EIGEN APPROACH FOR THE DIGITAL OPTICAL COMMUNICATION SYSTEM MODELLING AND SIMULATION

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Abstract - The procedure for the digital optical communication system modelling and simulation is presented. Each part of the optical system is modelled independently - optical transmitter, optical fiber and optical receiver. The program package MATLAB is used for the computer simulation. The simulation result is the eye pattern as well as the bit error rate.

1. INTRODUCTION

Optical transmitter, optical fiber and optical receiver are the basic elements of the optical communication system. In practice, it is appropriate to have a mathematical model of the optical communication system that could provide fast and exact estimation of the system parameters. The system parameters which have to be determined are: maximal length of the transmission line which can be achieved for the required transfer bit rate and the bit error rate, under what conditions PIN photodiode satisfies the requirements, the optimal gain of the avalanche photodiode etc. [1-2].

In the mathematical modelling procedure it is necessary to analyse each part of the optical communication system. The simulation program is accessible for the wide class of computers and users.

2. SYSTEM MODEL

In Fig. 1 the block diagram of the optical communication system is shown.

\[ H_0(f) = \frac{\sin(c\tau_0 f)}{\pi \tau_0} \]  (1)

where \( \tau_0 \) denotes the pulse width.

For the Gaussian pulse shape, the Fourier transformation is:

\[ H_0(f) = \exp(-\pi^2 f^2 \tau_0^2) \]  (2)

If the optical pulse were transmitted through the system shown in Fig.1, the pulse with the Fourier transformation \( H_0(f) \) would be obtained at the output. \( H_0(f) \) can be obtained as the product of the functions \( H_0(f) \) and \( H(f) \). In order to have a pulse shape in time domain, the inverse Fourier transformation is applied. For the input pulse, the inverse Fourier transformation is:

\[ h(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} H_0(f) \exp(j2\pi ft) df \]  (3)

The characteristics of the optical fiber are the fiber loss and the fiber bandwidth. The typical transfer function of the optical fibers is a Gaussian low-pass fiber shape, described as follows:

\[ H_0(f) = \exp(-\pi^2 f^2 \tau_f^2) \]  (4)

where \( \tau_f = \frac{1}{2B_f} \), \( B_f \) is a bandwidth of the optical fiber of length \( L \), which can be determined using the expression:

\[ B_f = \frac{B_e}{L}, \quad B_e \quad [\text{MHz}] \] is a bandwidth of a short fiber and could be obtained experimentally. The former expression can be used for \( L \leq L_e \), where \( L \) represents the length of the stationary state establishing.

The total transfer function at the end of the optical fiber is:

\[ S_2(f) = 10 \log_{10} \frac{1}{L} \exp(-\pi^2 f^2 \tau_f^2) \]  (5)

\[ \frac{1}{L} \exp(-\pi^2 f^2 \tau_f^2) \quad \frac{1}{L} \exp(-\pi^2 f^2 \tau_f^2) \]

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The part 10 \( -10 \) dB corresponds to the attenuation of the uniform optical power at the length \( L \) of the optical fiber, which has the total loss coefficient \( \gamma \). This modelling procedure neglects the dispersion in the optical fiber under the assumption that it hasn’t the greatest impact on the system performance.

The most important parameters of the optical receiver are bandwidth and noise. The crucial noise influence in the whole optical communication system comes down from the optical receiver consisting of photodiode quantum noise, photodiode thermal noise and the amplifier thermal noise.
Optical receiver modelling is made in the frequency domain, too. The Fourier transformation of the signal at the input of the optical receiver is determined using the expression:

$$S_2(f) = R_L \cdot P_0 \cdot D_1 \cdot H_0(f) \cdot H_1(f)$$  \hspace{1cm} (6)

$P_0$ is an input signal peak power. The coefficient $D_1$ is:

$$D_1 = \frac{y L}{10 \text{dBm}}$$

The incoming optical pulse provides generating of the photocurrent, which Fourier transformation is:

$$I(f) = R_2 \cdot \eta \cdot P_0 \cdot D_1 \cdot R_k \cdot M \cdot H_0(f) \cdot H_1(f)$$  \hspace{1cm} (7)

$R_k = \frac{m f}{h}$ is the photodiode conversion coefficient ($\eta$ is a photodiode quantum efficiency, $q$ is an electron charge, $h$ - Planck's constant, $f$ denotes a frequency of the optical carrier).

