magnetization vector \( \mathbf{M} \). The components of the Lagrangian elastic strain tensor \( E_0 \), namely, \( E_0 \) as well as \( E_{\text{mag}} \) are both incompatible and are referred to as elastic strain and magnetostriiction strain.

With these facts taken into account and the above assumptions (A1-A2) the constitutive equations for mechanical part of the stress tensor and the local magnetic field specialize into [4]:

\[
T = (c_{11} + c_2 E_0 + c_3 E_0^2) \text{tr} E_0 + 2c_6 E_0 + \ldots \tag{4}
\]

\[
H = c_6 M_0 + c_6 (E_0 \cdot M_0 + M_0 \cdot E_0) + \ldots \tag{5}
\]

where instead of magnetic induction field the internal magnetic field vector \( H = 0.5 \mathbf{L} : \mathbf{H} \) (opposing the local magnetic field vector under assumption (A1)) has been introduced by making use of tensorial representations for the proper orthogonal group [12]. In the above "magnetic" constitutive equation

\[
\mathbf{M}_0 = \mathbf{M} - \mathbf{M}_0,
\]

is the reversible magnetization vector. The antisymmetric second rank tensors \( \mathbf{H}, \mathbf{M}_0, \mathbf{M}_0 \) are made from the corresponding vectors \( \mathbf{H}, \mathbf{M}_0, \mathbf{M}_0 \) by means of the Ricci third rank permutation tensor \( \varepsilon \) defined in (X)-configuration in the following way \( (A \in \{r, H\}) \):

\[
\mathbf{H} = \varepsilon \cdot \mathbf{H} = -\mathbf{H}^T, \quad \mathbf{M}_0 = \varepsilon \cdot \mathbf{M}_0 = -\mathbf{M}_0^t. \tag{7}
\]

They are favored instead of the mentioned vectors for convenience and more compact representation. Of course, instead of (5) an equivalent formulation using cross products of vectors \( \mathbf{M}_0, \mathbf{M}_0 \) with symmetric second rank tensor \( E_0 \) is also possible [12]. Equation (4) is the generalized Hooke's law accounting for plastic strain induced mechanical anisotropy while the constitutive equation for internal magnetic field predicts magnetic anisotropy induced by the same cause.

The free energy function generating linear forms of (4) and (5) reads:

\[
F = \frac{1}{2} c_{11} \varphi^2 + \frac{1}{2} c_{12} \varphi^2 + \frac{1}{2} c_3 \varphi^2 + \alpha_0 \psi^2 + c_4 \psi^2 + c_5 \psi^2 + \ldots \tag{8}
\]

with the following proper and mixed invariants of its tensorial arguments (cf. [12]):

\[
i_1 = \text{tr} E_0, \quad i_2 = \text{tr} (E_0 \cdot E_0), \quad i_3 = \text{tr} (E_0^2), \quad i_4 = \text{tr} (E_0^2 - E_0^2), \quad i_5 = \text{tr} (E_0^2 - E_0^2)^2 \tag{9}
\]

In the sequel inverse forms of (4) and (5) will be useful. They can be written as follows:

\[
E_0 = \left( \gamma_1 + \gamma_2 E_0 + \gamma_3 E_0^2 \right) \text{tr} T + \ldots \tag{10}
\]

\[
M_0 = \gamma_2 H + \gamma_0 (E_0 \cdot H + H \cdot E_0) + \ldots \tag{11}
\]