Mean value of the photocurrent generated by a pulse in one bit interval $T_{bit}$ is as follows:

$$I_f = P_0 \cdot R_2 \cdot \eta \cdot D_1 \cdot R_k \cdot M$$

$H_0(f)$ and $H_1(f)$ for $f=0$ are equal to 1 and $R_0$ is a bit rate.

For the bit error rate determination it is necessary to know the mean power at the input of the receiver, denoted as $P_f$. The averaging of the received optical power must be done in a longer time interval, but since the line coding is applied and the numbers of logical ones and zeros are equal, the mean value at the photodetector input is:

$$P_f = P_0 \cdot \frac{R_2 \cdot \eta}{T_{bit}} \cdot D_1 \cdot 2$$  \hspace{1cm} (9)

The impact of the optical receiver on the signal shape is expressed by the low pass filter transfer function. Some of the most useful real functions are Nyquist function or raised cosine function that is:

$$H_2(f) = \frac{1 + \cos(\pi f T_2)}{2}, \text{ for } 0 \leq f \leq T_2$$  \hspace{1cm} (10)

### 2.1. RECEIVER NOISE MODELLING

Noise generated in the optical receiver is of the crucial influence on the system performance. The noise sources can be independent or in the relation with the signal. Since the noise generated in the optical receiver has the huge impact on the system performance, it will be analysed. The block diagram of the optical receiver is shown in Fig. 2.

![Block diagram of the optical receiver](image)

**Fig. 2.** Block diagram of the optical receiver

Photodiode is represented by the current generator of the photocurrent and noise a shunted capacitor $C_{ph}$ (circuit capacity). $R_L$ is an outer resistance at the input of the amplifier. The current generator of the thermal noise arising in $R_L$ is $I_T$. The receiver amplifier is presented by shunted connection of an input resistance and the capacitor and by an ideal amplifier stage of the gain $A$.

Noise arising in the amplifier is presented by the current thermal noise generator $I_{Th}$ and a voltage thermal noise generator $E_{Th}$ in a serial connection. Voltage at the output of the equaliser can be represented as:

$$v_{as}(t) = v(t) + v_{as}(t)$$  \hspace{1cm} (11)

$v_{as}(t)$ is the useful signal, $v(t)$ is the receiver total noise, which can be represented as:

$$v_{as}(t) = v(t) + v_{th}(t) + v_n(t)$$  \hspace{1cm} (12)

where:

- $v(t)$ - noise generated in the photodiode,
- $v(t)$ - thermal noise generated at the resistor $R_L$,
- $v_n(t)$ - noise generated by the input current generator in the equivalent scheme of the receiver amplifier,
- $v(t)$ - noise generated by the input voltage generator in the equivalent scheme of the receiver amplifier.

The total noise power at the output of the equaliser consists of the sum of the powers of the statistically independent noises:

$$v_{as}^2(t) = v(t)^2 + v_{as}^2(t) + v_n^2(t) + v(t)^2$$  \hspace{1cm} (13)

Photodiode total noise power, arising from quantum noise, is expressed as:

$$v_{as}^2(t) = 2q_i B R_i A^2$$  \hspace{1cm} (14)

where $q$ is an electron charge, $B$ is the bandwidth of the equivalent low pass noise filter, $R$ is the total resistance consisting of $R_L$ and $R_i$.

$$i^2 = R_k P_0 M^2 + x$$  \hspace{1cm} (15)

Thermal noise power at the output of the equaliser is:

$$v_{as}^2(t) = \frac{4 k B T}{R_L}$$  \hspace{1cm} (16)

$k_B$ is Boltzmann's constant, $T$ is a temperature in Kelvin's degrees.

Under the assumption that the sources $i_0$ and $e_0$ in the amplifier stage are independent and each generates the corresponding spectral power densities, the expressions for the noise power of these sources are as follows:

$$v(t) = 2G_i B R_i A^2$$  \hspace{1cm} (17)

$$v(t) = 2G_B A^2$$  \hspace{1cm} (18)

where $G_i$ and $G_B$ are spectral power densities of the current and voltage noise sources, respectively.

Bandwidths of the equivalent noise sources are:
\begin{align}
2B &= \frac{1}{\left| H_f(0)H_e(0) \right|^2} \int_{-\infty}^{\infty} \left| H_f(f)H_e(f) \right|^2 df \\
2B_e &= \frac{1}{\left| H_e(0) \right|^2} \int_{-\infty}^{\infty} \left| H_e(f) \right|^2 df
\end{align}

\( H_f(f) \) is an amplifier transfer function, \( H_e(f) \) is an equaliser transfer function.

In the procedure of the bit error rate determination, the assumption is that the condition probabilities \( P(1|0) \) and \( P(0|1) \) are equal, leading to the expression for the bit error rate:

\[ P_e = \frac{1}{2} \operatorname{erfc}\left( \frac{i_1}{\sqrt{2} (\sigma_0 + \sigma_1)} \right) \tag{21} \]

where \( \sigma_0^2 \) and \( \sigma_1^2 \) the variances of the current of the total noise, when "0" and "1" are received, respectively.

\[ i_1 = R_b P_1 M \tag{22} \]

\[ \sigma_0^2 = 4F_p k_T B_0 / R_L \tag{23} \]

\[ \sigma_1^2 = 2R_b R_i M^2 \sqrt{R_i B + 4F_p k_T B_0 / R_L} \]

3. SIMULATION RESULTS

The result of the computer simulation of the digital optical communication system, the eye diagram and the bit error rate could be obtained at the system output. The eye diagram is very useful and can enable observation and the analysis of the very important features such as intersymbol interference, optimal noise threshold, noise margins, jitter, or the pulse amplitude distortion. For the simulation of the eye diagram, the time discretized equivalent is used, i.e., sampling of the digital signal is done. The example of the digital optical transmission system is simulated, with the following transmitter parameters:

- transmission rate: \( R = 50 \text{ Mbit/s} \);
- transmitter peak power: \( P = 5 \text{ W} \).

The parameters of the optical fiber follows:

- input pulse width: \( T_p = 0.7 \ T_{\text{bit}} \);
- bandwidth of the unit length fiber: \( B_2 = 5 \text{ GHz} \);
- bandwidth of the optical fiber: \( B_1 = \frac{B_2}{L^2} \);
- time constant of the optical fiber transfer function: \( T_f = L^2 B_2 \);
- fiber length: \( L = 15 \text{ km} \);
- fiber loss: \( \rho = 0.3 \text{ dB/km} \).

Optical receiver parameter:

- receiver bandwidth: \( B = R_b \);
- time constant of the receiver transfer function: \( T_r = \frac{R}{B} \);
- photodiode conversion coefficient: \( R_k = 0.5 \text{ A/W} \);
- receiver noise temperature: \( 0K \leq T \leq 300 \text{ K} \).

As the illustration, in the interval \( 2T_{\text{bit}} \) for the input pulse sequence \( \{1,0,1,0,1,0,0\} \) the corresponding eye diagrams are shown in Figs. 3 and 4.

![Figure 3: Eye diagram when \( T_p = 0.7 \ T_{\text{bit}} \) in the interval \( 2T_{\text{bit}} \)](image)

![Figure 4: Eye diagram when \( T_p = 0.7 \ T_{\text{bit}} \) in the interval \( 2T_{\text{bit}} \)](image)

In Figs. 5 and 6 the simulation result representing the signal sequences at the input and at the output of the optical receiver are shown.

![Figure 5: Input and output signal sequences when \( T_p = 0.7 \ T_{\text{bit}} \)](image)
4. CONCLUSION

The procedure for modelling the digital optical communication system is proposed in this paper. The modelling is done in the frequency domain. The procedure is implemented into the computer program that enables observation the performance of the digital transmission with respect on eye diagram and bit error rate. These results enables determination of the maximum transmission length with required bit error rate, PIN photodiode performance, optimum avalanche photodiode gain etc. The results of the digital simulation are shown graphically.

REFERENCES