The relationships between sets \( \{c_1, \ldots, c_9\} \) and \( \{\gamma_1, \ldots, \gamma_9\} \) can be found as follows. Let us multiply (4) as well as (10) by the tensors \( I, E_0 \) and \( E_0 \) finding traces of both sides. If we introduce notations:

\[
s_1 = \text{tr} T, \quad s_2 = \text{tr} (E_0 \cdot \mathbf{T}), \quad s_3 = \text{tr} (E_0^2), \quad s_4 = \text{tr} (E_0^2 - E_0^2) \tag{12}
\]

\[

\]

\[
\gamma_2 = \text{tr} (E_0^2 - E_0^2) \]

\[
\gamma_0 = \text{tr} (E_0^2 - E_0^2 \cdot H)
\]

then such a procedure will provide required relationships between \( \{c_1, \ldots, c_9\} \) and \( \{\gamma_1, \ldots, \gamma_9\} \). Of course, the same procedure applied to (5) as well as (11) would connect sets \( \{c_7, \ldots, c_9\} \) and \( \{\gamma_7, \ldots, \gamma_9\} \).

Similarly, the evolution equations for plastic strain rate and residual magnetization rate are explicitly stated by the following formulators:

\[
D E_0 = \sum_{k=1}^{12} \delta_k G_{E_0}^k, \tag{13}
\]

\[
D M_0 = \sum_{k=1}^{8} \delta_k G_{M_0}^k, \tag{14}
\]

where tensor generators \( G_{E_0}^k \) and \( G_{M_0}^k \) are similar to those in (4 - 5). The scalar coefficients \( \{\delta_1, \ldots, \delta_{12}\} \) as well as \( \{\delta_1, \ldots, \delta_8\} \) depend on proper and mixed invariants of the tensors \( E_0, T \) and \( E_0, T \) according to assumptions (A1) and (A2). They are not written here for the sake of brevity and are listed explicitly in [4] on the basis of representation theory for tensor functions (cf. [12]).
It should be noted here that all the scalar coefficients in above constitutive relations (4)-(5) are functions of the principal invariants of the plastic strain tensor \( E_p \). For advanced magnetizations, a nonlinearity of the magnetic relationship (5) must be taken into account while Hooke's law (4) is always linear in the elastic strain for steels. Of course, if plastic strain itself is small, then the corresponding complete linearization of constitutive and evolution equations is straightforward which might be of interest especially if dynamic effects are considered i.e. wave equations of the linearized problem formalism (cf. [5]). Evolution equations then would reduce to Onsager-Caumass reciprocity relations.

### SMALL MAGNETO-VISCOPLASTIC STRAINS

Let us see what consequences could have an introduction of a generalized loading function \( \Omega \) with the following orthogonality properties (cf. [5]):

\[
D E_p = D A \frac{\partial \Omega}{\partial H} \quad \text{and} \quad D M_k = D A \frac{\partial \Omega}{\partial H},
\]

(15)

where the material time rate of a scalar function \( \Lambda \) vanishes if the yield function \( f \) (encompassing the corresponding elastic range) is either negative or zero (cf. eg. (3)). Suppose, for simplicity that the assumption \( (A2) \) still holds whereas the assumption \( (A1) \) is replaced by means of the following:

\( (A3) \) elastic and plastic strain, reversible and irreversible magnetization, as well as plastic strain rate and irreversible magnetization rate are all small of the same order.

Then we may assume the loading function in the following polynomial form

\[
\Omega = \frac{1}{2} \omega_1 \varepsilon^2 + \frac{1}{2} \omega_2 \sigma^2 + \frac{1}{2} \omega_3 \gamma^2
\]

leading by means of (15) into the following two evolution equations

\[
D E_p = D A \left[ \omega_1 \varepsilon + \omega_2 \sigma \right],
\]

(17)

\[
D M_k = D A \omega_3 H,
\]

(18)

whose simplicity follows from the above very special loading scalar function \( \Omega \). In solution, the free energy function \( F \) becomes very simplified as follows:

\[
F = \frac{1}{2} c_1 \varepsilon^2 + c_2 \sigma^2 + \frac{1}{2} \gamma^2 + F^*(E_p, M_k)
\]

(19)

where \( F^* \) would depend on proper and mixed invariants of \( E_p \) and \( M_k \). Such a function allows the following very special constitutive equations:

\[
T = c_{11} \varepsilon + 2 c_{12} \gamma^2, \quad \text{with} \quad c_1 \equiv \lambda, \quad c_2 \equiv \mu,
\]

(20)

\[
\mathbf{H} = c_3 \mathbf{M}_k, \quad \text{with} \quad c_3 \equiv \frac{1}{\sigma}
\]

(21)

Obviously, the inherent material constants are easily recognized to be Lamé constants as well as the constant of magnetic susceptibility (cf. [9]). It should be noted that if the tensor of magnetostriction constants \( \lambda \) is introduced into (20) then magnetostrictive process can be shown explicitly.

The situation described in this section could correspond to ferromagnetic induced by low-cycle fatigue of ferromagnetics. Such a process was investigated experimentally in the paper [10]. A cylindrical specimen of AISI 1018 was biaxially treated by push-pull test on MTS-810 servo-hydraulic testing machine such that total strain was periodic and triangularly shaped \( |\varepsilon| \leq 0.030 \) with cycle duration of 2 s. Magnetic induction due to piezomagnetic effect was also almost periodic with very slight changes with increase of relative number of cycles \( N_k/N \) (where \( N_k \) is number of cycles at failure) and calculation of phase delay with respect to strain with growth of accumulated plastic strain. Maxima and minima of \( \mathbf{B} \) are almost coincident with minima and maxima of the magnetic induction \( \mathbf{B} \). Thus, if plastic strain accumulation is calculated by means of

\[
\pi(t) := \int_0^t ||D E_p(t)|| dt
\]

(22)

then if uniaxial components of \( \mathbf{B} \) as well as \( \mathbf{H}, \mathbf{M}_k, \mathbf{M}_k \) are denoted by means of \( B_{13} \) as well as \( B_{11}, M_{11}, M_{11} \) the following memory-type equation

\[
B_{14}(t) := \int_0^t J(\sigma, t - \tau) D E_{14}(\tau) d\tau
\]

(23)

would describe fairly well the above explained experimental situation. Time differentiation of the above relationship gives rise to the expression:

\[
DB_{14}(t) := J(\sigma, 0) D E_{14}(t) + \int_0^t \frac{\partial}{\partial \tau} J(\sigma, t - \tau) D E_{14}(\tau) d\tau.
\]

(24)

In the above integro-differential equation the second term on the right hand side is responsible for the above mentioned change of time delay and the deflection of pure periodicity of \( B_{14}(t) \). Therefore, it is much smaller than the first part. On the other hand, if the constitutive equation \( B_{14} = \mu H_{14} \) (where \( \mu \) is magnetic permeability) is used, then we have

\[
DB_{14} = \frac{\mu}{\chi} (D M_{14} - D M_{14})
\]

(25)
if \( u/\chi = \text{const} \) which holds approximately only for magnitude of magnetic field much smaller than its saturation value. Since in the paper [10] the above splitting has not been made, a more specific comment on simultaneous zeros of \( D_{E} \)s and \( D_{M} \)s (following from (17) and (18) ) is not possible.

CONCLUDING REMARKS

Concluding this paper it is inevitable to compare the foregoing results with existing achievements in the field. The main contributions to viscoplasticity of ferromagnetic materials have been given by Maugin and his collaborators in [1, 13]. The principal assumptions accepted in our work are closer to the scope of the first of these two papers where

1. small strain case together with absence of exchange forces and gyromagnetic effects has been assumed,

2. the account on hysteresis effects has been given and

3. evolution equations derived by normality of plastic strain rate and residual magnetization rate onto a loading surface.

The main results of this paper might be summarized as follows:

1. in the case of finite plastic strains magnetic anisotropy induced by plastic strain is predicted by (14) where development of residual magnetization by mechanical terms is also evident;

2. the influence of magnetization on plastic strain rate is obtained even in the case of isotropic ferromagnetic materials;

3. the obtained relationships with couplings allow for magnetic measurements of inelastic phenomena but the measurements will show their order of magnitude and practical measurable of these phenomena;

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REFERENCES


